Journal of Applied Mathematics and Computational Mechanics 2015, 14(1), 79-90
www.amcm.pcz.pl p-ISSN 2299-9965
DOI: 10.17512/jamcm.2015.1.08 e-ISSN 2353-0588

INVESTIGATION OF NETWORKS WITH POSITIVE AND NEGATIVE MESSAGES, MANY-LINES QUEUEING SYSTEMS AND INCOMES

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Abstract. The analysis of an open Markov Queueing Network with positive and negative messages, many-lines queueing systems and incomes has been carried out. External arrivals to the network, service times of rates and probabilities of messages transition between queueing systems (QS) depend on time. A method for finding the expected incomes of the network systems, the expressions for the average number of messages at the systems has been proposed.

Keywords: *G-network, negative messages, many-lines systems, expected incomes, information systems and networks*

Introduction

In design and analysis of mathematical models of Information and Telecommunications Network (ITN) it is necessary to take into account characteristics of such models and possible influence of various destabilizing factors: sudden failures from viruses, loss of transmitted or processed data. Taking into consideration these factors, E. Gelenbe suggested using G-networks [1], in which in addition to the flow of ordinary (positive) messages are also considered additional Poisson streams of negative messages. In such networks, when the system receives the network message, the negative message destroys the positive message, if any is available in the system, reducing the number of positive messages in the system on an unit. In [2] an overview is given of the literature on G-networks and it is shown that the stationary probabilities of the states of the network have a multiplicative form (form product). It is important to note that in contact with viruses in the ITN due to loss of information or distortion it bears some of the costs or losses. Their inclusion can be realized by applying a model queueing network with incomes (HM-network) [3] with positive and negative messages. In [4] the HM-Markov network was investigated at the transient behaviour with positive and negative messages.

In such a network, in the transition of positive message of QS to another, the last gets some generally random income, and the income of the first QS is reduced accordingly by that amount. Contact with the negative message at the QS network is accompanied by consumption (loss, damage) for the QS. Negative messages coming in a system of a network, in which there is at least one positive message, instantly destroys one of them and causes loss of QS. Under the assumption of exponential service time distribution of positive messages, one can not worry about what kind of message is destroyed. After that the negative message immediately leaves the network or is destroyed, if at the queue there are no messages, and again after servicing positive messages receives some income.

Income from transitions at ITN may be, for example, online payments and commission from it, users remittances of online banking systems and other similar systems. The messages at the same time are requests of Internet users who use the services of online payments, and the negative messages can be viruses (or program disruptors Internet payment system) in such ITN or ICMP (Internet Control Message Protocol)-requests in the case of DDoS-attacks on the ITN of such a kind. Such network can be used in modeling of income changes, for example, at DDoS-attacks on such ITN or at the viruses penetration into it. This paper focuses on the G-networks with incomes, but with a many-lines queueing system. It is considered the case, when the intensity of the incoming streams of positive and negative messages and it services, and also the transition probabilities between QS messages depend on time. It is assumed that the incomes from the transitions between the states of the network are random variables with given mean values.

1. Finding the expected incomes of network systems

Consider an open G-network with n many-lines QS. In QS S_i from the outside (from the system S_0) is coming a Poisson stream of the positive messages with the intensity $\lambda_{0i}^{+}(t)$ and Poisson stream of negative messages with the intensity $\lambda_{0i}^{-}(t)$, $i=\overline{1,n}$. All flows of messages incoming to the network are independent. Let the system S_i contain m_i identical service lines, in each of which the service time of positive messages at QS S_i at moment time t distributed exponentially with parameter $\mu_i(t)$, $i=\overline{1,n}$. The positive message, serviced at S_i at moment time t, with the probability $p_{ij}^{+}(t)$ is sent to QS S_j as a positive message, with probability $p_{ij}^{-}(t)$ - as a negative message, and with probability $p_{i0}(t)=1-\sum_{j=1}^{n}\left(p_{ij}^{+}(t)+p_{ij}^{-}(t)\right)$ message leaves the network to the external environment (QS S_0), $i,j=\overline{1,n}$. HM-network with the ordinary messages, with time-dependent intensities of the incoming flow and service, and also the transition probabilities of messages of the QS network are considered in [5]. As the state of the network at time t we mean the vector $k(t)=(k,t)=(k_1(t),k_2(t),...,k_n(t))$, where $k_i(t)$ - count of messages at the moment time t at the system S_i, $i=\overline{1,n}$.

Consider the dynamics of income changes of a network system S_i. Denote by the $V_i(t)$ its income at moment time t. Let the initial moment time income of the system equal $V_i(0)=v_{i0}$. The income of its QS at moment time $t+\Delta t$ can be represented in the form

$$V_i(t+\Delta t)=V_i(t)+\Delta V_i(t,\Delta t), \tag{1}$$

where $\Delta V_i(t,\Delta t)$ - income changes of the system S_i at the time interval $[t,t+\Delta t)$, $i=\overline{1,n}$. To find its value we write down the value of the conditional probabilities of events that may occur during Δt and the income changes of its QS, associated with these events:

- with probability $\lambda_{0i}^{+}(t)\Delta t+o(\Delta t)$ at moment time t to the system S_i from the external environment will come positive message, which will bring an income to the amount of r_{0i}, where r_{0i} - random variable (RV), expectation (E) which is equals $M\{r_{0i}\}=a_{0i}$, $i=\overline{1,n}$;
- with probability $\lambda_{0i}^{-}(t)\Delta t+o(\Delta t)$ in the QS S_i at moment time t from the external environment will come a negative message, which will bring it an income (loss) in the amount of $-\bar{r}_{0i}$, where $\bar{r}_{0i}$ – RV with E $M\{\bar{r}_{0i}\}=\bar{a}_{0i}$, $i=\overline{1,n}$;
- if at the moment time t at the system S_i is located $k_i(t)$ of positive messages, then with probability $\mu_i(t)\min(k_i(t),m_i)p_{i0}(t)\Delta t+o(\Delta t)$ positive message comes out from the network to the external environment, while the total amount of income of QS S_i is reduced by an amount which is equal to $-R_{i0}$, where $M\{R_{i0}\}=b_{i0}$, $i=\overline{1,n}$;
- if at time t at the system S_i is located a positive customer, then after it has been serviced at QS S_i it is headed for S_j again as a positive message with probability $\mu_i(t)\min(k_i(t),m_i)p_{ij}^{+}(t)\Delta t+o(\Delta t)$, $i,j=\overline{1,n}, i\neq j$; in such a transition income of system S_i reduced by the amount r_{ij}, and income of system S_j increased by this amount, where $M\{r_{ij}\}=a_{ij}$, $i,j=\overline{1,n}, i\neq j$;
- with probability $\mu_i(t)\min(k_i(t),m_i)p_{ij}^{-}(t)\Delta t+o(\Delta t)$ a positive message, serviced at the QS S_i, at time t heads for the QS S_j as a negative message $i,j=\overline{1,n}, i\neq j$; by such transition the income of system S_i is reduced by the

amount $\bar{r}_{ij}$, and income of system S_j does not change, where $M\{\bar{r}_{ij}\}=\bar{a}_{ij}$, $i,j=\overline{1,n},\ i\neq j$;

– with probability $1-\sum_{j=1}^{n}\left[\lambda_{0j}^{+}(t)+\left(\lambda_{0j}^{-}(t)+\mu_j(t)\right)\min\left(k_j(t),m_j\right)\right]\Delta t+o(\Delta t)$ on time interval $[t,t+\Delta t)$ there will be no change of system S_i nothing is going to happen (not a positive customer or a negative customer is received and no customer is serviced), in this case, the total income of S_i may increase (decrease) to the amount of $r_i\Delta t$, where $M\{r_i\}=c_i$, $i=\overline{1,n}$.

Suppose that at any instant of time RV r_{0i}, R_{i0}, r_{ij}, $\bar{r}_{ij}$ does not depend on RV r_i. Then we obtain

$$\Delta V_i(t,\Delta t)=\begin{cases} r_{0i}+r_i\Delta t \text{ with probability } \lambda_{0i}^{+}(t)\Delta t+o(\Delta t),\\ -\bar{r}_{0i}+r_i\Delta t \text{ with probability } \lambda_{0i}^{-}(t)\Delta t+o(\Delta t),\\ -R_{i0}+r_i\Delta t \text{ with probability}\\ \quad \mu_i(t)\min\left(k_i(t),m_i\right)p_{i0}(t)\Delta t+o(\Delta t),\\ -r_{ij}+r_i\Delta t \text{ with probability}\\ \quad \mu_i(t)\min\left(k_i(t),m_i\right)p_{ij}^{+}(t)\Delta t+o(\Delta t),\ j=\overline{1,n},\ j\neq i,\\ r_{ji}+r_i\Delta t \text{ with probability}\\ \quad \mu_j(t)\min\left(k_j(t),m_j\right)p_{ji}^{+}(t)\Delta t+o(\Delta t),\ i=\overline{1,n}, i\neq j,\\ -\bar{r}_{ij}+r_i\Delta t \text{ with probability}\\ \quad \mu_i(t)\min\left(k_i(t),m_i\right)p_{ij}^{-}(t)\Delta t+o(\Delta t),\ j=\overline{1,n},\ j\neq i,\\ r_i\Delta t \text{ with probability } 1-\sum_{j=1}^{n}\left[\lambda_{0j}^{+}(t)+\left(\lambda_{0j}^{-}(t)+\mu_j(t)\right)\times\right.\\ \quad \left.\times\min\left(k_j(t),m_j\right)\right]\Delta t+o(\Delta t)\ . \end{cases} \tag{2}$$

As an approximation of the average value of expression $M\min(k_i(t),m_i)$ take $\min(N_i(t),m_i)$, i.e.

$$M\min\left(k_i(t),m_i\right)=\min\left(N_i(t),m_i\right), \tag{3}$$

where $N_i(t)$ ($N_i(t)=M\{k_i(t)\}$) - average number of messages (waiting and serviced) at the system S_i at moment time t, $i=\overline{1,n}$. These relations are obviously satisfied if, for all systems $\forall t\ \ k_i(t)>m_i$, because in that case $\min\left(k_i(t),m_i\right)=m_i$, $i=\overline{1,n}$, or all QS operate under low load conditions, i.e., when $\forall t\ \ k_i(t)\leq m_i$, $i=\overline{1,n}$.

Taking into account (3), similarly as in [4] for the mathematical expectation of income changes we can obtain:

$$
\begin{aligned}
M\{\Delta V_i(t,\Delta t)\} = \Big[& a_{0i}\lambda_{0i}^{+}(t) - \bar{a}_{0i}\lambda_{0i}^{-}(t) + c_i - \\
& - \mu_i(t)\min(N_i(t), m_i)\left(b_{i0}p_{i0}(t) + \sum_{j=1}^{n}\left(a_{ij}p_{ij}^{+}(t) + \bar{a}_{ij}p_{ij}^{-}(t)\right)\right) + \\
& + \sum_{\substack{j=1 \\ j\neq i}}^{n} \mu_j(t)\min(N_j(t), m_j) a_{ji}p_{ji}^{+}(t) \Big]\Delta t + o(\Delta t).
\end{aligned} \tag{4}
$$

Let's describe some special cases of this network operation and situations in which it can be used for modeling. A Distributed Denial of Service (DDoS) attack is an attempt to make an online service unavailable by overwhelming it with traffic from multiple sources. They target a wide variety of important resources, from banks to news websites, and present a major challenge to making sure people can publish and access important information, see [5].

Let us consider the case of a network attack, based on the nonfinite resources ("Building Capacity") of the attacked service on computers (servers) ITN. Attackers build networks of infected computers, known as "botnets", by spreading malicious software through emails, websites and social media. Once infected, these machines can be controlled remotely, without their owners' knowledge, and used like an army to launch an attack against any target. Some botnets are millions of machines strong.

If a great number of queries is organized, it's obvious that computers won't cope with and will be forced to refuse service or keep waiting unacceptably long. There may occur a situation that during some time period the number of the QS network can satisfy the conditions, when are $k_i(t) > m_i > 0$, i.e. $\min(N_i(t), m_i) = m_i$, $i \in X$, where X - set of numbers of QS, for which the given condition is satisfied. Let X_i - set of numbers of QS, associated with the numbers of the QS i, $i \in X$. In this situation, from (4) follows

$$
\begin{aligned}
M\{\Delta V_i(t,\Delta t)\} = \Big[& a_{0i}\lambda_{0i}^{+}(t) - \bar{a}_{0i}\lambda_{0i}^{-}(t) + c_i - \\
& - \mu_i(t)m_i\left(b_{i0}p_{i0}(t) + \sum_{j\in X_i}\left(a_{ij}p_{ij}^{+}(t) + \bar{a}_{ij}p_{ij}^{-}(t)\right)\right) + \\
& + \sum_{\substack{j\in X_i \\ j\neq i}} a_{ji}\mu_j(t)m_j p_{ji}^{+}(t) \Big]\Delta t + o(\Delta t),\ i = \overline{1, X}.
\end{aligned} \tag{5}
$$

In another situation, in the operation of computers infected with a malicious Trojan that allows attackers to remotely manage other people's computers without their owners (to manage the processing of requests and packets transmitted information), it will be activated after a random time for the attacker's command ("botnets") and destroys a large number of requests and packages. In this case, the simulation can be assumed that at some time intervals and for some systems the network average is observes bursts and the conditions $\min(N_i(t), m_i) = N_i(t)$, $i = \overline{1,n}$, $i \in Y$, where Y - set of QS numbers, which satisfy the condition described above. Let also Y_i - set of system numbers, associated with the number of the QS i, $i \in Y$. Then from (4) is follows that

$$M\{\Delta V_i(t,\Delta t)\} = \Bigg[a_{0i}\lambda_{0i}^{+}(t) - \overline{a}_{0i}\lambda_{0i}^{-}(t) + c_i - \\ - \mu_i(t)N_i(t)\left(b_{i0}p_{i0}(t) + \sum_{j\in Y_i}\left(a_{ij}p_{ij}^{+}(t) + \overline{a}_{ij}p_{ij}^{-}(t)\right)\right) + \\ + \sum_{\substack{j\in Y_i \\ j\neq i}} a_{ji}\mu_j(t)N_j(t)p_{ji}^{+}(t)\Bigg]\Delta t + o(\Delta t),\ i = \overline{1,Y}. \tag{6}$$

Describe how you can find the average number of customers in the QS network. So far as to the network there constantly arrive positive messages according to Poisson process of rate $\lambda_{0i}^{+}(t)$ and negative messages which constitute a Poisson process of rate $\lambda_{0i}^{-}(t)$, the average number of messages, received from outside the system S_i during time Δt equals $\left(\lambda_{0i}^{+}(t) + \lambda_{0i}^{-}(t)\right)\Delta t$, $i = \overline{1,n}$. Denote by $\rho_i(t)$ - the average number of occupied service lines in the system S_i at time t, $i = \overline{1,n}$. Then, it is obvious that $\mu_i(t)\rho_i(t)\Delta t + o(\Delta t)$ - the average number of messages have left the system S_i during time Δt, and $\sum_{\substack{j=1 \\ j\neq i}}^{n}\mu_j(t)\rho_j(t)p_{ji}^{+}(t)\Delta t + o(\Delta t)$ - the average number of positive messages received to S_i from the other QS during time Δt, $\sum_{\substack{j=1 \\ j\neq i}}^{n}\mu_j(t)\rho_j(t)p_{ji}^{-}(t)\Delta t + o(\Delta t)$ - the average number of negative messages received to S_i from others QS during time Δt, $i = \overline{1,n}$. Therefore

$$N_i(t+\Delta t) - N_i(t) = \left(\lambda_{0i}^{+}(t) + \lambda_{0i}^{-}(t)\right)\Delta t + \\ + \sum_{\substack{j=1 \\ j\neq i}}^{n}\mu_j(t)\rho_j(t)p_{ji}^{+}(t)\Delta t + \sum_{\substack{j=1 \\ j\neq i}}^{n}\mu_j(t)\rho_j(t)p_{ji}^{-}(t)\Delta t - \mu_i(t)\rho_i(t)\Delta t =$$

$$=\left[\lambda_{0i}^{+}(t)+\lambda_{0i}^{-}(t)+\sum_{\substack{j=1\\ j\neq i}}^{n}\mu_j(t)\rho_j(t)\left(p_{ji}^{+}(t)+p_{ji}^{-}(t)\right)-\mu_i(t)\rho_i(t)\right]\Delta t,\ i=\overline{1,n},$$

whence by $\Delta t\to 0$ follows a system of ordinary differential equations (ODE) for $N_i(t)$:

$$\frac{dN_i(t)}{dt}=\sum_{\substack{j=1\\ j\neq i}}^{n}\mu_j(t)\rho_j(t)\left(p_{ji}^{+}(t)+p_{ji}^{-}(t)\right)+ \\ +\lambda_{0i}^{+}(t)+\lambda_{0i}^{-}(t)-\mu_i(t)\rho_i(t),\ i=\overline{1,n}. \tag{7}$$

The value $\rho_i(t)$ is impossible to find exactly and therefore, as we have done before, we approximate it by the expression

$$\rho_i(t)=\begin{cases}N_i(t),\ N_i(t)\le m_i,\\ m_i, N_i(t)>m_i,\end{cases}=\min(N_i(t),\ m_i),\ i=\overline{1,n}.$$

Then the system of equations (7) takes the form

$$\frac{dN_i(t)}{dt}=\sum_{\substack{j=1\\ j\neq i}}^{n}\mu_j(t)\left(p_{ji}^{+}(t)+p_{ji}^{-}(t)\right)\min\left(N_j(t),m_j\right)- \\ -\mu_i(t)\min\left(N_i(t),m_i\right)+\lambda_{0i}^{+}(t)+\lambda_{0i}^{-}(t),\ i=\overline{1,n}. \tag{8}$$

It is a system of inhomogeneous linear ODE with discontinuous right-hand sides. It should be addressed by dividing the phase space into a number of areas and finding solutions to each of them. System (8) can be solved, for example, using the tools of computer mathematics Maple or Mathematica [6, 7].

We introduce the notation $v_i(t)=M\{V_i(t)\}$, $i=\overline{1,n}$. From (1) and (4) we obtain inhomogeneous linear first order ODEs for incomes of systems in the network:

$$\frac{dv_i(t)}{dt}=-\mu_i(t)\min\left(N_i(t),m_i\right)\left(b_{i0}p_{i0}(t)+\sum_{j=1}^{n}\left(a_{ij}p_{ij}^{+}(t)+\overline{a}_{ij}p_{ij}^{-}(t)\right)\right)+ \\ +\sum_{\substack{j=1\\ j\neq i}}^{n}a_{ji}p_{ji}^{+}(t)\mu_j(t)\min\left(N_j(t),m_j\right)+a_{0i}\lambda_{0i}^{+}(t)-\overline{a}_{0i}\lambda_{0i}^{-}(t)+c_i,\ i=\overline{1,n}.$$

Setting the initial conditions $v_i(0) = v_{i0}$, $i = \overline{1,n}$, you can find the expected incomes of systems in the network. Thus

$$v_i(t) = v_{i0}(0) + \int_0^t f_i(\tau)\,d\tau\,, \tag{9}$$

where

$$f(\tau) = -\mu_i(\tau)\min(N_i(\tau), m_i)\left(b_{i0}p_{i0}(\tau) + \sum_{j=1}^{n}\left(a_{ij}p_{ij}^{+}(\tau) + \overline{a}_{ij}p_{ij}^{-}(\tau)\right)\right) +$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{n} a_{ji}p_{ji}^{+}(\tau)\mu_j(\tau)\min(N_j(\tau), m_j) + a_{0i}\lambda_{0i}^{+}(\tau) - \overline{a}_{0i}\lambda_{0i}^{-}(\tau) + c_i\,,\ i = \overline{1,n}\,.$$

For two special cases when we have the formulas (5) and (6), we obtain, respectively,

$$\begin{aligned} v_i(t) = v_{i0}(0) + \int_0^t \Bigg[& -m_i\mu_i(\tau)\left(b_{i0}p_{i0}(\tau) + \sum_{j\in X_i}\left(a_{ij}p_{ij}^{+}(\tau) + \overline{a}_{ij}p_{ij}^{-}(\tau)\right)\right) + \\ & + \sum_{\substack{j\in X_i\\ j\neq i}} a_{ji}\mu_j(\tau)m_j p_{ji}^{+}(\tau) + a_{0i}\lambda_{0i}^{+}(\tau) - \overline{a}_{0i}\lambda_{0i}^{-}(\tau)\Bigg]d\tau + c_i t\,,\ i = \overline{1,X}\,, \end{aligned} \tag{10}$$

and

$$\begin{aligned} v_i(t) = v_{i0}(0) + \int_0^t \Bigg[& -\mu_i(\tau)\left(b_{i0}p_{i0}(\tau) + \sum_{j\in Y_i}\left(a_{ij}p_{ij}^{+}(\tau) + \overline{a}_{ij}p_{ij}^{-}(\tau)\right)\right)N_i(\tau) + \\ & + \sum_{\substack{j\in Y_i\\ j\neq i}} a_{ji}\mu_j(t)N_j(t)p_{ji}^{+}(t) + a_{0i}\lambda_{0i}^{+}(\tau) - \overline{a}_{0i}\lambda_{0i}^{-}(\tau)\Bigg]d\tau + c_i t\,,\ i = \overline{1,Y}\,, \end{aligned} \tag{11}$$

and $N_i(t)$ can be found from the system of ODE

$$\begin{aligned} \frac{dN_i(t)}{dt} = & \sum_{\substack{j=1\\ j\neq i}}^{n} \mu_j(t)\left(p_{ji}^{+}(t) + p_{ji}^{-}(t)\right)N_j(t) - \\ & -\mu_i(t)N_i(t) + \lambda_{0i}^{+}(t) + \lambda_{0i}^{-}(t)\,,\ i = \overline{1,n}\,. \end{aligned} \tag{12}$$

3. Examples of finding the expected incomes

Example 1. Consider the following model of ITN, Figure 1, when the network receives the negative messages (viruses). QS S_4 corresponds to a central service facility (which may be a central server or a cluster of central (main) computer or group of computers), if $m_4 > 1$, system S_i corresponds to a group of the peripheral computers, if $m_i > 1$, $i = \overline{1,3}$.

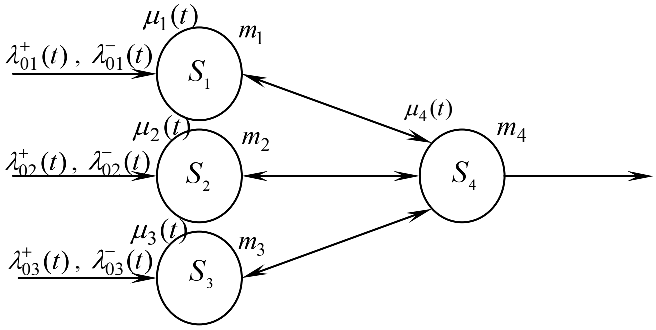

Fig. 1. ITN model

We define the network settings. Let $T = 10$, the intensity of input flow of positive and negative messages equal respectively, $\lambda^+_{01}(t) = 0.1\sin^2 t$, $\lambda^+_{02}(t) = 0.7\cos^2 t$, $\lambda^+_{03}(t) = 0.2\sin^2 t$, $\lambda^+_{04}(t) = 0$, $\lambda^-_{01}(t) = 0.1\cos^2 t$, $\lambda^-_{02}(t) = 0.7\sin^2 t$, $\lambda^-_{03}(t) = 0.2\cos^2 t$, $\lambda^-_{04}(t) = 0$. The intensity of service messages equal $\mu_1(t) = 3\sin^2 2t$, $\mu_2(t) = 0.5\cos^2 t$, $\mu_3(t) = \cos^2 t$, $\mu_4(t) = 3$. The count of service lines equals $m_1 = 3$, $m_2 = 4$, $m_3 = 1$, $m_4 = 4$. We define the transition probabilities of positive messages in the form of $p^+_{14}(t) = \cos^2 t$, $p^+_{24}(t) = \sin^2 t$, $p^+_{34}(t) = \cos^2 4t$, $p^+_{41}(t) = 0.1\sin^2 3t$, $p^+_{42}(t) = 0.2\cos^2 t$, $p^+_{43}(t) = 0.5\sin^2 3t$, and the transition probabilities in the form of negative messages $p^-_{14}(t) = \sin^2 t$, $p^-_{24}(t) = \cos^2 t$, $p^-_{34}(t) = \sin^2 4t$, others equal zero. Then the probability of the messages from the network to the external environment from the QS S_4 equals $p_{40}(t) = 1 - 0.2\cos^2 t - 0.5\cos^2 3t - 0.1\sin^2 3t$. Let the values we need for mathematical expectations equal: $a_{01} = 100000$, $a_{02} = 200000$, $a_{03} = 300000$, $\overline{a}_{01} = 10000$, $\overline{a}_{02} = 30000$, $\overline{a}_{03} = 50000$, $b_{40} = 300000$, $a_{14} = 200000$, $a_{24} = 150000$, $a_{34} = 300000$, $a_{41} = 300000$, $a_{42} = 200000$, $a_{43} = 500000$, $\overline{a}_{14} = 350000$, $\overline{a}_{24} = 200000$, $\overline{a}_{34} = 400000$, $c_1 = 10000$, $c_2 = 20000$, $c_3 = 30000$, $c_4 = 70000$, other equals zero. Suppose also that incomes at the initial time is zero, i.e. $v_{i0}(0) = 0$, $i = \overline{1,n}$. The package of mathematical calculations "Mathematica" found a numerical solution of the ODE system (8)

[6] for the average number of messages in the network systems. Then, using (9) found the expected incomes of the QS network. Figure 2 shows a graph of the expected incomes of systems in the network, built in the package "Mathematica":

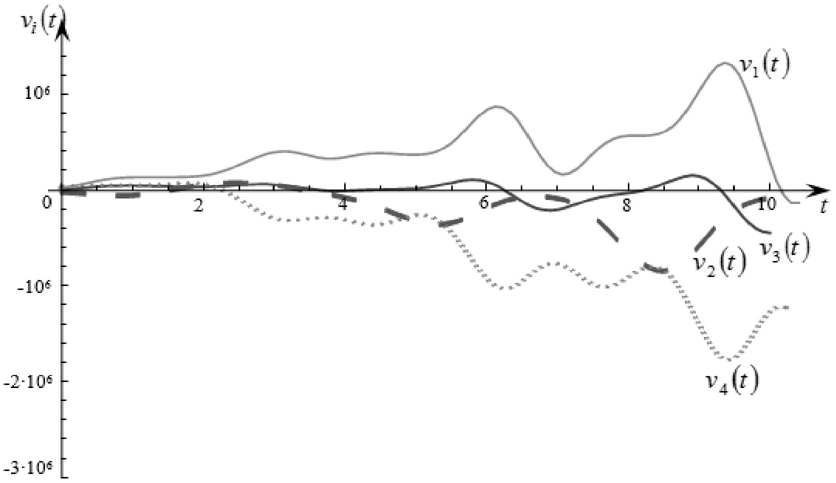

Fig. 2. Income changes at QS S_i, $i=\overline{1,4}$

Example 2. Consider the model of the ITN from Example 1 in the case of computer DDoS-attacks on the central server over a time interval when the attack on the server is particularly strong. In this case, we can assume that in the peripheral network QS there are no observed queues in the average, i.e. $\min(N_i(t),m_i)=N_i(t),\ i=\overline{1,n-1}$, and the central QS operates under high loads and it is satisfied by $\min(N_n(t),m_n)=m_n$. System (8) in this case can be rewritten as

$$\begin{cases} \dfrac{dN_i(t)}{dt}=-\mu_i(t)N_i(t)+\mu_4(t)m_4\left(p_{4i}^+(t)+p_{4i}^-(t)\right)+\lambda_{0i}^+(t)+\lambda_{0i}^-(t),\ i=\overline{1,3}, \\ \dfrac{dN_4(t)}{dt}=\sum\limits_{i=1}^{3}\mu_i(t)N_i(t)-\mu_4(t)m_4, \end{cases} \tag{13}$$

and the expressions for the system incomes in the network, taking into account (10) and (11), have the form

$$v_i(t)=\int_0^t\left[-\mu_i(\tau)\left(a_{i4}p_{i4}^+(\tau)+\overline{a}_{i4}p_{i4}^-(\tau)\right)N_i(\tau)+\right.$$
$$\left.+a_{4i}\mu_4(\tau)m_4p_{4i}^+(\tau)+a_{0i}\lambda_{0i}^+(\tau)-\overline{a}_{0i}\lambda_{0i}^-(\tau)\right]d\tau+c_it\,,\ i=\overline{1,3}\,, \tag{14}$$

$$v_4(t)=\int_0^t\left[-m_4\mu_4(\tau)\left(b_{40}p_{40}(\tau)+\sum_{j=1}^{3}\left(a_{4j}p_{4j}^{+}(\tau)+\overline{a}_{4j}p_{4j}^{-}(\tau)\right)\right)+\right.$$

$$\left.+\sum_{\substack{j=1\\ j\neq i}}^{3}a_{j4}\mu_j(\tau)N_j(\tau)p_{j4}^{+}(\tau)\right]d\tau+c_4t\,. \quad (15)$$

Let the intensity of the input flow of positive and negative messages respectively equal $\lambda_{01}^{+}(t)=\sin^2 t$, $\lambda_{02}^{+}(t)=\cos^2 t$, $\lambda_{03}^{+}(t)=0.1\cos^2 t$, $\lambda_{01}^{-}(t)=\cos^2 t$, $\lambda_{02}^{-}(t)=\sin^2 t$, $\lambda_{03}^{-}(t)=0.2\cos^2 t$, $\lambda_{04}^{+}(t)=\lambda_{04}^{-}(t)=0$, and the service intensity equals $\mu_1(t)=\sin^2 t$, $\mu_2(t)=\cos^2 t$, $\mu_3(t)=0.2\sin^2 t$, $\mu_4(t)=0.1$. Let the count of service lines equal $m_1=1$, $m_2=3$, $m_3=2$, $m_4=2$. The transition probabilities of positive messages given in the form $p_{14}^{+}(t)=p_{24}^{+}(t)=p_{34}^{+}(t)=0.2$, $p_{41}^{+}(t)=p_{42}^{+}(t)=p_{43}^{+}(t)=0.02$, and the transition probabilities of negative messages in the form of $p_{14}^{-}(t)=p_{24}^{-}(t)=p_{34}^{-}(t)=0.5$, $p_{41}^{-}(t)=p_{42}^{-}(t)=p_{43}^{-}(t)=0.001$, other probabilities equal zero. Then the probability of the messages from the network to the external environment from teh QS S_4 equals $p_{40}(t)=0.937$.

Let income at the initial time equal $v_{i0}=0$, $i=\overline{1,n}$. Then using (13)-(15), in the package "Mathematica" numerical solutions were obtained for the average number of messages and the expected incomes of network QS and graphs of incomes of these systems.

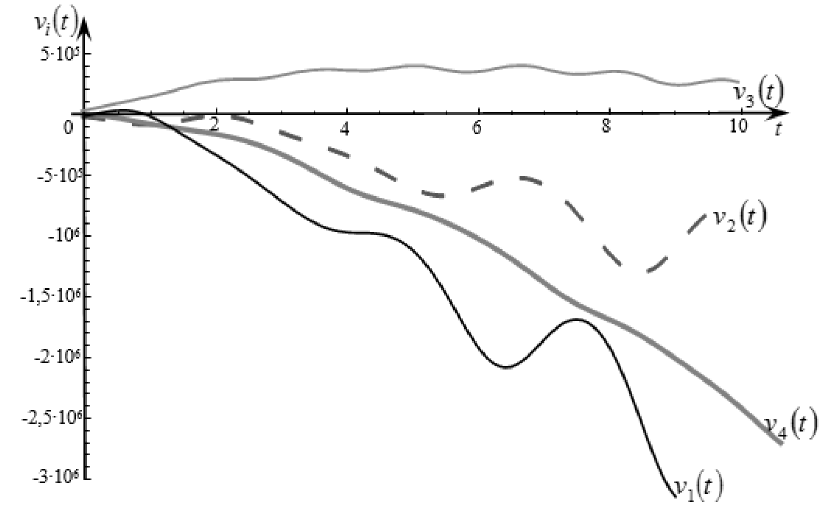

Fig. 3. Income changes at QS S_i, $i=\overline{1,4}$

The central server is configured as a high load regime and the number of messages to it much more than service channels of the server. In the peripheral computers there are no observed queueing requests in the average. Comparing the income charting of network QS (see Figs. 2 and 3), we can make the following conclusions. According to the graph it is clear that the costs in the case of powerful computer attacks on the central server exceed costs in the case, where viruses are present in the ITN (negative messages). The central server in the case of a powerful computer attack costs are rising faster. But it should be noted that in a state when at the ITN is circulating viruses, it also incur losses to the central server and peripheral computers. Computer Peripheral spending more when it is a powerful DDoS-attack, than in the case of the presence of viruses in the network.

Conclusion

In the article there was investigated an open Markovian G-network with incomes and many-lines queueing systems. The intensity of the incoming streams of positive and negative messages, the intensity of its service and the transition probabilities between messages at QS are time-dependent. The incomes from the state transition network is a random variable with given mean values. A method of finding the expected incomes in many-lines network systems were described. The obtained results can be used for the modeling of income changes at the ITN at DDoS-attacks on it, and also for forecasting of costs, considering the viruses penetration. Further studies in this direction will be associated with finding of expected incomes at such networks with different types of messages.

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