JAM CM Journal of Applied Mathematics and Computational Mechanics 2023, 22(3), 43-56
www.amcm.pcz.pl p-ISSN 2299-9965
DOI: 10.17512/jamcm.2023.3.04 e-ISSN 2353-0588

ELECTROSURGICAL RESECTION OF COLORECTAL POLYPS – MATHEMATICAL MODELLING OF PROCESSES DURING MEDICAL TREATMENT

Jarosław Siedlecki

Department of Mathematics, Czestochowa University of Technology
Czestochowa, Poland
jaroslaw.siedlecki@pcz.pl

Received: 11 July 2023; Accepted: 25 September 2023

Abstract. The colonoscopic electrosurgical polypectomy is a very popular surgical procedure in which the colon polyps are removed. In this work, the mathematical description of the electrical and thermal processes proceeding during this procedure has been proposed. The mathematical model contains the specification of the considered domain's geometry, the system of the partial differential equations that governs heat transfer in the considered particular sub-domains (i.e. polyp, colon and electrode) with the adequate initial-boundary conditions, the system of the differential equations for determination of the electrical potential distribution in the tissue sub-domains, and the definition of the Arrhenius tissue damage integral. Next, the example results of numerical simulations for the proper and incorrect positions of the polyp in the colon are presented. The conclusions are also provided. The proposed research can be helpful for the surgeons to choose the optimal set parameters of the electric current during the endoscopy procedure.

MSC 2010: *35K05, 35Q79*
Keywords: *electrosurgical polypectomy, Pennes bio-heat transfer model, biological tissue heating, tumor*

1. Introduction

An increasingly widespread gastrointestinal disease are polyps in the large intestine (colon). These polyps slowly grow inside the intestine and can become tumors (approximately 15 percent of polyps). From the medical point of view, there are many treatments including, for instance, the colonoscopic electrosurgical polypectomy [1-4] in which the colon polyp can be removed directly. This method is an invasive method of treatment (and is treated as a surgical procedure) and simultaneously an effective method for the prevention of colorectal cancer. The endoscopic instruments such as: snares, clips, forceps and electrosurgery generator units (ESU) have been used by doctors for colorectal polypectomy. Exact understanding

the characteristics of these instruments and their proper use according to the geometry of the colorectal polyp will enable endoscopists to be able to perform effective procedure. The colonoscopic polypectomy is a continuously evolving therapy that allows one to reduce the risk of the colorectal cancer. Gastroenterologists should be thoughtful and proficient in techniques for endoscopic success. Difficult to reach polyps require various endoscopic tricks for successful resection.

Knowledge of the course of thermal and electrical processes occurring during this short-term treatment makes it possible to analyse them in detail. Mathematical models and numerical simulations describing the ongoing processes can also be helpful. In works [5, 6], the authors undertook the development of such mathematical models and presented examples of simulation results. The influence of current parameters and its duration on the course of the polypectomy, especially in order to determine the degree of the tissue damage, was investigated.

In this work, a similar problem is considered, in which during the electrosurgical polypectomy, the head of polyp may inevitably touch the intestinal wall, and it can lead to inefficient heating of the part of the colon wall and the unexpected burn (i.e. the tissue destruction) may occur.

2. Statement of the problem

In the electrosurgical polypectomy procedure [1-4], a polypectomy snare (wire loop) is passed over the stalk of the polyp and is tightened around the polyp. The snares are made from different metals and have different sizes (usually 0.3-0.5 mm in diameter) and different shapes of a continuous wire loop (usually 20-25 mm in diameter). The snare is placed within a flexible sheath (a typical size is of 3-5 mm and length is about 2 m). The wire and sheath are affixed to a moving-parts handle at the end of the device controlled by the surgeon who opens and tightens the wire loop. In addition, the handle has an electrical connector that allows one to connect the snare wire with an active cord to the electrosurgical generator unit. In the case of the monopolar snares, a high frequency electrical current (usually in the range of 300 kHz to 1 MHz) is turned on on the electrosurgical generator unit for a very short moment of time. At this time, the current flows from the electrosurgical unit to the wire loop (which is the first electrode). Next, through the polyp-colon tissues (which are in contact with the wire loop) and through other tissues of the patient's body to the second electrode placed, for example, on the patient's back. The second electrode is also connected to the electrosurgical generator unit.

The passage of the electric current through the living tissues generates heat (so-called the Joule heating effect) and, as result, the temperature within these tissues increases. Here, the temperature rise is expected to be correspondingly high, which causes the protein denaturation and the tissue coagulation [7, 8], which results in the closing of the wound, the stopping of the bleeding and cutting off the polyp from the colon wall.

The highest concentration of the electrical current flowing in the tissues occurs in a narrow channel – here, i.e. in the squeezed base of the polyp (this is only the place preferred during the electrosurgical polypectomy procedure). Furthermore, such a problem may occur in the case when the head of the polyp is in a very narrow contact with the colon wall (this case must be avoided, because it can lead, e.g., to the colon perforation). In both cases, the Joule heating effect is a very intensive. The local increase in temperature (over approximately 60°C) of the tissue domains may lead to an irreversible cellular damage of tissue. This process is called the process of thermal coagulation and may cause changes in the structure of the tissues. If the current stops flowing then the warm tissues cause further heat dissipation to the adjacent tissues. In Figure 1, the scheme of surgical operation in the polypectomy is presented. Also, one can see the problematic case that may occur during the current flow in this procedure.

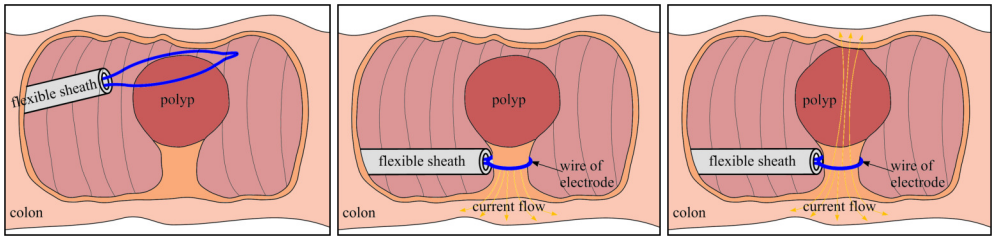

Fig. 1. Scheme of surgical operation in the polypectomy

An important issue is to describe the processes taking place during the electrosurgical polypectomy procedure using the mathematical models. Here, three different models (the electrical potential model, the model of thermal processes proceeding in the biological tissues and the wire, the tissue damage model) are introduced, but all the models are essentially conjugated together. The aim of the present work is determine the spatial transient-state temperature distribution in the polyp-colon-wire system and the thermal damage degree of the biological tissues. From the mathematical point of view, the heat transfer proceeding in the vital tissues is usually described by the very popular Pennes equation [8-10]. This equation contains two heat sources associated with the metabolism in tissues and the blood perfusion. While the heat transfer in the wire loop (that squeezes the stalk of polyp) is described by the Fourier equation. When the electric current flows through tissues, the Joule heating effect takes place (the quantity of the heat source generation depends on the gradient of the electric field potential), and then the heat source is introduced into the Pennes equation as an additional term. One of the electrodes is the wire loop, while the ground electrode is placed on the patient's back skin (usually). Accurately taking into account such a geometry model is very complicated, and the simplified model of the considered geometrical domain (i.e. assuming that the ground potential is placed on the outer surface of the intestine) is considered, and simultaneously the smaller difference of the electric potentials between the first electrode (wire) and grounding on the 'virtual' surface of the intestine are assumed.

If the electric current stops flowing through tissues then the heat generation in tissues ends. However, heat continues to spread to the neighbouring tissue domains and may cause further tissue destruction. The degree of the tissue damage is estimated on the base of the value of the Arrhenius damage integral [11-13].

3. Mathematical model

The considered complex geometrical problem is simplified to the axially-symmetrical problem. Two cases of the non-contact and contact polyp head with the inner surface of the colon are analyzed. In Figure 2, the geometrical models of the colon-polyp-wire domains are illustrated. One can see the three types of sub-domains: Ω_1 – the colon tissue, Ω_2 – the polyp tissue, Ω_3 – the wire loop electrode. The outer surfaces Γ_{b1}, Γ_{b2} and Γ_{b3} of the particular sub-domains are in direct contact with the gaseous environment present in the colon. The surfaces Γ_{1-2}, Γ_{1-3} and Γ_{2-3} are the interfacial surfaces (the ideal contact between different sub-domains is assumed). The surface Γ_z is a part of the outer surface of the colon, while the surfaces Γ_0 and Γ_r are the 'virtual cutting surface' of the colon wall and the symmetry axis of the considered sub-domains, respectively.

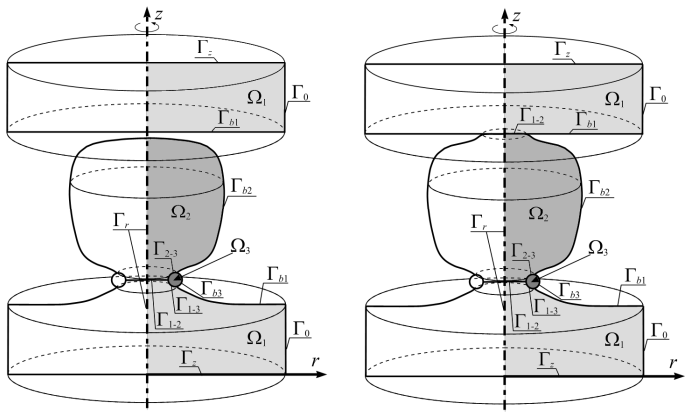

Fig. 2. The considered domain (longitudinal cross-section) of the polyp-colon-wire system cases of the non-contact and contact polyp head with the inner surface of the colon

Here, equations of the mathematical model in the 2D cylindrical axi-symmetrical coordinates system are given. First, the governing equations related to the bio-heat transfer model are presented. The temperature field in all considered sub-domains is described by the system of equations [5, 6, 9]

$$\mathbf{x}\in\Omega_e:\; c_e(T)\rho_e(T)\frac{\partial T_e(\mathbf{x},t)}{\partial t}=\nabla\cdot\left[\lambda_e(T)\nabla T_e(\mathbf{x},t)\right] + Q_{per\,e}(\mathbf{x},t)+Q_{met\,e}(\mathbf{x},t)+Q_{Joule\,e}(\mathbf{x},t) \tag{1}$$

where sub-index $e = 1, 2, 3$ identifies the particular sub-domains (1 – colon tissue, 2 – the polyp tissue, 3 – the wire loop electrode), T [°C] is the temperature, $\mathbf{x}$ [m], t [s] denote geometrical coordinates and time, while c [J/(kg K)], ρ [kg/m^3], λ [W/(m K)] are the thermophysical parameters, i.e. the specific heat, the density and the thermal conductivity, respectively. In the above system of equations, the terms $Q_{met\,e}$ [W/m^3], for $e = 1, 2$, are the capacities of volumetric internal heat sources associated with the metabolic heat in tissues and can be treated as a temperature-dependent function or a constant value, while in the wire loop electrode domain, the source $Q_{met\,3}(\mathbf{x},t) = 0$ is assumed, of course. Assuming that the colon and polyp tissues are fed by a large number of uniformly spaced capillary blood vessels, hence the volumetric internal heat sources associated with blood perfusion $Q_{per\,e}$ [W/m^3] in the Pennes model can be treated as

$$Q_{per\,e}\left(\mathbf{x},t\right)=\begin{cases}c_{blood}\rho_{blood}G_{blood\,e}\left(\mathbf{x},t\right)\left[T_{blood}-T_e\left(\mathbf{x},t\right)\right], & e=1,2\\ 0 & e=3\end{cases} \tag{2}$$

where c_{blood} is the blood specific heat, ρ_{blood} is the blood density and T_{blood} is the average blood temperature of the body and G_{blood} is the blood perfusion rate in the tissue [m$^3_{(blood)}$/(s m$^3_{(tissue)}$)], which depends on the tissue thermal damage [14]

$$G_{blood\,e}\left(\mathbf{x},t\right)=G_{blood0\,e}\cdot\begin{cases}1+25\Psi_e\left(\mathbf{x},t\right)-260\left(\Psi_e\left(\mathbf{x},t\right)\right)^2 & \text{for } 0\le\Psi_e\left(\mathbf{x},t\right)\le 0.1\\ \exp\left(-\Psi_e\left(\mathbf{x},t\right)\right) & \text{for } \Psi_e\left(\mathbf{x},t\right)>0.1\end{cases} \tag{3}$$

in which G_{blood0} is the baseline blood perfusion and Ψ is the value of the Arrhenius integral (see Eq. (9) in the next part of the work).

The system of equations (1) is supplemented by the set of initial-boundary conditions. On the interfacial surfaces between all sub-domains, the continuity conditions are assumed, while the non-flux boundary condition on the boundary Γ_r is given. Moreover, on the external surfaces of the sub-domains, the Dirichlet-type (on the boundaries Γ_0 and Γ_z) and the Robin-type (on the outer surfaces of the sub-domains: Γ_{b1}, Γ_{b2} and Γ_{b3}) boundary conditions are assumed. In mathematical notation, these boundary conditions have the form

$$\begin{aligned}
&\mathbf{x}\in\Gamma_{k-l}:\ \begin{cases}-\lambda_k\left[\mathbf{n}\cdot\nabla T_k\left(\mathbf{x},t\right)\right]=-\lambda_l\left[\mathbf{n}\cdot\nabla T_l\left(\mathbf{x},t\right)\right]\\ T_k\left(\mathbf{x},t\right)=T_l\left(\mathbf{x},t\right)\end{cases},\\
&\qquad\qquad (k-l)\in\left\{(1-2),(1-3),(2-3)\right\}\\
&\mathbf{x}\in\Gamma_r:\ \mathbf{n}\cdot\nabla T_e\left(\mathbf{x},t\right)\big|_{\mathbf{x}\in\Gamma_r}=0,\ e=1,2\\
&\mathbf{x}\in\left\{\Gamma_0,\Gamma_z\right\}:\ T_1\left(\mathbf{x},t\right)=T_{tissue}\\
&\mathbf{x}\in\Gamma_{b\,e}:\ -\lambda_e\left[\mathbf{n}\cdot\nabla T_e\left(\mathbf{x},t\right)\right]=\alpha_e\left[T_e\left(\mathbf{x},t\right)-T_{amb}\left(t\right)\right],\ e=\ 1,2,3
\end{aligned} \tag{4}$$

where **n** is the outward-point unit normal vector to boundary, α [W/(m^2 K)] is the convective heat transfer coefficient and T_{amb} [°C] can be treated as the gas temperature in the colon or $T_{amb} = T_{tissue}$. The initial conditions are as follows

$$t = 0:\ T_e(\mathbf{x},t)\big|_{t=0} = T_{init\,e}(\mathbf{x}),\ e = 1,2,3 \tag{5}$$

Here, one can assume that the initial temperature T_{init} is equal to the average temperature T_{tissue} of the human body.

In Eq. (1), the term Q_{Joule} related to the Joule heating effect [9, 15] inside the tissue sub-domains is present. Here, in this model, the electrical current flows in the interval $t \in [0, t_{heating}]$ and the heating in this time interval takes place. The heat generation rate per unit volume Q_{Joule} is mainly related to the electric potential field as

$$Q_{Joule\,e}(\mathbf{x},t) = \begin{cases} H(t_{heating} - t)\,\sigma_e \left|\nabla\varphi_e(\mathbf{x})\right|^2 & \text{if } e = 1,2 \\ 0 & \text{if } e = 3 \end{cases} \tag{6}$$

where φ [V] is the electrical potential, σ [S/m] is the electrical conductivity, while H is the Heaviside step function which describes the current flow over time (so-called the duty cycle of the electrosurgical generator unit).

In order to determine $\nabla\varphi_e(\mathbf{x})$, the knowledge of the electrical potential distribution in the tissue sub-domains is required. To achieve this, a system of differential equations should be solved (the model of electrical potential) [5, 9, 15]

$$\mathbf{x} \in \Omega_e:\ \nabla\cdot\left[\sigma_e(\mathbf{x})\nabla\varphi_e(\mathbf{x})\right] = 0,\ e = 1,2 \tag{7}$$

supplemented by the adequate boundary conditions. On the wire loop electrode $\mathbf{\Omega}_3$ that is in the contact with the tissue sub-domains (the surfaces Γ_{1-3}, Γ_{2-3}), the voltage U_1 is applied. The second/ground electrode is usually placed on the skin e.g. of the patient's back. Here, certain simplification has been adopted that on the 'virtual cutting surface' of the colon wall Γ_0 and on the outer surface of colon Γ_z (assuming that the analyzed part of the colon is in ideal contact with other human tissues), the average intermediate value of the electrical potential U_2 is assumed. From the computational point of view, to calculate the potential gradient value, the difference in potentials $U_1 - U_2$ is essential. Hence, it was assumed that $U_2 = 0$. Between both tissue sub-domains $\mathbf{\Omega}_1$ and $\mathbf{\Omega}_2$, i.e. on the surface Γ_{1-2}, the continuity condition (4-th kind boundary condition) is applied, while on the remaining boundaries Γ_{b1}, Γ_{b2}, Γ_r of the analyzed domain that are not in contact with the wire electrode, the electrical insulation condition is assumed. To summarize, all boundary conditions can be written as

$$\begin{aligned}
&\mathbf{x}\in\left\{\Gamma_{1-3},\Gamma_{2-3}\right\}:\ \varphi_e\left(\mathbf{x}\right)=U_1,\ e=1,2\\
&\mathbf{x}\in\left\{\Gamma_0,\Gamma_z\right\}:\ \varphi_1\left(\mathbf{x}\right)=U_2\\
&\mathbf{x}\in\Gamma_{1-2}:\ \begin{cases}\sigma_1\left[\mathbf{n}\cdot\nabla\varphi_1\left(\mathbf{x}\right)\right]=\sigma_2\left[\mathbf{n}\cdot\nabla\varphi_2\left(\mathbf{x}\right)\right]\\ \varphi_1\left(\mathbf{x}\right)=\varphi_2\left(\mathbf{x}\right)\end{cases}\\
&\mathbf{x}\in\left\{\Gamma_{b1},\Gamma_{b2},\Gamma_r\right\}:\ \mathbf{n}\cdot\nabla\varphi_e\left(\mathbf{x}\right)=0,\ e=1,2
\end{aligned}\tag{8}$$

The solution of the system of heat transfer equations (1) leads to the determination of the spatio-temporal temperature distribution $T\left(\mathbf{x},t\right)$ in the sub-domains. The significant increase in the temperature in certain parts of the polyp and colon tissues above a certain threshold can lead to irreversible cellular damage (e.g. the tissue denaturation). In this order, the tissue damage model based on the Arrhenius model [10, 12, 13] is introduced. The Arrhenius damage integral allows one to determine the degree of the tissue damage, and this integral is defined as

$$\Psi_e\left(\mathbf{x},t\right)=\int_0^t\xi_e\,\exp\left(-\frac{\Delta E_e}{R_g\cdot\left(T_e\left(\mathbf{x},\tau\right)+273\right)}\right)d\tau,\ e=1,2 \tag{9}$$

where ξ [1/s] is the pre-exponential/frequency factor, ΔE [J/mol] is the activation energy and R_g [J/(mol K)] is the universal gas constant. One can note that the value of this integral depends on the knowledge of the spatio-temporal temperature distribution (in other words, it depends on the tissue temperature history at the selected point **x** of tissue). Based on the Arrhenius model, the calculated values of integral over time at a specific point of the tissue domain are used to determine a probability of cell death $P_e\left(\mathbf{x},t\right)=\left(1-\exp\left(-\Psi_e\left(\mathbf{x},t\right)\right)\right)\cdot 100\%$. For instance, if the value of Ψ is equal to 1 then a probability of cell death is about 63%, while if the value of Ψ equals 4.6, then a probability of cell death is about 99%, and it is assumed that the thermal damage is almost total. From a patient's treatment point of view, the area of tissue damage should be limited to minimal one in the adequate place and simultaneously, it must be effective for medical procedure.

In the case of the considered problem in the 2D cylindrical axi-symmetrical coordinates, the general coordinates **x** should be replaced by $\mathbf{x}=\{r,\,z\}$ in all of the above equations. Additionally, then the two operators for the new coordinates take the forms (for e = 1, 2, 3)

$$\nabla\cdot\left[\lambda_e\left(T\right)\nabla T_e\left(\mathbf{x},t\right)\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_e\left(T\right)\frac{\partial T_e\left(r,z,t\right)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda_e\left(T\right)\frac{\partial T_e\left(r,z,t\right)}{\partial z}\right) \tag{10}$$

$$\left|\nabla\varphi_e\left(\mathbf{x}\right)\right|^2=\left(\frac{\partial\varphi_e\left(r,z\right)}{\partial r}\right)^2+\left(\frac{\partial\varphi_e\left(r,z\right)}{\partial z}\right)^2 \tag{11}$$

From the computation point of view, the governing equations of the presented mathematical model are coupled. First, the electric potential field distribution in the tissue sub-domains should be determined. Next, on the basis of the gradient values of the electric potential distribution, the internal heat source in tissues (the Joule heating) should be calculated. Then the spatio-temporal temperature field in the polyp-colon-wire system can be determined, and finally, the values of the Arrhenius damage integral and the tissue damage degrees associated with them are calculated.

4. Results of simulation

The proposed mathematical model in the previous section has been solved numerically using the Control Volume Method – CVM (known also as the Finite Volume Method – FVM) [5]. Here, the detailed description of this method is omitted, but the reader can find the details in works [16, 17]. Finding the analytical solution for the mathematical model seems to be impossible for the complex geometry of the considered problems.

As mentioned in the introduction, the influence of the contact area of the polyp head with the inner surface of the large intestine on the course of the thermal and electrical processes proceedings in the considered sub-domains in the selected moments of time is analyzed. Three selected geometric cases of the polyp-colon system are considered:

– case I: without contact of the polyp head with the large intestine,
– case II: with contact on a small area,
– case III: with contact on a large area.

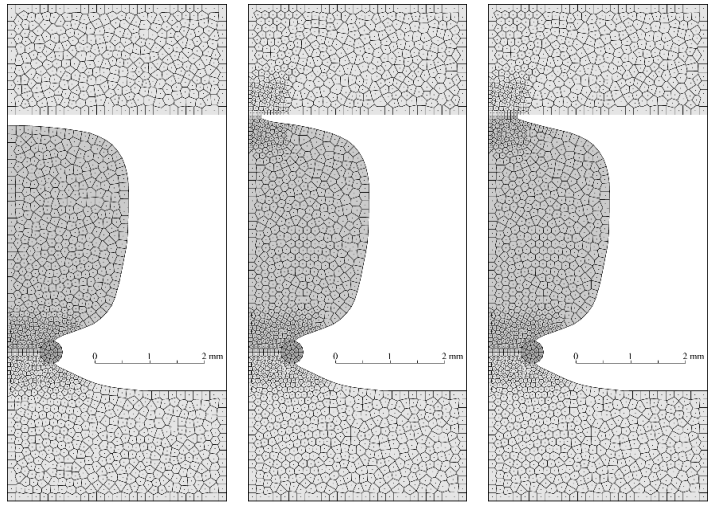

Fig. 3. Discretisation of considered domain – three cases

The Voronoi discretisations of three cases of the colon-polyp-wire sub-domains (such as the 2D cylindrical cross-sections) are presented in Figure 3. In the case II,

the radius of contact is about 0.237 mm, and then the area of contact is about 0.237 mm^2. While, in the case III, the radius of contact is about 0.522 mm and the area of contact is about 0.856 mm^2. In the cross-section, the control volumes have shapes of the convex polygons, but in reality each control volume has the shape of a ring.

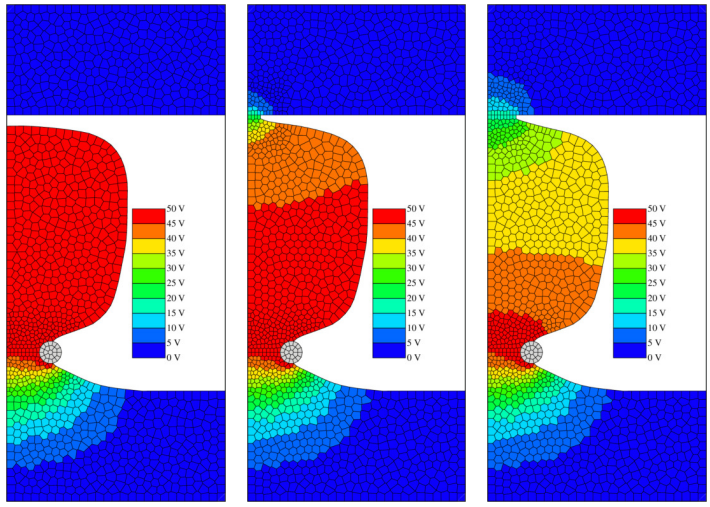

Fig. 4. Electric potential field distribution φ for three cases of domains

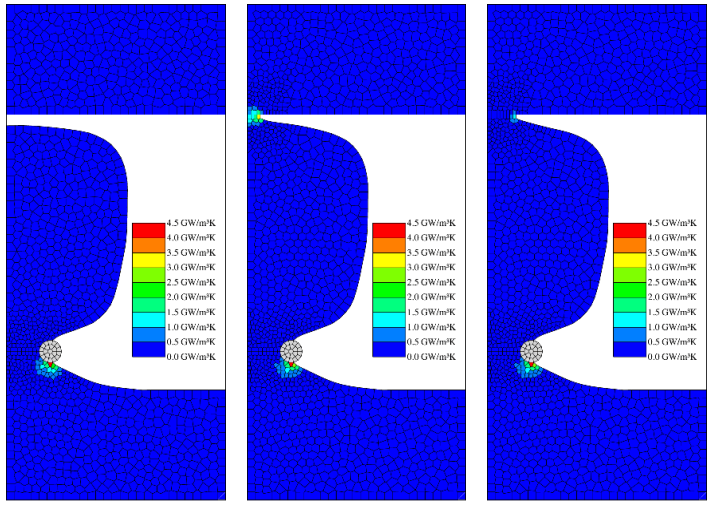

Fig. 5. Internal heat source distribution $\sigma|\nabla\varphi|^2$ for three cases of domains

The following thermophysical parameters of particular sub-domains have been assumed to perform sample calculations [18, 19]: $\lambda_1 = \lambda_2 = 0.55$ W/(m K) (for current frequency of $f = 400$ kHz), $\lambda_3 = 15$ W/(m K), $c_1 = c_2 = 1041$ J/(kg K), $c_3 = 500$ J/(kg K), $c_{blood} = 3650$ J/(kg K), $\rho_1 = \rho_2 = 3655$ kg/m^3, $\rho_3 = 8000$ kg/m^3, $\rho_{blood} = 1069$ kg/m^3, $G_{blood0\,1} = G_{blood0\,2} = 0.538 \cdot 10^{-3} \cdot s^{-1}$, $Q_{met\,1} = Q_{met\,2} = 684$ W/m^3, $\sigma_1 = \sigma_2 = 0.25$ S/m,

$\sigma_3 = 1.45 \cdot 10^6$ S/m, $\xi_1 = \xi_2 = 3.1 \cdot 10^{98}$ 1/s, $\Delta E_1 = \Delta E_2 = 6.27 \cdot 10^5$ J/mol, $R_g = 8.314$ J/(mol K). In the case of the colon and polyp tissues, the same values of parameters have been taken into account, because the parameters for the polyp tissue are unknown in the literature. In simulations, the following parameters are also used: $T_{blood} = 37°C$, $T_{init} = 37°C$, $T_{amb} = 37°C$, $\alpha_1 = \alpha_2 = \alpha_3 = 10$ W/(m^2 K), $U_1 - U_2 = 50$ V, $t_{heating} = 0.5$ s.

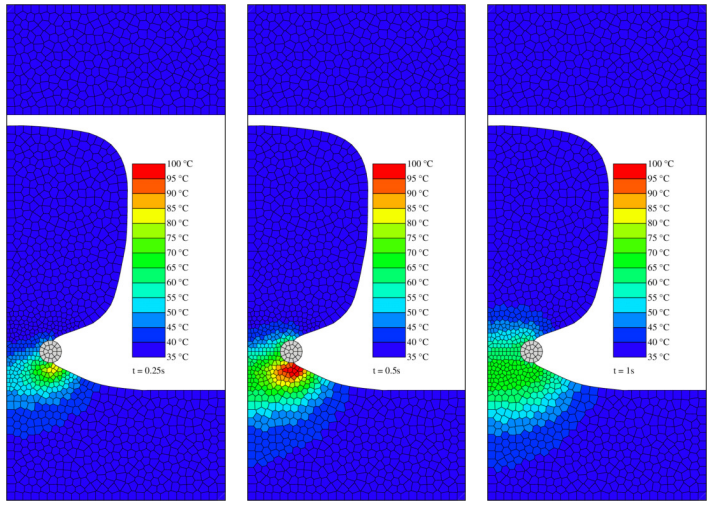

Fig. 6. Temperature distribution at differential moment of time – case I

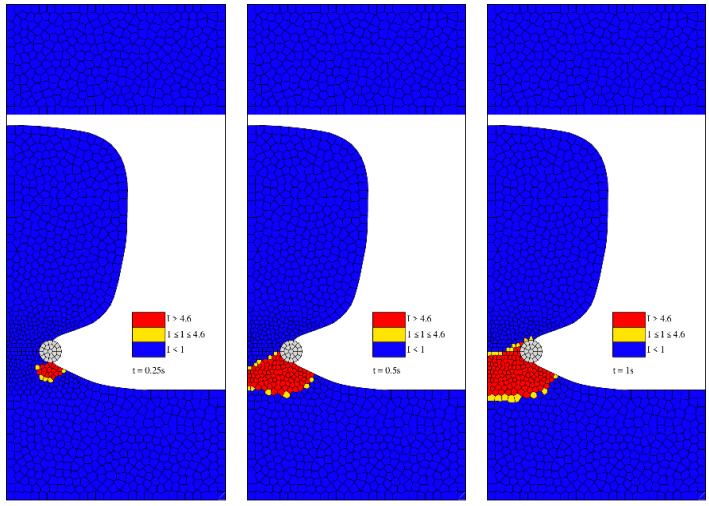

Fig. 7. Degrees of the tissue damage at differential moment of time – case I

In Figure 4, the electric potential distributions for three geometric cases of the polyp-colon system are presented, while in Figure 5 the internal heat source distributions for these cases are shown.

In the next figures, the temperature fields and the tissue damage distributions (the values of the Arrhenius integral Ψ) at the selected moment of the simulation times t_{simul} = {0.25 s, 0.5 s, 1 s} are presented. Figures 6 and 7 refer to the case I, Figures 8 and 9 refer to the case II and Figures 10 and 11 refer to the case III, respectively. It should be pointed out that for time $t > t_{heating}$, the current does not flow by tissues (the Joule heating does not take place), but the thermal diffusion in the polyp-colon-wire sub-domains takes the place, and the heat is still transferred to neighbouring parts of tissues and the process of tissue damage is growing.

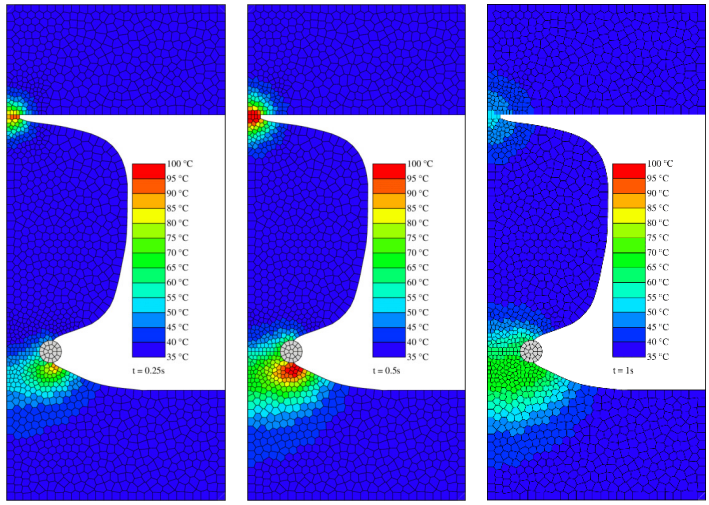

Fig. 8. Temperature distribution at differential moment of time – case II

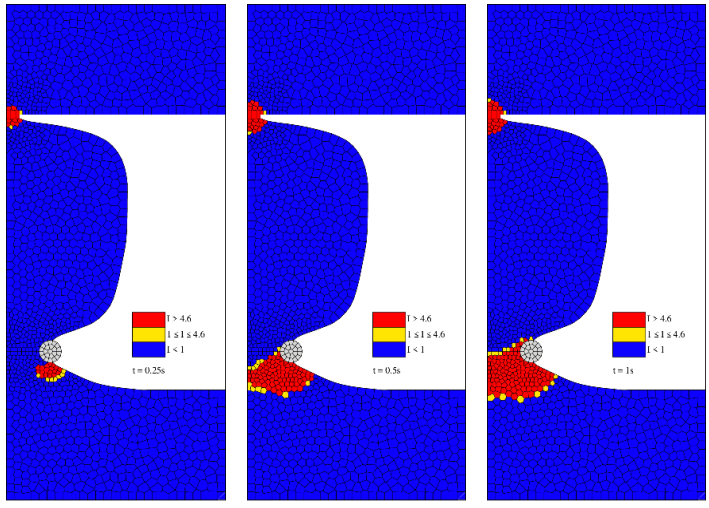

Fig. 9. Degrees of the tissue damage at differential moment of time – case II

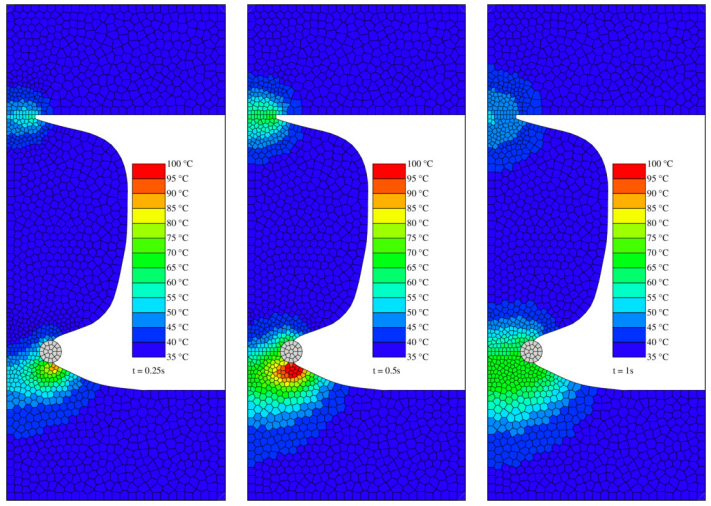

Fig. 10. Temperature distribution at differential moment of time – case III

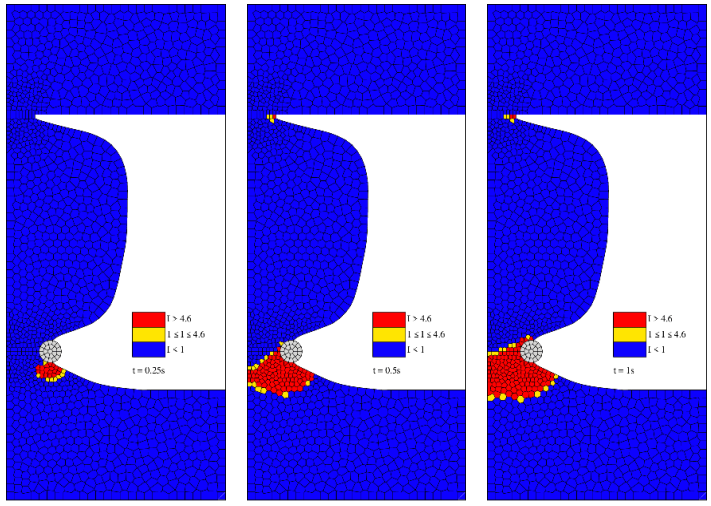

Fig. 11. Degrees of the tissue damage at differential moment of time – case III

5. Conclusion

The mathematical description of three physical problems (i.e. related to the thermal processes proceeding in the living biological tissues and the domain of wire electrode, the distribution of the electrical potential in the domain of polyp-colon and the determination of the degree of tissue damage) have been proposed and investigated. It is worth mentioning that these models are connected together. This work is a continuation of the previous researches [5, 6], in which it was extended to atypical (incorrect) cases of the polyp location in the large intestine during surgery.

Many variants of the simulations that depended on the contact area of the polyp head with the inner surface of the large intestine have been carried out. In this work, three selected cases: without contact, with contacts on small and larger areas were presented. The most expected position of the polyp during the current flow is no-contact of the polyp head with any part of the large intestine. But, when such accidental contact occurs even on a small area, then the current flows through the contact surface. Analysing various cases of the contact areas, it can be seen that if the contact area is smaller, then the electric potential gradients near this contact have the higher values and simultaneously the temperature of the tissues in the neighbourhood of this contact increases. Increased tissue temperature (especially on the side of the intestine) can lead to their damage and even eventually lead to the colon perforation. It can also be seen that if the area of such contact is large (i.e. exceeds the cross-sectional area at the height of the polyp's squeeze by the wire loop), then the Joule heating effect of the tissues in the neighbourhood of this contact is weaker – however, such a situation is also undesirable and can be dangerous for the patient.

The obtained simulation results (i.e. the distributions of temperature in the system of tissues and wire loop domains and the distributions of the tissue damage degree) can help doctors understand the physical problems occurring during the electrosurgical polypectomy. These results can be very interesting both from the mathematical and medical practice points of view. Some doctors do not have much mathematical experience, especially in modeling such physical processes.

The presented illustrative results may be useful for the endoscopist in making the decision just before cutting the polyp (the execution of the endoscopy procedure). Here, the most important parameters for the doctor are: the appropriate choice of the optimal heating time and the parameters of the electric current depending on the position of the polyp in the colon.

The considered problem in this work has been solved numerically using the Finite Volume Method in which the Voronoi tessellation has been used for the discretization of the axially-symmetrical shapes of the particular sub-domains. This kind of tessellation allows one to accurately reconstruct the shapes of the sub-domains (of course, with simplified shapes). Also, the application of the FVM enables very good numerical approximation of the mathematical equations presented in the work. The simulations were performed in the authorial software. The proposed mathematical model and results of the numerical simulations can allow for the creation of safer clinical procedures and better treatment of the patients.

References

[1] Cotton, P.B., & Williams, Ch.B. (2008). *Practical Gastrointestinal Endoscopy: The Fundamentals*. A John Wiley & Sons.

[2] Engin, O. (2015). *Colon Polyps and the Prevention of Colorectal Cancer*. Springer International Publishing.

[3] Haycock, A., Cohen, J., Saunders, B.P., Cotton, P.B., & Williams, Ch.B. (2014). *Cotton and Williams' Practical Gastrointestinal Endoscopy, The Fundamentals*. 7th ed. Willey Backwell.

[4] Gordon, P.H., & Nivatvongs, S. (2007). *Principles and Practice of Surgery for the Colon, Rectum, and Anus*. Third Edition. Taylor & Francis Group.

[5] Ciesielski, M., Siedlecki, J., & Janik, M.K. (2020). Mathematical modelling of thermal and electrical processes during electrosurgical resection of colorectal polyps. *International Journal of Engineering Science*, *154*, 103351.

[6] Ciesielski, M., Siedlecki, J., & Janik, M.K. (2020). Mathematical modelling of thermal and electrical processes in the polyp-colon system during electrosurgical polypectomy. *International Journal of Heat and Technology*, *38*(4), 808-816.

[7] Martínez, C.A.R., & et al. (2016). Modeling of Electric Field and Joule Heating in Breast Tumor during Electroporation. In: *13th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE)*, Mexico, City. Mexico. September 26-30.

[8] Pennes, H.H. (1948). Analysis of tissue and arterial blood temperature in the resting human forearm. *Journal of Applied Physiology*, *1*, 93-122.

[9] Majchrzak, E., Dziatkiewicz, G., & Paruch, M. (2008). The modeling of heating a tissue subjected to external electromagnetic field. *Acta of Bioengineering and Biomechanics*, *10*, 29-37.

[10] Arkin, H., Xu, L.X., & Holmes, K.R. (1994). Recent developments in modeling heat transfer in blood perfused tissues. *IEEE Transactions on Biomedical Engineering*, *41*, 97-107.

[11] Jasinski, M. (2018). Modelling of thermal damage process in soft tissue subjected to laser irradiation. *Journal of Applied Mathematics and Computational Mechanics*, *17*(2), 29-41.

[12] Qin, Z., Balasubramanian, S.K., Wolkers, W.F., Pearce, J.A., & Bischof, J.C. (2014). Correlated parameter fit of Arrhenius model for thermal denaturation of proteins and cells. *Annals of Biomedical Engineering*, *42*(12), 2392-2404.

[13] Paruch, M. (2018). Identification of the degree of tumor destruction on the basis of the Arrhenius integral using the evolutionary algorithm. *International Journal of Thermal Sciences*, *130*, 507-517.

[14] Singh, S., & Repaka, R. (2018). Parametric sensitivity analysis of critical factors affecting the thermal damage during RFA of breast tumor. *International Journal of Thermal Sciences*, *124*, 366-374.

[15] Corovic, S., & et al. (2013). Modeling of electric field distribution in tissues during electroporation. *BioMedical Engineering OnLine*, *12*, 16.

[16] Ciesielski, M., Mochnacki, B., & Siedlecki, J. (2016). Simulations of thermal processes in tooth proceeding during cold pulp vitality testing. *Acta of Bioengineering and Biomechanics*, *18*(3), 33-41.

[17] Ciesielski, M., & Mochnacki, B. (2018). Hyperbolic model of thermal interactions in a system biological tissue-protective clothing subjected to an external heat source. *Numerical Heat Transfer, Part A: Applications*, *74*(11), 1685-1700.

[18] Hasgall, P.A., & et al. (2018). *IT'IS Database for thermal and electromagnetic parameters of biological tissues*, Version 4.0. DOI: 10.13099/VIP21000-04-0.

[19] Gabriel, C. (1996). *Compilation of the Dielectric Properties of Body Tissues at RF and Microwave Frequencies*. Report N.AL/OE-TR-1996-0037, Occupational and environmental health directorate, Radiofrequency Radiation Division. Brooks Air Force Base, Texas (USA).