ASSOCIATION AUTOMATA FUNCTIONING ANALYSIS

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Abstract. In this aricle we analyze a mechanism for automatic reactions to environment changes. In this case, an association automaton should react not only upon a known situation structure but should also create new associations with their properties (attributes). We assume that all perceptions can be sent to all so-called nodes of association. Every association is represented by its name. Every association is evoked by a set of generators based on different sets of attributes. The analysis regards the life, creation and annihilation of associations and the influences on their changes in perception.

Introduction

An association automaton is a net which can be described by a 3-tuple $AS = \{S,P,B\}$, where WS - association nodes set, P - perceptor, B - association background. The net has connections between the preceptor, all the nodes and the background. Every node can be represented by a 3-tuple [1, 2] describing its structure and states of vitality: $WS(i) = \{\Sigma(i),G(i),T(i)\}$, where $\Sigma(i)$ - alphabet of i-th associa-tion attributes, $G(i)$ - association generators, $T(i)$ - time function of association disappearance [3]. The situation where a preceptor is connected in a directed way and the background is connected in an undirected way with all the association nodes is difficult to clearly present [4]. Obviously, the nodes are connected among them in an undirected way. Hence, we present this complicated system of connections with overlapping areas representing specified structure elements (layers). The direction of activation of these layers is described by the following implication form: $P\rightarrow B$, $P\rightarrow WS(i)$, $B\rightarrow WS(i)$, $WS(i) \rightarrow WS(j), i,j = 1,2,\ldots n$.

1. Association probability distribution analysis

Let us start with perception distribution. Observed attributes are sent to the association background. The observation process can be simulated with the elimination method whose effects are presented in Figure 1. This method consists in exploiting a given discrete, empiric probability distribution of attributes $p(j)$, where j - number of attributes, generating random values $c(j)$ and checking condition $c(j) \le p(j)$ [5].

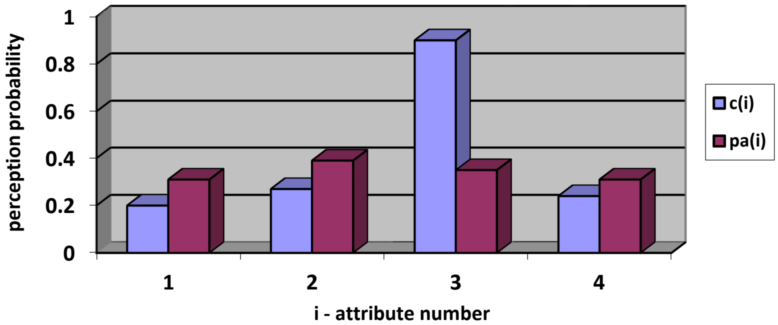

Fig. 1. Attribute choice with help of elimination method: attribute acceptation under condition: *if* $c(j) \le p(j)$ then $\alpha(j) = 1$ *else* $\alpha(j) = 0$. Here *at*(1), *at*(2) and *at*(4) have been chosen

The associations probability density is estimated by exploiting the probability of their creation:

$$ps(i) = 1/lg(i) \sum_{k=1}^{lg(i)} \prod_{j=1}^{la} \alpha(j)\beta(i,j,k)\text{, or} \tag{1}$$

$$ps(i) = 1/lg(i) \sum_{k=1}^{lg(i)} \prod_{j=1}^{la} p(j)\beta(i,j,k)$$

where:
i - association number, *j* - attribute number, *k* - association generator number, *ps*(*i*) - generation probability of *i*-th association, *lg*(*i*) - generator number for *i*-th association, *ls* - current number of evoked associations, *la* - attribute number in alphabet Σ, $\alpha(j)$ - perception coefficient of *j*-th attribute (inherent in the background), binary variable {0,1}, $\beta(i,j,k)$ - coefficient of participation *j*-th attribute in an argument set of *k*-th generators and for *i*-th association, it is binary variable {0,1}; *if* $at(j) \in g(i,k)$ *then* $\beta(i,j,k) = 1$ *otherwise* $\beta(i,j,k) = 0$, where $g(i,k)$ - *k*-th generator for *i*-th association.

Obviously, we have situations when the preceptor sent to the background attributers does not evoke any associations. In this case

$$\mathrm{P}\left(\bigcup_{i=1}^{ls} NS(i)\right) \le 1 \tag{2}$$

where *NS*(*i*) - *i*-th association.

The process of association appearance is presented in Figure 2.

For the given state of background associations, activation is realized in one of several stages (before the next stage background is enriched with associations activated in the previous stage). Therefore, in the simulation process we have to separate and count the stages:

$$B = B\bigcup_{d=1}^{ld} NS^{(d)} \tag{3}$$

where: d - number of association stage, ld - depth of association, $NS^{(d)}$ - set of new associations generated on stage d.

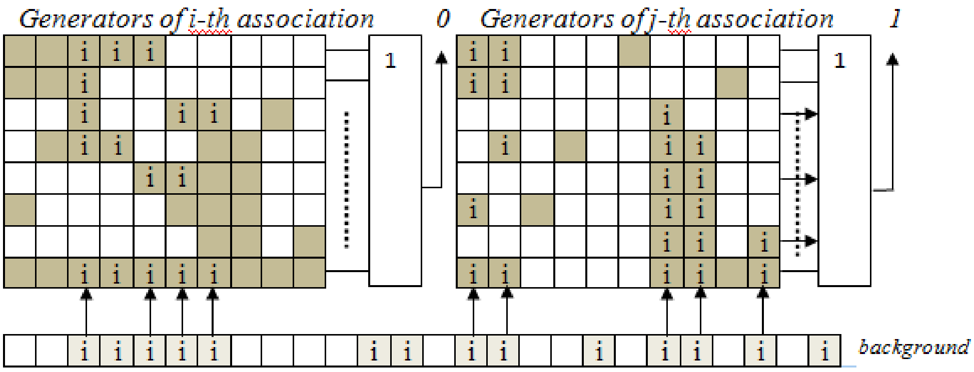

Fig. 2. Conception associations generation procedure; - i-th lack of association, - j-th - association appearance

We do it with the help of an algorithmic structure, assuming that until all associations for a given background state are generated, new associations cannot be included in the background register. It is adequate to perform one step of association depth $ld = ld + 1$. At the beginning, we assume that the background elements are sent to all the association nodes and the issues of these nodes are blocked according to the possibilities of a supplement background state and sending new associations to the next nodes. Hence, even if the background contains the sum of alphabets of all association nodes $\bigcup_{i=1}^{n}\sum_{i}$, the first stage is equal to the first depth step. After this operation before the next stage, the background is supplemented by new associations which appear in the previous stage (Figs 3, 4).

After a new association generation which is absent in the association background, will be sent to it with the characteristic probability function $pa(j) = ps(k)$, where: j - number of association, k - sequenced attribute number in the background. If such an association is present in the background structure, its probability function will be corrected in the following way: $pa(j) = pa(j) + ps(k) - pa(j)ps(k)$. The probability distribution will be corrected after every stage regarding the actual vector of attributes with stable levels of utility probabilities.

The association generation process consists of stages whose number is defined by chosen criteria [5-7]. Limit MP (product of association number and maximal association depth), the limit of attributes number (wool-gathering), limit of associations number, time of association vitality (lack of refreshing processes), limit of generations stages etc. can be chosen as criteria.

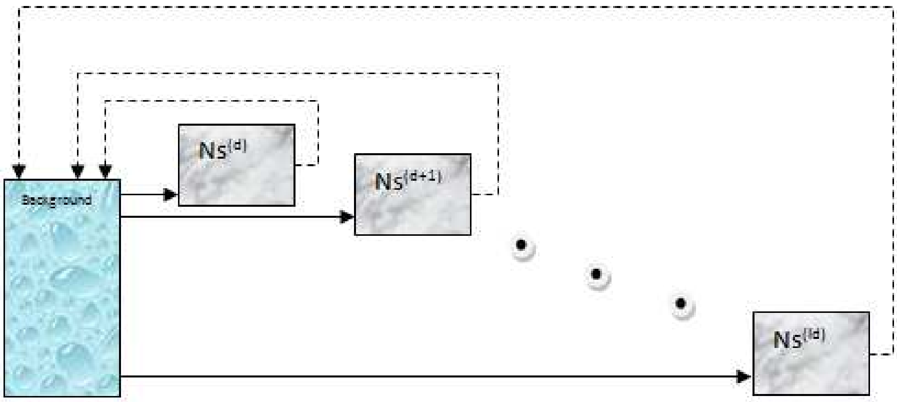

Fig. 3. Stages of association depth step - background is supplemented by new associations generated in previous stage (dashed lines)

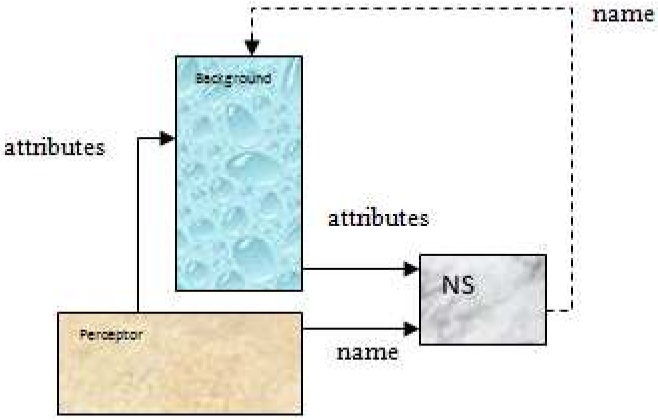

Fig. 4. New associations achieved from external environment

2. Association automata functioning algorithm

This algorithm consists of the following stages:

1. Data initiation
   - attributes and associations in background
   - parameters of attributes and association vitality in background
   - present associations
   - arguments of declared association generators
   - parameters of attributes and associations vitality beyond background
   - parameters of stop criteria of automaton functioning
2. Start of association generation process, *stage:* = 1
3. Perception phase - background of new attributes supplement
4. Association generations

5. Corrections and checking of associations vitality beyond background
6. Background supplementation with newly-generated associations
7. Background elements probability distribution corrections
8. Corrections and checking of background elements vitality
9. Stop functioning automaton criteria checking, *stop* or back to new stage *etap*=*etap*+1 (item 3.)
10. Stop.

Using the above description, we can investigate the activation process of automaton elements and the stages of probability distribution modification of the background arguments. In the first stage, the element structure of the background and nodes is defined (Fig. 5).

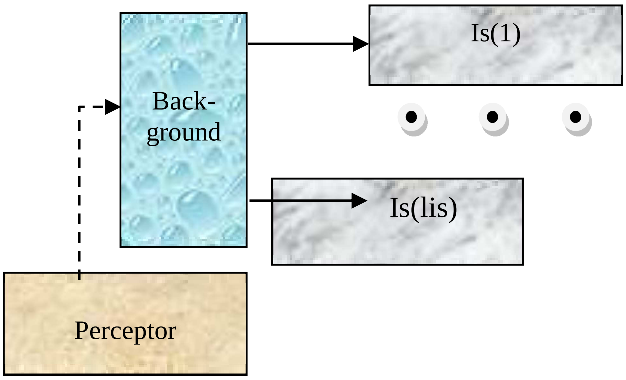

Fig. 5. Initial association structure - starting stage (without dashed lines), *Is*(*i*) - initial association node, *lis* - number of nodes

For example, when in the background we have four attributes (or association names), the probability distribution is created for them and it can look like in Figure 6. The perception phase consists in sending new or the same elements to the perceptor and from it to the background. The probabilities of the same elements are summed ("overlapped"), according to the above-presented rule, using the expressions: $pa(j) = pa(j) + pp(k) - pa(j)pp(k)$, where $pp(k)$ - attribute k perception probability. It leads to modification of the distribution, as for instance it gives the effect presented in Figure 7.

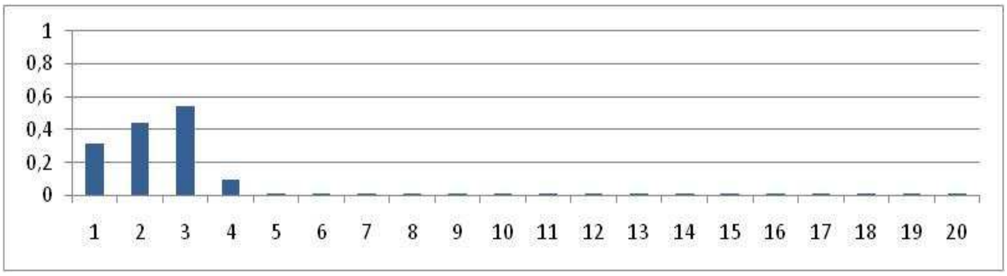

Fig. 6. Probability distribution of background elements for initial data

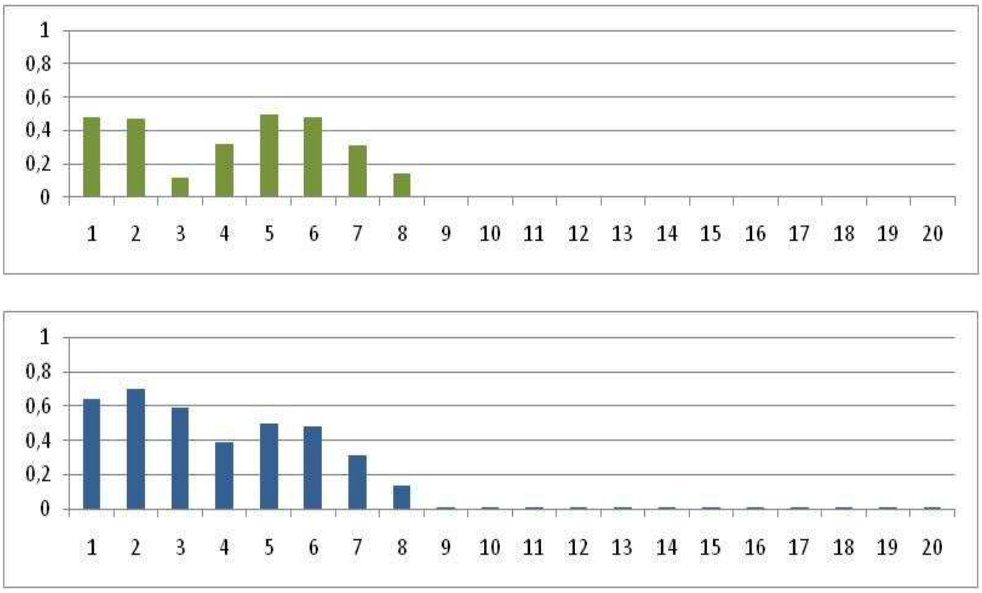

Fig. 7. Probability distribution for perception attributes (upper diagram) and result of probabilities modifications according to initial data from Figure 10 (lower diagram)

In Figure 7 attributes $a(1)$ - $a(8)$ appear as a result of perception (which refers to activation of the connection presented in form of a dashed line in Figure 5). The first four elements appear again. It induces probabilities modifications (in initial data phase). The remaining elements take over the probability characteristics from the perceptor (because $pa(i) = 0$ in left part of equation).

Let us go to the generation association phase on the basis of the background attributes and hitherto existing associations (Fig. 8). The procedure of vitality association corrections is connected with reading present, refreshing and remembering time parameters. The vitality correction is based on the appearing association (or attribute) moment.

The stage of background supplementation based on the scheme is presented in Figure 9. The new associations depth steps are created directly or indirectly. Let us introduce the definition of a new association step.

Definition: *A new association step is created by a minimum of one generation in a set of existing but not activated association nodes beyond the background.*

Generation is realized on the basis of arguments consisting of activated associations beyond the background and attributes from the background. Usually we have (Fig. 10):

$$\exists_{i,d}\{Ns(i)=\arg(G(d))\}\wedge\exists_{j}\{at(j)\in B; at(j)=\arg(G(d))\} \tag{4}$$

where $\arg(G(d))$ - generator argument for new step association d.

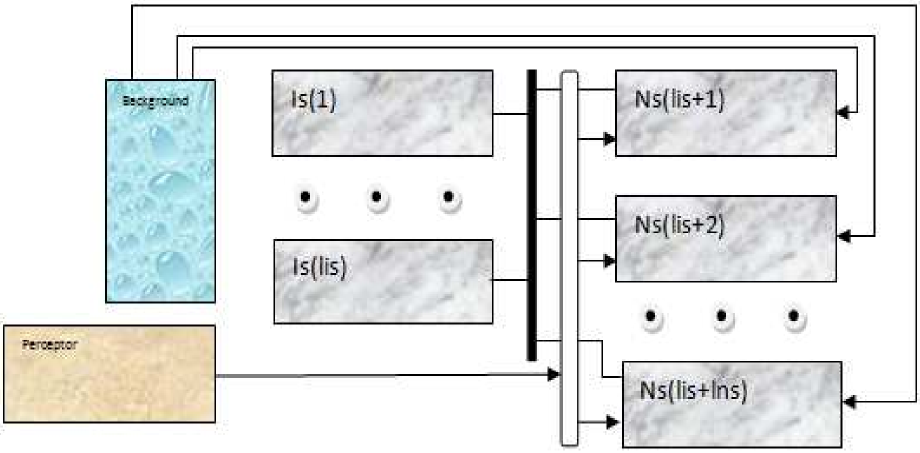

Fig. 8. Idea scheme for new associations *NS*(*i*) (*lns* - new associations number) generation

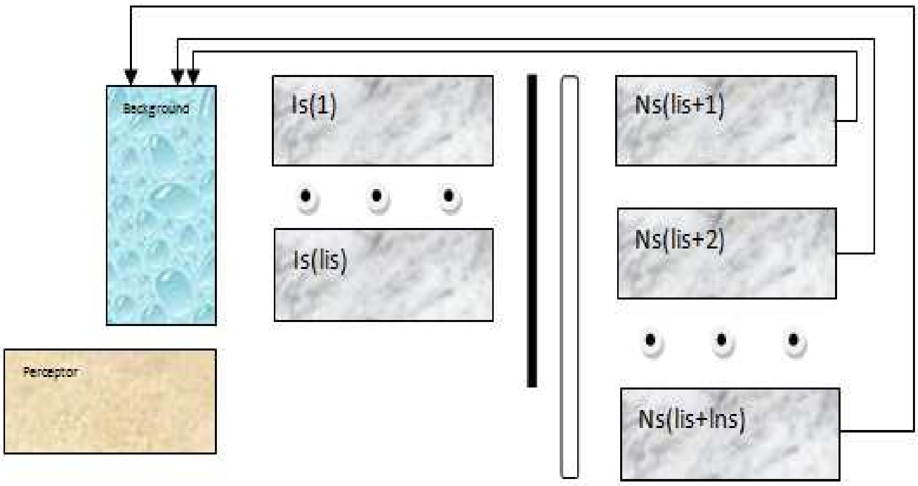

Fig. 9. Parts scheme activated for sending new associations to background

An indirectly created new association step engages only the attributes or associations from the perceptor and nodes beyond the background (Fig. 11)

$$\exists_{i,d}\{Ns(i)=\arg(G(d))\}\wedge \text{not}\exists_j\{at(j)\in B; at(j)=\arg(G(d))\} \tag{5}$$

The indirection infers from the undirected exploitation of background elements which causes a chronological previous step of associations. In both cases, the perceptor optionally partakes in the creation of a new step of associations (dashed line in Fig. 11).

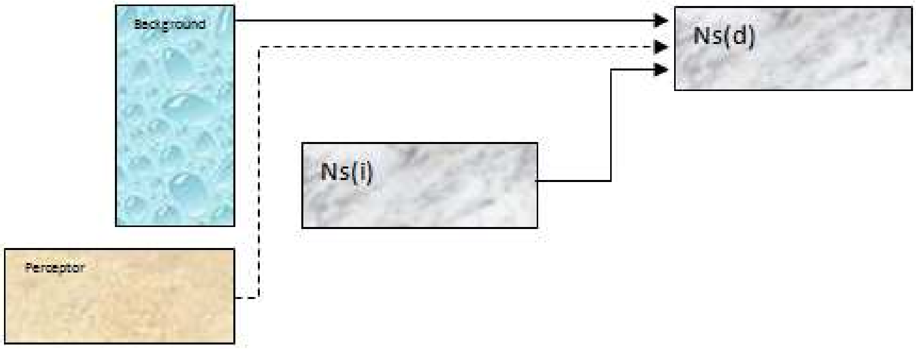

Fig. 10. Direct way of creating new association step

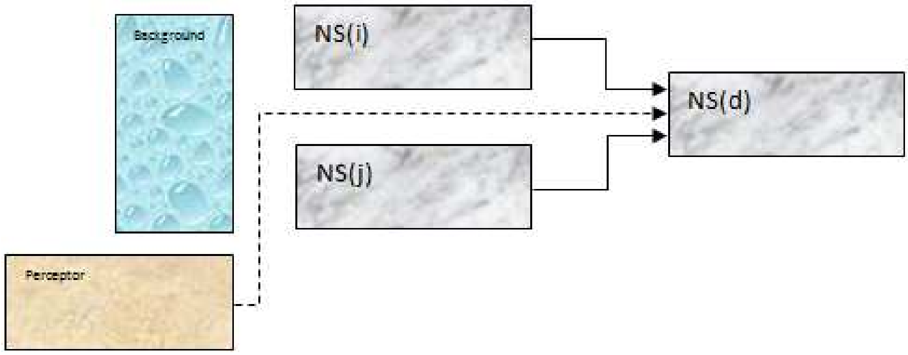

Fig. 11. Direct way of creating new association step

3. Feature of elements of association automata functioning and structure

The perception of attributes and associations mutually overlap similarly to background elements. It is connected with probability modification relating to the environment elements influence. To perform it we use the typical Büchi automata description convention [1, 8].

$$ap \wedge \Diamond ap, \text{ where } ap = \text{observed attribute},$$

$\Diamond$ - eventually.

Therefore, perception is realized according the rule:

$$P = O(ap(1) \wedge \Diamond ap(1)) \wedge \ldots \wedge O(ap(|\Sigma|) \wedge \Diamond ap(|\Sigma|)) \tag{6}$$

where O - next.

We can describe it as the following perception of attributes or associations independent of their previous appearance. Besides the probability distribution correction for the given background elements, refreshing procedures can be realized simultaneously. Adding elements to the background is performed by the convention:

$$B = \Diamond\, (ap(1) \wedge \Diamond ap(1)) \wedge \ldots \wedge \Diamond (ap(|\Sigma|) \wedge \Diamond ap(|\Sigma|)) \tag{7}$$

where $|\Sigma|$- the number of alphabet elements for all associations.

It can be interpreted as follows: element $ap(i)$ absent in the background and not sent from the perceptor or present in background and was sent or not from the preceptor.

The generation of association is realized on the basis of perception, background contents and other associations:

$$(Ns,i) \models c \quad \text{iff } (\exists_{j;} \forall_{k=1,\ldots,la(i,j)} \ \arg(g(i,j,k)) \in (P \vee B \vee NS) \tag{8}$$

where c - creation (association evocation), $la(i,j)$ - the number of arguments of j-th generator for i-th association, NS - set of hitherto existing associations, $\arg(g(i,j,k))$ - k-th argument of j-th generator for i-th association.

We can describe it as an association appearing if and only if when all the arguments for even one generator are activated (are sent to a given node by perceptor, background or other existing associations).

The number of associations increments when:

$$\{(Ns,i) \ \models c \ \text{ and } \ (Ns,i) \notin NS\} \Rightarrow ls = ls+1. \tag{9}$$

The creation of a new association is described by:

$$(Ns,i) \ \models \ nc \ \ \text{iff } \{(\forall_{k=1,\ldots,la(i,1)} \ \arg(g(i,1,k)) \in (B \vee NS) \ \text{ and } (Ns,i) \in (P)\} \tag{10}$$

where nc - new association creation.

We describe it as follows: the first generator is created with the help of arguments obtained from the background and hitherto existing associations, and the name of the new association is sent from the perceptor. In the proposed association, the following rule functions in the automata:

Rule: The association name is sent from the perceptor to the nodes and only then to the background:

$$(Ns,i) \ \models \ nc \ \Rightarrow P \rightarrow (Ns,i) \Rightarrow (Ns,i) \rightarrow B \tag{11}$$

The addition of a new feature (new attribute) to the association in the proposed automaton is connected with increasing the number of its generators on $lg(i)$ (which was earlier presented) and can be described as follows:

$$(G,j) \ \models nc \ \text{ iff } \forall_{j=1,\ldots,lg,\ k=1,\ldots,la(i,j)} \ \arg(g(i,j+lg,k)) = \arg(g(i,j,k)) \text{ and}$$

$$\arg(g(i,j+lg,k+1)) = nat(i) \tag{12}$$

where $nat(i)$ - new attribute (feature) of i-th association.

In this case the number of new generators will be greater by one than their number in the based set.

$$\forall_{j=lg+1,\dots,2lg}\ la(i,j) = la(i,j-lg)+1. \tag{13}$$

If the new attribute contains only one generator of a given association node then new generators will not be created:

$$(G,j) \nvDash nc \quad \text{if } \exists_{j,k}\ nat(i) = \arg(g(i,j,k)) \tag{14}$$

The association depth step is incremented when:

$$\{(Ns,i) \vDash nc \ \text{ and } \ (\exists_j \exists_k \exists_{l_{\max}} \arg(g(i,j,k)) = Ns(l_{\max})\} \Rightarrow d(i) = d(l_{\max})+1 \tag{15}$$

where l_{max} - number of hitherto activated associations with the greatest depth step $d(l)$, - always.

This situation is presented as follows: the depth of association is always incremented when a newly activated association based on even one argument existing and previously activating an association. We chose the argument with the greatest depth step $d(l)$. The new depth step is assigned to the newly created association. Hence, we can finally ascertain that: $d = \max_{p=1,\dots,ls}\{d(p)\}$. The sequence of mutually evoking up-to-date associations $AS \in NS$ create a so-called profile or tree of association depth ((Fig. 12):

$$(AS) \vDash u \ \ AS = \cup\{(Ns,i) \vDash mc\} \tag{16}$$

where: u - up-to date, mc - mutual creation.

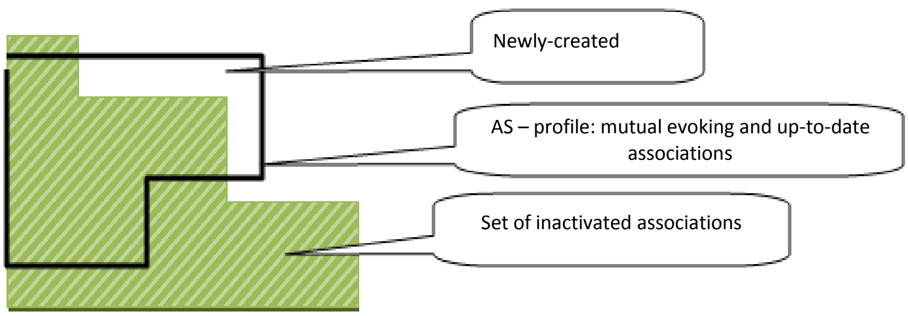

Fig. 12. Profile of association depth as a sum of new and old associations in mutually created and evoked structure

Conclusions

New association depth steps are created directly or indirectly. Let us introduce the definition of a new association step. Using the above description we can investigate the activation process of automaton elements and stages of probability distribution modification of background arguments.

The proposed automata can be systematically structurally developed and enriched with new attributes inside the nodes. The probabilistic characteristics of particular parts (background, nodes etc.) elements (attributes, associations) dynamically changes during the exploitation process.

In aim to refer to natural memory and association systems [5], the parameters of association vitality and the external factors influencing them are introduced.

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