COMBING UP METHOD FOR CREATING FINAL OBJECTS LOCATION ON BASE OF RANKING LISTS

Henryk Piech, Tomasz Chłopaś

Czestochowa University of Technology, Poland, hpiech@adm.pcz.czest.pl

Abstract. Multi-attribute classification (ordering) problem concerns assignment of objects (tasks) to some predefined and preference-ordered decision classes [1, 2]. In considered situation objects are described by a finite sets of locations. Disposing several objects ranking lists it is possible exploit information about distribution. It was assumed that distribution information base on so called neighborhoods structures. These structures can approve or disapprove judgments unanimity prepared by experts (agents, responders, algorithms). Disposing distribution characteristics we define set of criteria. So, it is challenging problem which can be solved on base of preference theory. Reaching preference approach by neighborhood elements we exploit addition information, which help compromise achievement. Proposed method of objects ordering based of considering preference relations during process of lists combining up along their positions. Such approach enlarge scale of divergences in these relations.

Introduction

The information permitting to move forward solution process is called preferential information [3-5]. It is acquisition, construction of the preference model, exploitation of preference structure involve a single or multiply decision makers (DM) in the solution process. We can mention about classical rough sets theory [6-9], in which don't take into account monotonicity constrains or classes. It is based on the assumption that objects having the same description are indiscernible (similar) with respect to available information. In our approach we find distribution details which help to find supporting decision divergences. It can be shown that frequencies of objects appearing used for estimating probabilities are the maximum estimators supporting hypothesis of final objects locations [10-12]. Using different variant of criteria hierarchy or important sequences we obtain different final lists location. Hence, it is important to estimate level of compromise according given (chosen) criterion. Proposed conception give possibilities of realizing scheduling problem in dynamic version i.e. when new objects (tasks) succeed and extend data set and chosen dominating objects (tasks) are removed from ranking lists. Such convention is similar to maximum likelihood method [13-15]. It can lead to statistical problem of isotonic regression, which is solved by the optimal objects reassignment problem

[16, 17]. Instead of estimating risk level [18] we try to obtain compromise solution. Characteristics of neighborhood elements [19, 20] are reflecting in frequencies vectors and their gradients in given bandwidths. Ordered gradients are exploited as hierarchy criteria. We consider also the upward (belong at least concrete class) and downward (belong at most concrete class) union of classes. It help to develop hierarchy criteria sequence. The presentation is organized as follows. In section 1 a brief performance of ranking list and neighborhood elements. Then description of method combing up convention and their characteristics. First of all we base on Lorenze coefficients according to frequencies. In second section we introduce additional criteria like gradient and its bandwidth parameter. We define criteria hierarchy and enrich description by example of concrete distribution situation.

1. Conception of method based on lists combing up

As was mentioned above, we dispose ranking lists of length n in which are dislocated n objects coded from $k = 0,...,n-1$: φ_k (Table 1).

Table 1. Ranking lists of objects

1	4	7	0	9	6	3	5	8	2
4	5	2	8	6	1	7	3	0	9
1	3	8	5	0	9	7	6	2	4
2	5	9	0	7	6	1	3	4	8
7	9	6	5	0	1	4	3	2	8
8	3	4	7	1	2	0	6	5	9
9	8	3	7	5	6	2	0	4	1

Our conception consist in combing up the set of list starting from first position (first column in Table 1) on lists. During coming process we observe frequency $f(k,s)$ of appearing object in particular "combed" position.

$$f(k,s+1) = f(k,s) + del\_f(k,s), \quad f(k,0) = 0$$
$$f(k,s+1) = \sum_{i=1}^{s} del\_f(k,i)$$

where:
$f(k,s)$ - frequency of object k appearing on s stage of combing lists up,
$del\_f(k,s)$ - frequency increment on s stage of combing lists up.
Result of this analysis are presented in Table 2 and Figure 1.
Using obtained results we can compare chosen objects on base of sums of frequencies in all stages (for all objects) what can be treated as Lorenze preference values (Fig. 2):

$$Lv(k) = \sum_{i=1}^{n} f(k,i) = \sum_{i=1}^{n} \{i * del\_f(k, n-i+1)\}$$

where:
$Lv(k)$ - Lorenze preference value for object k,
i - stage number,
n - number of stages.

Table 2. Results of combing up method realization

Frequency of appearing objects in sequenced stages												
	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10		Lv
object 0	0	0	0	2	4	4	5	6	7	7		35
object 1	2	2	2	2	3	5	6	6	6	7		41
object 2	1	1	2	2	2	3	4	4	6	7		32
object 3	0	2	3	3	3	3	4	7	7	7		39
object 4	1	2	3	3	3	3	4	4	6	7		36
object 5	0	2	2	4	5	5	5	6	7	7		43
object 6	0	0	1	1	2	5	5	7	7	7		35
object 7	1	1	2	4	5	5	7	7	7	7		46
object 8	1	2	3	4	4	4	4	4	5	7		38
object 9	1	2	3	3	4	5	5	5	5	7		40

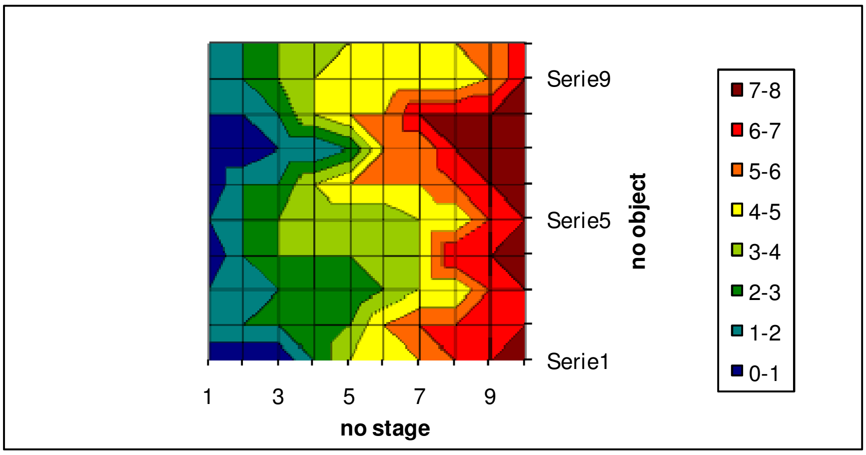

Fig. 1. Lorenze levels for all stages

Resulting Lorenze preferences are depicted in Figure 3.

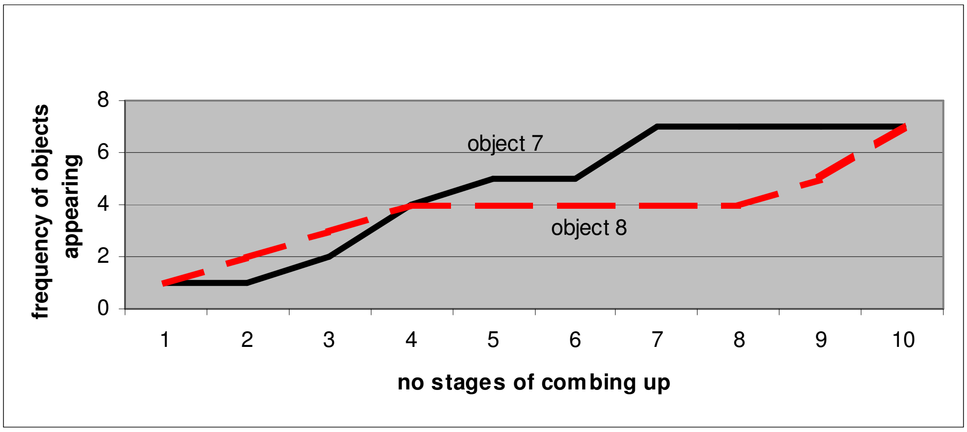

Fig. 2. Comparing two objects frequences during lists combing up: $Lv(7) = 46$; $Lv(8) = 38$

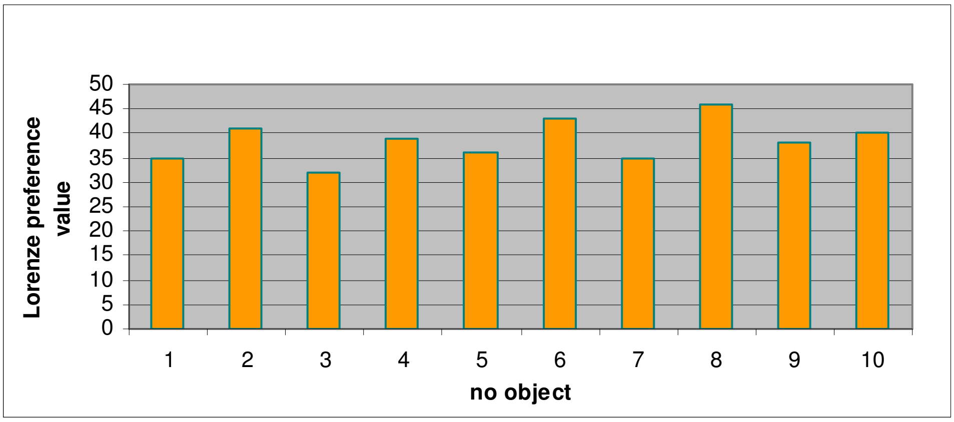

Fig. 3. Lorenze preferences after lists combing up for 10 objects

2. Additional frequency characteristics and their exploitation

Using frequency table it is possible to define maximal frequency gradients for all objects. To do it we should chose bandwidth: *bw* consist of concrete number of positions. Shifting bandwidth along frequency elements for every object are finding location with maximum gradient value (Table 3, Figures 4 and 5):

$$mg(k) = \max_{1 \le i \le n-bw+1} grad(f(k,i), f(k,i+bw-1)) = \max_{1 \le i \le n-bw+1} (f(k,i+bw-1) - f(k,i))$$

Location of $mg(k)$ is: $lmg(k) = \{i, mg(k)\}$.

Table 3. Finding maximum gradient frequency

	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10		mg
object 0	0	0	0	2	4	4	5	6	7	7		4
object 1	2	2	2	2	3	5	6	6	6	7		3
object 2	1	1	2	2	2	3	4	4	6	7		3
object 3	0	2	3	3	3	3	4	7	7	7		4
object 4	1	2	3	3	3	3	4	4	6	7		3
object 5	0	2	2	4	5	5	5	6	7	7		3
object 6	0	0	1	1	2	5	5	7	7	7		4
object 7	1	1	2	4	5	5	7	7	7	7		3
object 8	1	2	3	4	4	4	4	4	5	7		3
object 9	1	2	3	3	4	5	5	5	5	7		2

All objects are classified according *mg* parameter.

$Cl_t = \{k \in U : mg(k) = t\}$

where U - set of objects
then union classes can be defined [18]:

$Cl^{\geq}_t = \{k \in U : mg(k) \geq t\}$
$Cl^{\leq}_t = \{k \in U : mg(k) \leq t\}$

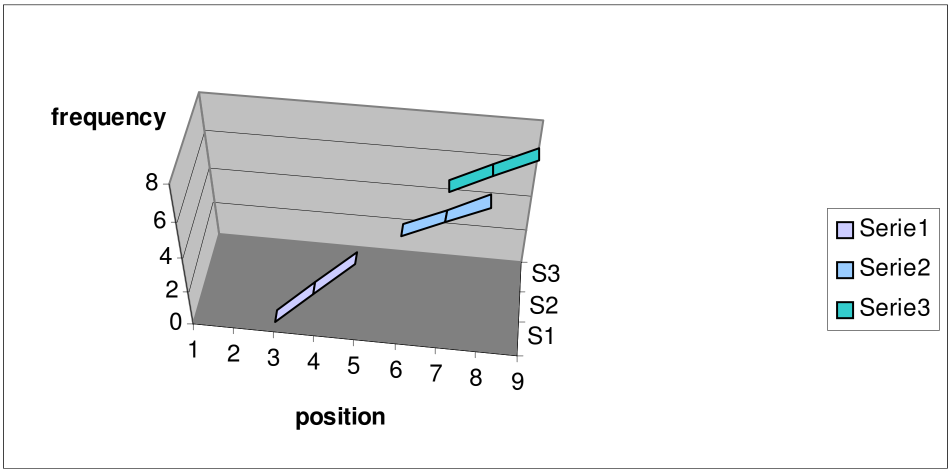

Fig. 4. Three most important gredients for object "0"

To define precisely hypothesis of final object k location are searching maximal increment in given object bandwidth (Table 4).

$lh(k) = \{lmg(k) + i : \max_{0 \le i \le bw-1} (f(k, lmg(k) + i) - f(k, lmg(k) + i - 1)\}, f(k, 0) = 0$ for every k

Obviously: $lmg(k) \le lh(k) \le (lmg(k) + \text{bw} - 1)$.

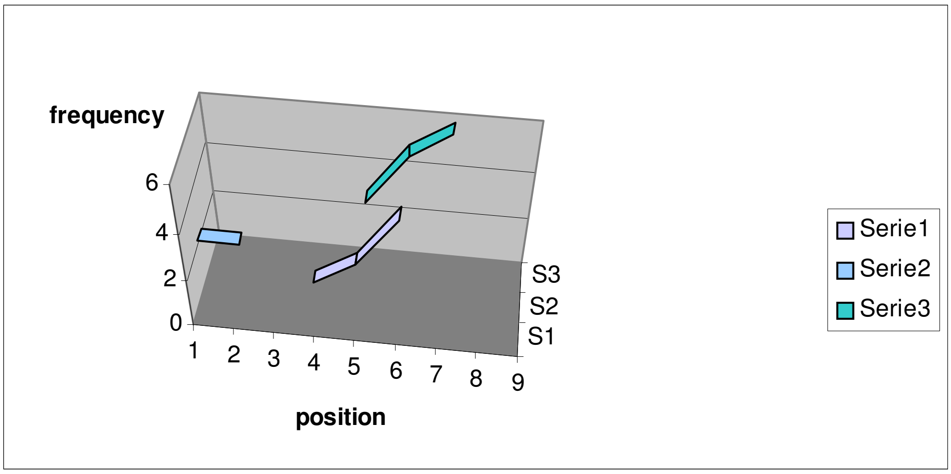

Fig. 5. Three most important gradients for object "9"

Table 4. Maximal increments in bandwidth objects location

	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10
object 0	0	0	0	2	4	4	5	6	7	7
object 1	2	2	2	2	3	5	6	6	6	7
object 2	1	1	2	2	2	3	4	4	6	7
object 3	0	2	3	3	3	3	4	7	7	7
object 4	1	2	3	3	3	3	4	4	6	7
object 5	0	2	2	4	5	5	5	6	7	7
object 6	0	0	1	1	2	5	5	7	7	7
object 7	1	1	2	4	5	5	7	7	7	7
object 8	1	2	3	4	4	4	4	4	5	7
object 9	1	2	3	3	4	5	5	5	5	7

Let's define different variants of classification hierarchy considering forming final list of objects location:

- $\max Lv(k)$
- $\max Lv(k) \supseteq mg(k)$
- $mg(k) \supseteq lh(k)$
- $lh(k)$

Symbol ⊒ in above notation means that criterion on the left hand side is more important then on the right hand side. When appear several objects with the same value of *mg*(*k*) it is possible to define next value of this parameter for given objects: *mg'*(*k*), *mg"*(*k*), … (Table 5).

Table 5. Supporting decision with help of next parameter values

	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10		mg	mg'	mg"
object 0	0	0	0	2	4	4	5	6	7	7		4	2	2
object 1	2	2	2	2	3	5	6	6	6	7		3	2	1
object 2	1	1	2	2	2	3	4	4	6	7		3	2	2
object 3	0	2	3	3	3	3	4	7	7	7		4	3	3
object 4	1	2	3	3	3	3	4	4	6	7		3	2	2
object 5	0	2	2	4	5	5	5	6	7	7		3	2	2
object 6	0	0	1	1	2	5	5	7	7	7		4	3	1
object 7	1	1	2	4	5	5	7	7	7	7		3	3	2
object 8	1	2	3	4	4	4	4	4	5	7		3	2	2
object 9	1	2	3	3	4	5	5	5	5	7		2	2	2

Next bandwidths *mg*(*k*), *mg'*(*k*), *mg"*(*k*) (adequate colors: white, shadow and dark) mutual overlapping each other in Table 5. Hence, final objects location can be formed according follow classification hierarchy: *mg*(*k*) ⊒ *mg'*(*k*) ⊒ *mg"*(*k*) ⊒ *lh*(*k*). Supporting results of optimization : max *Lv*(*k*) we create follow final objects list location (Table 6):

Table 6. Final list of objects location on base of criterion: max *Lv*(*k*)

pos1	pos2	pos3	pos4	pos5	pos6	pos7	pos8	pos9	pos10
7	5	1	9	3	8	4	0	6	2

On base of optimization: *mg*(*k*) ⊒ *lh*(*k*) we obtain final objects sequence (Table 7):

Table 7. Final list of objects location referring to *mg*(*k*) ⊒ *lh*(*k*)

pos1	pos2	pos3	pos4	pos5	pos6	pos7	pos8	pos9	pos10
4	9	7	5	0	6	1	3	2	8

And using optimization: *mg*(*k*) ⊒ *mg'*(*k*) ⊒ *mg"*(*k*) final list will be following (Table 8):

Table 8. Final list of objects location regarding: *mg*(*k*) ⊒ *mg'*(*k*) ⊒ *mg"*(*k*)

pos1	pos2	pos3	pos4	pos5	pos6	pos7	pos8	pos9	pos10
1	5	2	0	9	6	7	3	4	8

When the proposed hypothesis object location is already occupied then we can chose follows strategy:
- for chosen according actual considered criterion object we go to the next criterion in hierarchy and find location for the same object
- or we go to the next object and find its location.

Referring to topology of ranking lists and bandwidth range we can try to define probability of exploitation of hypothesis of object location:

$$ph(k,i) = p(k)*p(mg(k))*p(i = lh(k))$$

where:

$p(k) = Lv(k)/\sum_{j=1}^{n} Lv(j)$

$p(mg(k)) = 1/(n - 2)$

$p(i = lj(k)) = 1/bw$

Referring to gradient frequency values we can try to define probability of using of hypothesis of object location:

$$ph(k,i) = pmg(k)*pg(i = lh(k))$$

where:

$pmg(k) = mg(\mathrm{k})/\sum_{j=1}^{n} mg(j)$

$pg(i = lh(k)) = (f(k,i) - f(k,i - 1))/mg(k)$

or

$$ph(k,i) = (pmg(k))^{r}*(pmg'(k))^{s}*(pmg''(k))^{t}*pg(i= lh(k))$$

where:

r - number of objects with the same values of $mg(*)$,

s - number of objects with the same values of $mg(*)$ and $mg'(*)$ (respectively),

t - number of objects with the same values of $mg(*)$, $mg'(k)$ and $mg''(*)$ (respectively).

Examples for precise explanation presented above description:

Table 9a. Example 1 about exploiting $mg(k)$, $mg'(k)$, $mg''(k)$

mg	mg'	mg''		mg	mg'	mg''		mg	mg'	mg''
4	2	2		4	2	2		4	2	2
3	2	1		3	2	1		3	2	1
3	2	2		3	2	2		3	2	2
4	3	3		4	3	3		4	3	3

3	2	2		3	2	2		3	2	2
3	2	2		3	2	2		3	2	2
4	3	1		4	3	1		4	3	1
3	3	2		3	3	2		3	3	2
3	2	2		3	2	2		3	2	2
2	2	2		2	2	2		2	2	2

$mg(0) = mg(3) = mg(6) = 4$
$r = 3$
$mg'(3) = mg(6) = 3$
$s = 2$
$mg''(3) = 3$
$t = 1$

Table 9b. Example 2 about exploiting $mg(k)$, $mg'(k)$, $mg''(k)$

mg	mg'	mg"		mg	mg'	mg"		mg	mg'	mg"
4	2	2		4	2	2		4	2	2
3	2	1		3	2	1		3	2	1
3	2	2		3	2	2		3	2	2
4	3	3		4	3	3		4	3	3
3	2	2		3	2	2		3	2	2
3	2	2		3	2	2		3	2	2
4	3	1		4	3	1		4	3	1
3	3	2		3	3	2		3	3	2
3	2	2		3	2	2		3	2	2
2	2	2		2	2	2		2	2	2

$mg(1) = mg(2) = mg(4) = mg(5) = mg(7) = mg(8) = 3$
$r = 6$
$mg(1) = mg(2) = mg(4) = mg(5) = mg(8) = 3s = 2$
$s = 5$
$mg(2) = mg(4) = mg(5) = mg(8) = 3s = 2$
$t = 4$

3. Accumulation objects in neighborhood centers

Accumulation objects process is realized by summing all objects in concrete neighborhood and placing them in its center. It is possibilities to use increments of objects location on lists set.

Using presented above method based on accumulation objects convention, we researches frequencies of appearing objects in sequence of positions (stages of combing up) (Table 10) but Lorenze values are less distinguished according to lack of accumulations (variance levels are in dependencies: 17<18).

Table 10. Results of combing up method realization after objects accumulation

	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10		Lv
object 0	0	0	0	4	4	4	7	7	7	7		40
object 1	2	2	2	2	2	6	6	6	6	7		41
object 2	1	1	2	2	2	4	4	4	7	7		34
object 3	0	3	3	3	3	3	3	7	7	7		39
object 4	3	3	3	3	3	3	4	4	7	7		40
object 5	0	2	2	5	5	5	5	7	7	7		45
object 6	0	0	1	1	1	5	5	7	7	7		34
object 7	1	1	1	5	5	5	7	7	7	7		46
object 8	4	4	4	4	4	4	4	4	4	7		43
object 9	3	3	3	3	5	5	5	5	5	7		44

Finally we obtain list which is very similar to list in Table 6 (Table 11).

Table 11. Final list of objects location after accumulation

pos1	pos2	pos3	pos4	pos5	pos6	pos7	pos8	pos9	pos10
7	5	9	8	1	0	4	3	6	2

So, this modification don't yield better effects, because the estimator differentiation don't increase (generally in statistical sense).

Conclusions

1. Combing ranking lists up we relative judgment problem bring about challenging preference problem with full exploitation of location objects information.
2. Lorenze value and gradient parameter are the most effective features for support decision about final objects location
3. Using accumulation convention based on neighborhoods we don't obtain expected results in aspect of support of objects location decision making.

References

[1] Dembczyński K., Greco S., Słowiński R., Methodology of rough-set-based classification and sorting with hierarchical structure of attributes and criteria, Control and Cybernetics 2002, 31, 4.

[2] Słowiński R., Greco S., Matarazzo B., Axiomatization of utility and decision rule preference models for multiple-criteria classification problem under partial inconsistency with the dominance principle, Control and Cybernetics 2002, 1005-1035.

[3] Finch H.A., Confirming power of observations metricized for decisions among hypotheses, Philosophy of Science 1999, 27, 391-404.

[4] Piech H., Analysis of possibilities and effectiveness of combine rough set theoryand neibourhood theories for solving dynamic scheduling problem, IEEE Computer Society, P3674, Washington, Tokyo 2009, 296-302.

[5] Słowiński R., Brzezińska I., Greco S., Application of Bayesian Confirmation Measures for Mining Rules from Support Confidence Pareto Optimal Set, ICAISC LNSC, 4029, Heilderberg 2006, Springer, 1018-1026.

[6] Greco S., Matarazzo B., Słowiński R., Stefanowski J., An algorithm for induction of decision rules with dominance principle, Rough Sets and Current Trends in Computing, LNAI, Springer - Verlag, Berlin 2005, 304-313.

[7] Greco S., Matarazzo B., Słowiński R., Extension of rough set approach to multicriteria decision support, INFOR 2000, 38, 161-196.

[8] Greco S., Matarazzo B., Słowiński R., Rough sets theory for multicriteria decision analysis, European J. of Operational Research 2001, 129, 1-47.

[9] Pawlak Z., Sugeno M., Decision rules Bayes, rule and rough, new decisions in rough rets, Springer - Verlag, Berlin 1999.

[10] Kohler W.H., A preliminary evolution of the critical path method for scheduling tasks on multiprocessor systems, IEEE Trans. Comput. 1975, 24, 1235-1238.

[11] Roy B., Main sources of inaccurate determination, uncertainty and imprecision in decision models, Mathematical and Computer Modeling 1989, 12, 10/11, 1245-1254.

[12] Talbi E.D., Geneste L., Grabot B., Prévitali R., Hostachy P., Application of optimization techniques to parameter set-up in scheduling, Computers in Industry 2004, 55, 2, October, 105-124.

[13] Greco S., Matarazzo B., Słowiński R., Assessing the Quality of Rules with a New Monotonic Interestingness Measure Z, Artificial Intelligence and Soft Computing, Springer - Verlag 2008, 556-565.

[14] Roy B., Bouyssou D., Aide Multicriteria a la Decision: Methodes et Cas, Economica, Paris 1993.

[15] Namagashi M., Kansei Engineering: A new ergonomic customer-oriented technology for product development, International Journal of Industrial Ergonomics 1995, 15, 1, 3-11.

[16] Brzezińska I., Greco S., Słowiński R., Mining Pareto-optimal rules with respect to support and anti-suport, Engineering Applications of Artificial Inteligence 2007, 20(5), 587-600.

[17] Greco S., Matarazzo B., Słowiński R., Axiomatic characterization of a general utility function and its particular cases in terms of conjoint measurement and rough sets decision rules, European J. of Operational Research 2003.

[18] Greco S., Matarazzo B., Słowiński R., Customer Satisfaction Analysis based on Rough Set Approach, ZfB 77, H. 3, Gabler Verlag 2007, 325-339.

[19] Jaroń J., Systemic Prolegomena to Theoretical Cybernetics, Scient. Papers of Inst. of Techn. Cybernetics 25, Wrocław 1975.

[20] Nikodem J., Autonomy and Cooperation as Factors Dependability in Wireless Sensor Network, IEEE Computer Society P3179, 406-413, 2008-08-09.