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NUMERICAL MODELING OF TISSUE COAGULATION DURING LASER IRRADIATION CONTROLLED BY SURFACE TEMPERATURE

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Abstract. The numerical analysis of thermal process proceeding in biological tissue during a laser irradiation controlled by surface temperature of tissue is presented. Heat transfer in the tissue is assumed to be transient and one-dimensional. The internal heat sources resulting from laser irradiation based on the Beer law are taken into account. Perfusion rate and effective scattering coefficient are treated as dependent on tissue injury. At the stage of numerical computations the boundary element method has been used. In the final part of paper the results obtained are shown.

Introduction

Laser irradiation on biological tissue often leads to the temperature elevation that can cause irreversible damage by the alteration of thermophysical and optical properties of tissue. In some medical applications a laser can affect both targeted tissue and surrounding healthy tissue what may become critical in the certain irradiation procedures in tissues such as heart, vessel walls and brain [1].

One of the possible ways to minimize and control a thermal damage during photocoagulation is the use of temperature feedback based on surface temperature. In this concept the constant surface temperature of tissue is maintained within a prescribed tolerance below the threshold for vaporization, desiccation and charring and this temperature controls laser delivery.

In this paper the tissue is regarded as a homogeneous domain with perfusion rate coefficient and effective scattering coefficient dependent on tissue damage expressed by the Arrhenius injury integral, while the remaining parameters are assumed to be constant [2, 3].

1. Governing equations

A transient heat transfer in biological tissue is described by the Pennes equation in the form [1-6]

$$0 < x < L: \quad c\dot{T} = \lambda T_{,ii} + Q_{perf} + Q_{met} + Q_{las} \tag{1}$$

where λ [$\mathrm{Wm^{-1}K^{-1}}$] is the thermal conductivity, c [$\mathrm{Jm^{-3}K^{-1}}$] is the volumetric specific heat, Q_{perf}, Q_{met} and Q_{las} [$\mathrm{Wm^{-3}}$] are the heat sources connected with the perfusion, metabolism and laser radiation, respectively, T is the temperature, t is the time and x denotes the spatial co-ordinate. In equation (1):

$$\dot{T} = \frac{\partial T}{\partial t}, \quad T_{,ii} = \nabla^2 T \tag{2}$$

In the current work the metabolic heat source heat is assumed as a constant value while the parameters appearing in the perfusion heat source (perfusion rate) and in the laser heat source (effective scattering coefficient) are treated as dependent on tissue injury expressed by the Arrhenius integral [2, 5, 6]

$$\theta(x) = \int_0^{t^F} A \exp\left[-\frac{\Delta E}{RT(x,t)}\right] \mathrm{d}t \tag{3}$$

where A is the pre-exponential factor [$\mathrm{s^{-1}}$], ΔE is the activation energy of the reaction [J $\mathrm{mole^{-1}}$] and R is universal gas constant [J $\mathrm{mole^{-1}\ K^{-1}}$].

The accepted criterion for complete tissue necrosis is [2, 5, 6]

$$\theta(x) \geq 1 \tag{4}$$

The relation between effective scattering coefficient and the tissue injury integral is as follows [1]

$$\mu_s'(\theta) = \mu_{s\,nat}' \exp(-\theta) + \mu_{s\,den}'[1 - \exp(-\theta)] \tag{5}$$

where $\mu'_{s\ nat}$ and $\mu'_{s\ den}$ [$\mathrm{m^{-1}}$] denote the effective scattering coefficient for native and destructed (denaturated) tissue, respectively.

The definition of laser heat source is based on the Beer's law [1]

$$Q_{las}(x,t) = \mu_t' I_0 \exp(-\mu_t' x)\, p(t) \tag{6}$$

where I_0 [$\mathrm{Wm^{-2}}$] is the surface irradiance of laser and $p(t)$ is the function equal to 1 when the laser is on and equal to 0 when the laser is off while μ'_t [$\mathrm{m^{-1}}$] is the attenuation coefficient defined as

$$\mu_t' = \mu_a + \mu_s' \tag{7}$$

where μ_a [$\mathrm{m^{-1}}$] is the absorption coefficient.

The perfusion heat source is as follows

$$Q_{perf}(x,t) = c_B G_B \left[T_B - T(x,t)\right] \tag{8}$$

where G_B [(m^3_{blood} s^{-1})/(m^3_{tissue})] is the blood perfusion rate, c_B [$Jm^{-3}K^{-1}$] is the volumetric specific heat of blood while T_B denotes the artery temperature [4-6].

According to the necrotic changes in tissue, the blood perfusion coefficient is defined as

$$G_B = G_B(\theta) = G_{B0} w(\theta) \tag{9}$$

where G_{B0} is the initial perfusion rate and, while the function of θ is assumed as a polynomial one [2, 5]

$$w(\theta) = \sum_{j=1}^{3} m_j \theta^{j-1} \tag{10}$$

where m_j are the coefficients.

Equation (1) is supplemented by the boundary condition on tissue surface subjected to a laser irradiation (an external surface)

$$x = 0: \quad q(x,t) = \alpha(T - T_{amb}) \tag{11}$$

where α [$Wm^{-2}K^{-1}$] is the convective heat transfer coefficient and T_{amb} is the temperature of surrounding, while on the internal tissue surface ($x = L$) the adiabatic condition is assumed.

The initial distribution of temperature is also known

$$t = 0: \quad T = T_p \tag{12}$$

2. Boundary element method

The problem has been solved using the 1st scheme of the BEM for 1D transient heat diffusion [7]. This method for the equation (1) and transition $t^{f-1} \to t^f$ leads to the formula

$$T(\xi,t^f) + \left[\frac{1}{c}\int_{t^{f-1}}^{f} T^*(\xi,x,t^f,t) q(x,t)\,\mathrm{d}t\right]_{x=0}^{x=L} =$$

$$= \left[\frac{1}{c}\int_{t^{f-1}}^{f} q^*(\xi,x,t^f,t) T(x,t)\,\mathrm{d}t\right]_{x=0}^{x=L} + \int_0^L T^*(\xi,x,t^f,t^{f-1}) T(x,t^{f-1})\,\mathrm{d}x + \tag{13}$$

$$+ \frac{1}{c}\int_0^L [Q_{perf} + Q_{met} + Q_{las}] \int_{t^{f-1}}^{t^f} T^*(\xi,x,t^f,t)\,\mathrm{d}t\,\mathrm{d}x$$

where T^* is the fundamental solution:

$$T^*(\xi, x, t^f, t) = \frac{1}{2\sqrt{\pi a(t^f - t)}} \exp\left[-\frac{(x-\xi)^2}{4a(t^f - t)}\right] \tag{14}$$

where ξ is the point in which the concentrated heat source is applied and $a = \lambda/c$.

The heat flux resulting from the fundamental solution is equal to

$$q^*(\xi, x, t^f, t) = -\lambda T^*_{,i} n_i = \frac{\lambda(x-\xi)}{4\sqrt{\pi}[a(t^f - t)]^{3/2}} \exp\left[-\frac{(x-\xi)^2}{4a(t^f - t)}\right] \tag{15}$$

while $q(x, t) = -\lambda T_{,i} n_i$.

Assuming that for $t \in [t^{f-1}, t^f]$: $T(x, t) = T(x, t^f)$ and $q(x, t) = q(x, t^f)$ one has the following form of equation (13)

$$\begin{aligned} &T(\xi, t^f) + g(\xi, L)q(L, t^f) - g(\xi, 0)q(0, t^f) = \\ &= h(\xi, L)T(L, t^f) - h(\xi, 0)T(0, t^f) + p(\xi) + z(\xi) \end{aligned} \tag{16}$$

where

$$h(\xi, x) = \frac{\operatorname{sgn}(x-\xi)}{2} \operatorname{erfc}\left(\frac{|x-\xi|}{2\sqrt{a\Delta t}}\right) \tag{17}$$

and

$$g(\xi, x) = \frac{\Delta t}{\sqrt{\pi\lambda c}} \exp\left[-\frac{(x-\xi)^2}{4a\Delta t}\right] - \frac{|x-\xi|}{2\lambda} \operatorname{erfc}\left(\frac{|x-\xi|}{2\sqrt{a\Delta t}}\right) \tag{18}$$

while

$$p(\xi) = \frac{1}{2\sqrt{\pi a\Delta t}} \int_0^L \exp\left[-\frac{(x-\xi)^2}{4a\Delta t}\right] T(x, t^{f-1}) \,\mathrm{d}x \tag{19}$$

at the same time

$$z(\xi) = \int_0^L [Q_{perf} + Q_{met} + Q_{las}] g(\xi, x) \,\mathrm{d}x \tag{20}$$

For $\xi \to 0^+$ and $\xi \to L^-$ one obtains the system of equations

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} q(0,t^f) \\ q(L,t^f) \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} T(0,t^f) \\ T(L,t^f) \end{bmatrix} + \begin{bmatrix} p(0) \\ p(L) \end{bmatrix} + \begin{bmatrix} z(0) \\ z(L) \end{bmatrix} \tag{21}$$

The solution of (21) determines the boundary temperatures for time ^{f} (for $x = 0$ and $x = L$) and next the boundary heat flux for $x = 0$ is calculated (c.f. equation (11). Finally, the temperatures at the internal points can be found using the formula

$$\begin{aligned} T(\xi,t^f) &= h(\xi,L)T(L,t^f) - h(\xi,0)T(0,t^f) - \\ &- g(\xi,L)q(L,t^f) + g(\xi,0)q(0,t^f) + p(\xi) + z(\xi) \end{aligned} \tag{22}$$

3. Results of computations

In computations the following values of tissue parameters have been assumed: $\lambda = 0.609$ Wm^{-1} K^{-1}, $c = 4.18$ MJm^{-3} K^{-1}, $Q_{met} = 245$ Wm^{-3}, $G_{B0} = 0.00125$ (m$^3_{blood}$ s^{-1})/(m$^3_{tissue}$), $\mu_a = 40$ m^{-1}, $\mu'_{s\ nat} = 1000$ m^{-1}, $\mu'_{s\ den} = 4000$ m^{-1} and $L = 35$ mm while for the blood $c_B = 3.9962$ MJm^{-3} K^{-1} and $T_B = 37$°C [1, 5, 6]. The parameters of Arrhenius injury integral are: $A = 3.1\ 10^{98}$ s^{-1}, $\Delta E = 6.27 \cdot 10^5$ J mole^{-1} and $R = 8.314$ J mole^{-1} K^{-1} and the coefficients appearing in the $w(\theta)$ function are as follows [2, 5]

$$\begin{aligned} &0 < \theta \le 0.1: \quad m_1 = 1, \quad m_2 = 25, \quad m_3 = -260 \\ &0.1 < \theta \le 1: \quad m_1 = 1, \quad m_2 = -1, \quad m_3 = 0 \end{aligned} \tag{23}$$

The values of these coefficients for the interval from 0 to 0.1 correspond to the increase of perfusion rate caused by vasodilatation, while for interval from 0.1 to 1 they reflect blood decrease as the vasculature begins to shut down (thrombosis).

Tissue domain has been divided into 100 elements and the time step equals $\Delta t = 0.1$ s.

For the boundary condition (11) the following input data have been introduced: $\alpha = 8$ Wm^{-2} K^{-1} and $T_{amb} = 20$°C while the initial distribution of temperature has been assumed as the constant one $T_p = 37$°C.

In order to provide the "constant" surface temperature equal to 80°C within a prescribed tolerance ±3°C, the function $p(t)$ has been used (c.f. equation (6)). That's mean the value of function is going to 0 (laser off) when the temperature at $x = 0$ is greater than 83°C and 1 when the surface temperature is lower than 77°C.

The laser irradiation I_0 has been assumed as equal to 3 W cm^{-2}.

In Figure 1 the course of controlled surface temperature and the increase of damage depth are presented, while in Figures 2 and 3 the changes of tissue injury integral, effective scattering coefficient μ'_s and blood perfusion rate G_B at selected points from the domain considered are shown.

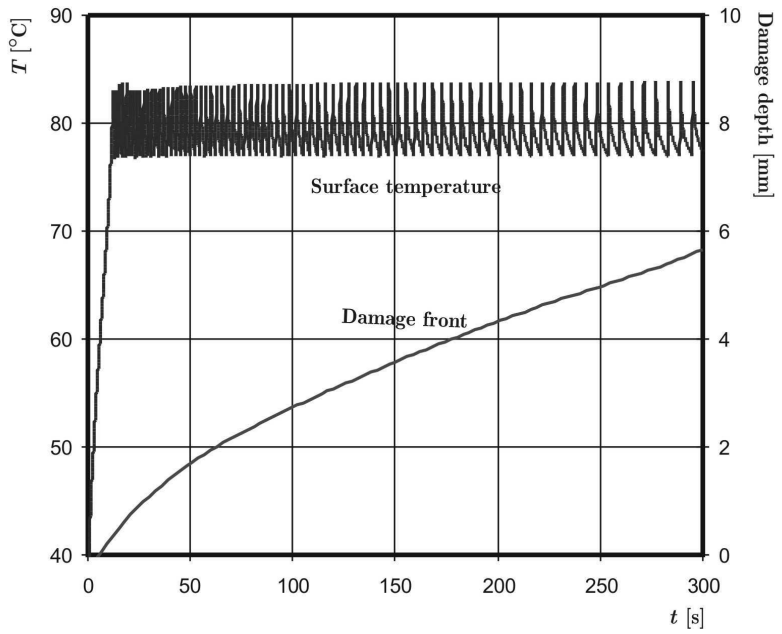

Fig. 1. Course of surface temperature and damage front

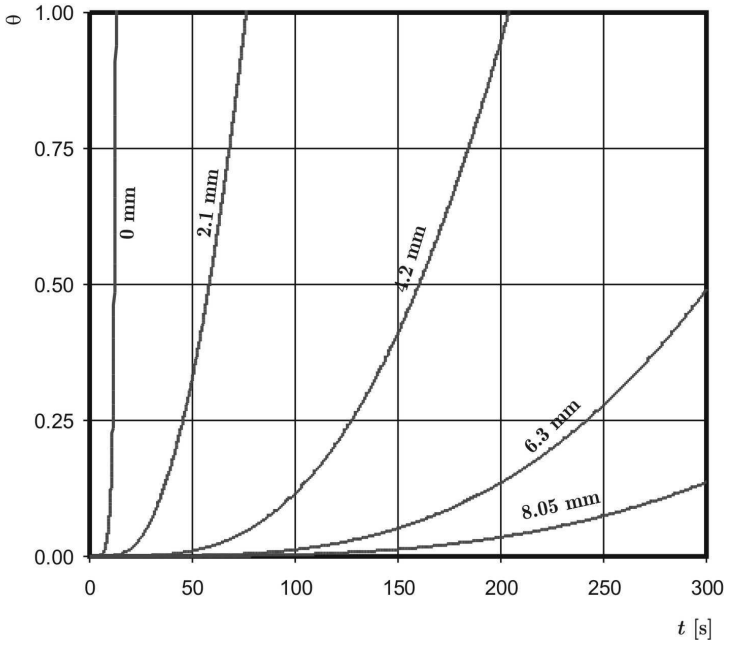

Fig. 2. Changes of tissue injury integral θ

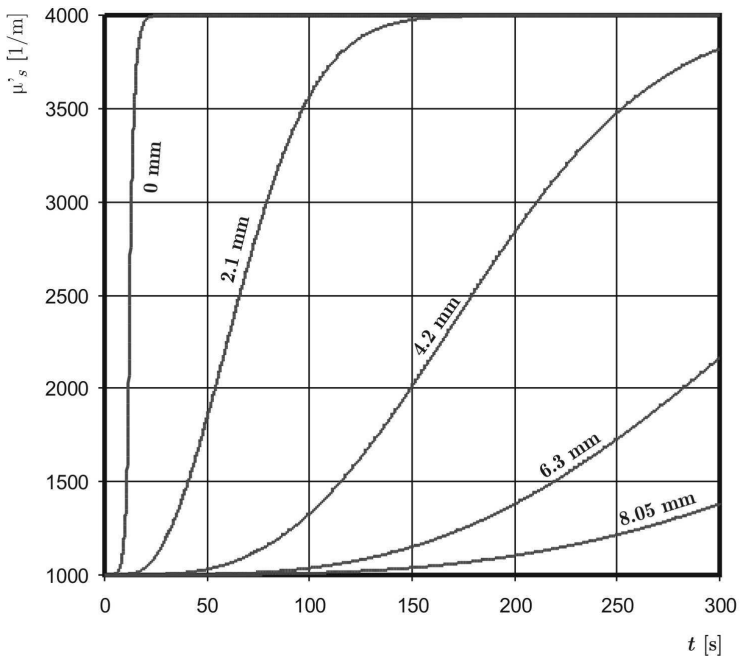

Fig. 3. Changes of effective scattering coefficient μ'_s

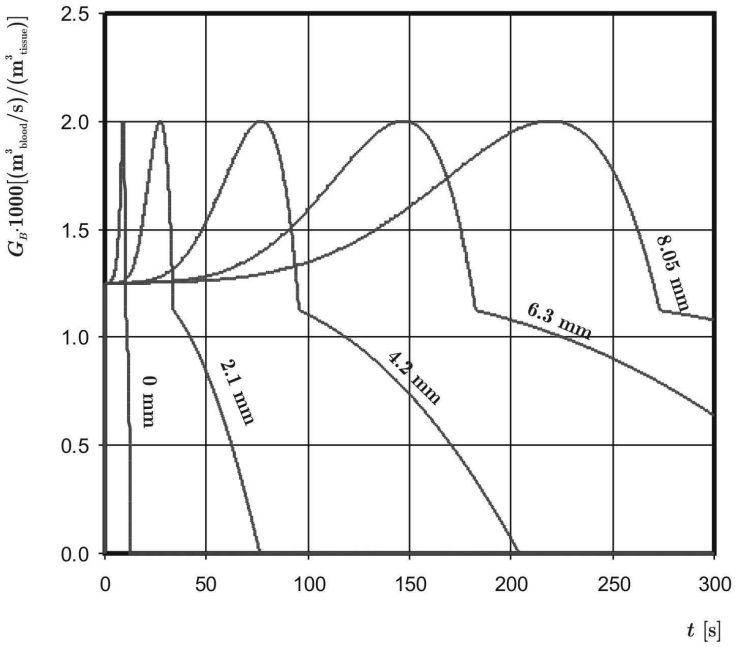

Fig. 4. Changes of blood perfusion rate G_B

Final remarks

The process of tissue destruction begins after 12.9 seconds and after 300 seconds the damage depth was determined to be equal to 5.6 mm (c.f. Figure 1). The process of tissue coagulation causes changes both in perfusion rate and effective scattering coefficient. In particular (c.f. Figure 3), the initial increase of perfusion rate caused by vasodilation and the drop of perfusion to zero at the points $x = 0$, 2.1 and 4.2 mm are clearly visible. It should be pointed out that the value of effective scattering coefficient corresponding to the denaturated tissue ($\mu'_{s\ den}$) was reached only at the points $x = 0$ and $x = 2.1$ (c.f. Figure 2).

Changes in parameters values during coagulation have also visible effect on the tissue cooling (c.f. Figure 1). At the beginning of the process the temperature of the tissue drops from over 83°C to below 77°C within 1 second, while at the end of the process in more than 6 seconds. Time of heating from 77 to 83°C is the same (about 0.5 second) throughout the process.

The proposed model is closer to the real conditions of a coagulation process in living tissue than the classical Pennes equation with constant values of thermal and optical parameters. Constant surface temperature during laser irradiation which may be preferred in some medical applications allows one to control thermal damage of tissue.

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