SENSITIVITY ANALYSIS OF BIOHEAT TRANSFER IN HUMAN CORNEA SUBJECTED TO LASER IRRADIATION. PART 1: VARIATION OF OPTICAL PARAMETERS

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Abstract. The numerical analysis of thermal process proceeding in the human cornea due to a laser irradiation is presented. Heat transfer in the cornea is assumed to be transient and one-dimensional. The internal heat sources resulting from laser irradiation based on the Beer law are taken into account. The paper deals with sensitivity analysis of temperature field with respect to the optical and thermophysical parameters of cornea. At the stage of numerical computations the boundary element method has been used. In the final part of paper the results obtained are shown.

Introduction

Cornea is transparent, dome-shaped window covering the front of the eye, responsible for focusing light rays to the back of the eye and providing 2/3 of the eye's focusing power. It is an extremely sensitive structure because there are more nerve endings in the cornea than anywhere else in the body. There are no blood vessels in it and cornea contains 78% of water.

The cornea is the dominant absorber of UV wavelengths in human eye. It absorbs between 20% for UVA (315÷400 nm) and 100% for UVC (100÷280 nm) band. The radiation throughout the UV band is under investigation with regard to its possible long-term mutagenic and carcinogenic potentials. Other possible damages of cornea are the keratoconjunctivitis and acute photochemically induced inflammation (called snow blindness or welder's flash) [1].

The absorption coefficient is the main optical parameter of cornea in the UV and its value depends on the wavelength and could change from person to person. Moreover it is probably one of the most discussed coefficients in the literature on account of many difficulties with the determination of its exact value. It should be pointed out that a scope of its values is from about 2300 cm^{-1} to 20 000 cm^{-1} and more for 193 nm wavelength corresponding to ArF excimer laser. There are many reasons of such kind of variance like tissue hydration or ageing. Some researches suggest that the ablation process during the laser irradiation has effect in dynamic changes of tissue absorptivity [2, 3].

It is also known that other cornea parameters like the thermal conductivity, density or specific heat tend to change its value due to ageing [5].

Most of the sight diseases, like the nearsightedness, the farsightedness or the astigmatism are resulting from improper cornea shape. It is also known that the cornea could change its shape due to ageing - it flattens about the age of 60. The methods of corrections, apart from most popular glasses consist in changing of cornea's shape. With nearsighted people, the goal is to flatten the too-steep cornea; with farsighted people, a steeper cornea is desired, while astigmatism correction is reached by smoothing an irregular cornea into a more normal shape.

Many of such kind of cases could be subjected to a laser sight correction like in PRK method (photorefractive keratectomy) or LASIK (laser-assisted in situ keratomileusis). Others methods are various kind of hard contact lens and cornea transplantation.

All the aspects mentioned above pointing out the significant differences in human cornea from person to person, which should be taken into consideration in an investigation of temperature field during laser irradiation. One of the possible mathematical tools applied in such cases is the sensitivity analysis. The sensitivity information may be used, among others, to analyze the influence of the parameters change on the final solution of the problem considered. Additional tasks required to determine the sensitivity functions result from differentiation of the assumed equation describing bioheat transfer with respect to the parameter, which means that the number of additional sensitivity tasks corresponds to the number of parameters with respect to which the sensitivity analysis is performed [6-9].

There are some differences between shape sensitivity analysis and sensitivity analysis with respect to the thermophysical or optic parameters. Among other thing the concept of material derivative must be applied in the sensitivity analysis with regard to shape parameters. Due to this feature the present work is in two parts: 1st part of the paper deals with the variation of thermal conductivity, volumetric specific heat and absorption coefficient while the 2nd part concerns the temperature analysis according to the changes in geometrical parameters.

1. Governing equations

A description of the transient heat transfer in human cornea is based on the Pennes formulation [4-7]

$$L_0 < x < L_1: \quad c\dot{T} = \lambda T_{,ii} + Q_{perf} + Q_{met} + Q_{las} \tag{1}$$

where λ [$\mathrm{Wm^{-1}K^{-1}}$] is the thermal conductivity, c [$\mathrm{Jm^{-3}K^{-1}}$] is the volumetric specific heat, Q_{perf}, Q_{met} and Q_{las} [$\mathrm{Wm^{-3}}$] are the heat sources connected with the perfusion, metabolism and laser radiation, respectively, T is the temperature, t is the time and x denotes the spatial co-ordinate, while L_0 and L_1 correspond to the external (anterior) and internal (posterior) surface of cornea. In equation (1):

$$\dot{T} = \frac{\partial T}{\partial t}, \quad T_{,ii} = \nabla^2 T \tag{2}$$

Taking into account the properties of cornea the perfusion and metabolic sources can be neglected ($Q_{perf} = Q_{met} = 0$), while the definition of laser heat source is basing on the Beer law [4, 5]

$$Q_{las}(x) = \mu_a I_0 \exp(-\mu_a x) \tag{3}$$

where μ_a [m^{-1}] is the absorption coefficient and I_0 [Wm^{-2}] is the irradiation on the anterior surface of cornea.

The equation (1) is supplemented by the boundary condition on the external surface of the domain considered [4]

$$x = L_0: \quad q(x,t) = \alpha(T - T_{amb}) + \varepsilon\sigma(T^4 - T_{amb}^4) + E \tag{4}$$

where α [$Wm^{-2}K^{-1}$] is the convective heat transfer coefficient, T_{amb} is the temperature of surrounding, ε is the emissivity of cornea, σ [$Wm^{-2}K^{-4}$] is the Stefan-Boltzman constant, while E [Wm^{-2}] denotes the heat flux loss due to tear evaporation.

On the internal surface of cornea the constant heat flux is assumed [4, 6, 7]

$$x = L_1: \quad q(x,t) = -\lambda T_{,i} n_i = q_1 \tag{5}$$

The initial distribution of temperature is also known

$$t = 0: \quad T = T_p \tag{6}$$

2. Sensitivity analysis - direct approach

Sensitivity analysis of bioheat transfer has been carried out both with respect to the optical (absorption coefficient) and the thermophysical parameters (thermal conductivity and volumetric specific heat). These parameters are denoted by p_s, $s =$ $= 1,2,3$, this means $p_1 = \mu_a$, $p_2 = \lambda$ and $p_3 = c$.

If the direct approach of sensitivity analysis is applied then the equation (1) and the conditions (4), (5) and (6) are differentiated with respect to the parameter p_s [8, 9].

Taking into account the bioheat transfer equation (1) one has

$$\frac{\partial c}{\partial p_s}\dot{T} + c\frac{\partial \dot{T}}{\partial p_s} = \frac{\partial \lambda}{\partial p_s}T_{,ii} + \lambda\frac{\partial T_{,ii}}{\partial p_s} + \frac{\partial Q_{las}}{\partial p_s} \tag{7}$$

Let us assume that

$$U^s = \frac{\partial T}{\partial p_s}, \quad \dot{U}^s = \frac{\partial U}{\partial t}, \quad U^s_{,ii} = \frac{\partial T_{,ii}}{\partial p_s} \tag{8}$$

while (c. f. equation (1))

$$T_{,ii} = \frac{c}{\lambda}\dot{T} - \frac{Q_{las}}{\lambda} \tag{9}$$

Introducing (8) and (9) into (7) we have [6-9]

$$c\dot{U}^s = \lambda U^s_{,ii} + \frac{\partial \lambda}{\partial p_s}\left[\frac{c}{\lambda}\dot{T} - \frac{Q_{las}}{\lambda}\right] - \frac{\partial c}{\partial p_s}\dot{T} + \frac{\partial Q_{las}}{\partial p_s} \tag{10}$$

where (c. f. equation (3))

$$\begin{aligned} \frac{\partial Q_{las}}{\partial p_s} &= \frac{\partial}{\partial p_s}\left[\mu_a I_0 \exp(-\mu_a x)\right] = \\ &= \frac{\partial \mu_a}{\partial p_s} I_0 \exp(-\mu_a x) - \frac{\partial \mu_a}{\partial p_s}\mu_a I_0 x \exp(-\mu_a x) = \\ &= \frac{\partial \mu_a}{\partial p_s} I_0 (1-\mu_a x)\exp(-\mu_a x) \end{aligned} \tag{11}$$

Finally, the equation (10) takes a form [6-8]

$$c\dot{U}^s = \lambda U^s_{,ii} + Q^s_V \tag{12}$$

where sensitivity source function $Q^s{}_V$ is defined as

$$\begin{aligned} Q^s_V = \frac{\partial \lambda}{\partial p_s}\frac{1}{\lambda}\left[c\dot{T} - \mu_a I_0 \exp(-\mu_a x)\right] - \frac{\partial c}{\partial p_s}\dot{T} + \\ + \frac{\partial \mu_a}{\partial p_s} I_0 (1-\mu_a x)\exp(-\mu_a x) \end{aligned} \tag{13}$$

In particular, we have

$$Q^\lambda_V = \frac{1}{\lambda}\left[c\dot{T} - \mu_a I_0 \exp(-\mu_a x)\right] \tag{14}$$

and

$$Q^c_V = -\dot{T} \tag{15}$$

while

$$Q_V^{\mu_a} = \frac{\partial \mu_a}{\partial p_s} I_0 (1 - \mu_a x) \exp(-\mu_a x) \tag{16}$$

Differentiating the boundary - initial conditions we obtain (c. f. equation (4))

$$x = L_0: \quad Q^s(x,t) = \alpha U^s + 4\sigma\varepsilon T^3 U^s \tag{17}$$

on the anterior (external) surface of cornea, and (c.f. equation (5))

$$x = L_1: \quad Q^s(x,t) = Q_1^s = -\frac{1}{\lambda}\frac{\partial \lambda}{\partial p_s} q_1 \tag{18}$$

on the posterior (internal) surface, while (c. f. equation (6))

$$t = 0: \quad U_p^s = 0 \tag{19}$$

It should be pointed out that Q^s in equations (17) and (18) is defined as [6, 8]

$$Q^s = -\lambda U_{,i}^s n_i \tag{20}$$

3. Results of computations

The 1D task is considered. The primary and also the additional problem resulting from the sensitivity analysis have been solved using 1st scheme of the BEM for 1D transient heat diffusion [10].

In computations the following values of tissue parameters have been assumed: $\lambda = 0.58$ Wm^{-1} K^{-1}, $c = 4.3869$ MJm^{-3} K^{-1}, $L_1 - L_0 = 600$ μm and $\mu_a = 2300$ cm^{-1}.

For the boundary condition (4) the following input data have been introduced: $\alpha = 10$ Wm^{-2} K^{-1}, $T_{amb} = 25$°C, $\varepsilon = 0.975$, $\sigma = 5.67\cdot 10^{-8}$ Wm^{-2} K^{-4}, $E = 40$ Wm^{-2}, while for internal surface the non-flux condition has been accepted.

Two cases of laser irradiation have been analysed: constant and time - varying irradiation. In both cases $I_0 = 10$ kWm^{-2} while duration of laser impulse was 1 s with 1s off (for time - varying irradiation).

A parabolic distribution of the temperature has been assumed between 33.6°C on anterior surface of cornea and 33.8°C on posterior surface for time $t = 0$. The tissue domain has been divided into 50 elements and the time step equals $\Delta t = 0.1$ s.

In Figure 1 the heating curves on the anterior (external) and posterior (internal) surface of the cornea are presented, while on Figures 2-4 the successive sensitivity functions are shown. The values of parameters on those figures are multiplayed by: $\Delta\mu_a = 300$ cm^{-1} (Fig. 2), $\Delta\lambda = 0.1$ Wm^{-2} K^{-1} (Fig. 3) and $\Delta c = 1$MJm^{-3} K^{-1} (Fig. 4).

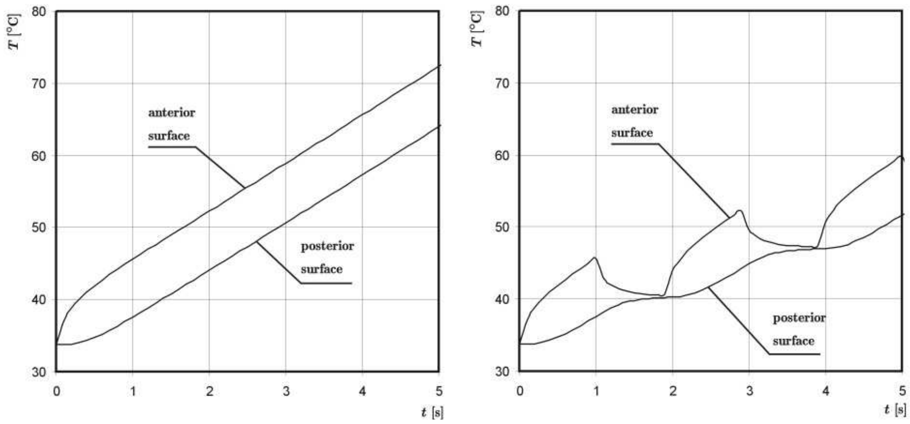

Fig. 1. Heating curves

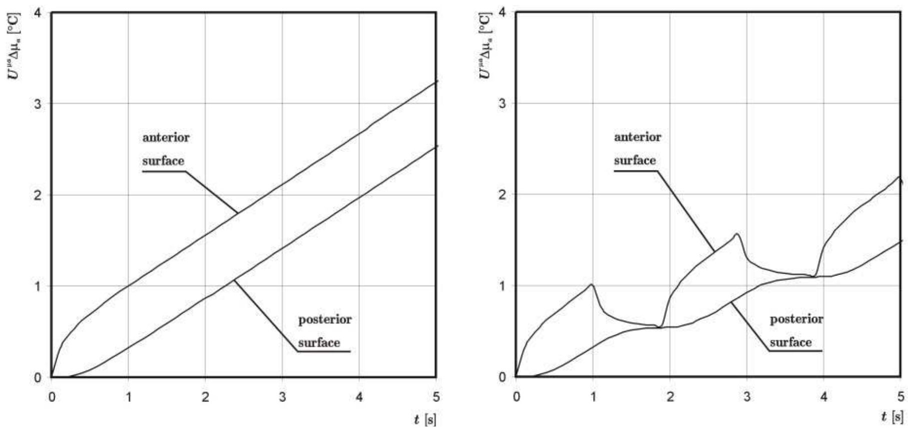

Fig. 2. Course of sensitivity function of absorption coefficient μ_a

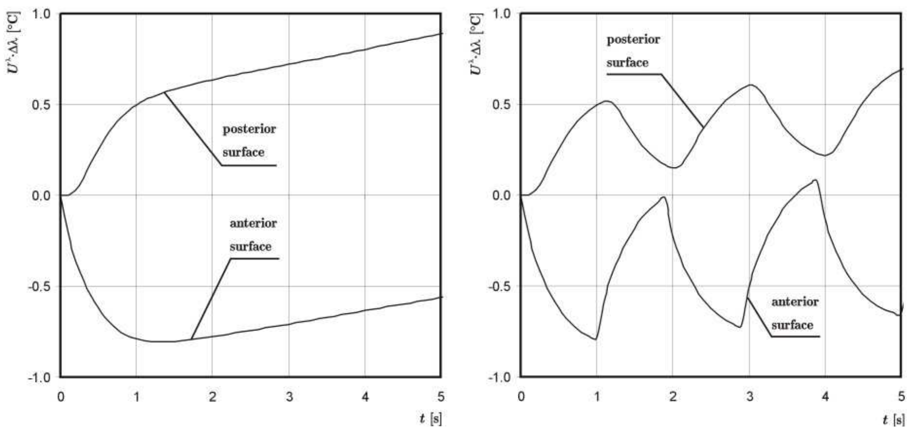

Fig. 3. Course of sensitivity function of thermal conductivity λ

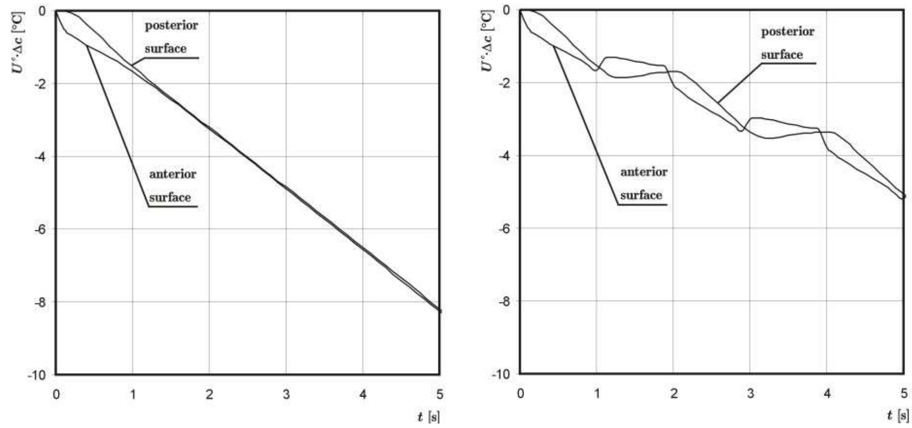

Fig. 4. Course of sensitivity function of volumetric specific heat c

Final remarks

The irradiation I_0 = 10 kWm^{-2} causes that the temperature in the cornea after 5 seconds is between 64 (anterior surface) and 72.3°C (posterior surface) for constant laser impulse. In case of time-varying irradiation the temperature is from the scope from 51.5 to 59.8°C. Those temperatures suggest that possible irreversible damage of tissue could be expected, although for modulated impulse the temperatures are below 60°C which is the border temperature assumed for destruction of tissue [5, 6].

The sensitivity studies show that the major parameter influencing on the temperature level in the cornea is the volumetric specific heat: the increase of basic value of this parameter by 23% (Δc = 1 MJm^{-3} K^{-1}, Fig. 4) causes fall in the temperature to more than 8 degrees for constant irradiation while for time - varying irradiation the fall is about 5°C.

Variations of absorption coefficient (Fig. 2) have been assumed as about 13% ($\Delta\mu_a$ = 300 cm^{-1}) of basic value. As results show the increase of coefficient have effect in rising of temperatures about 2.5÷3°C after 5 seconds for constant impulse and 1.5÷2°C in case of modulated irradiation.

A change of the thermal conductivity has the weakest influence on the values of temperatures (Fig. 3). Both for the constant and time – varying irradiation, for assumed $\Delta\lambda$ = 0.1 Wm^{-2} K^{-1} (about 17% of basic value) increase of parameter causes fall in temperatures on anterior surface of the cornea to about 0.5°C, while on the posterior surface temperature rise of 0.75°C.

Summing up, variations of optic (absorption coefficient) and thermophysical parameters (thermal conductivity and volumetric specific heat) of cornea have visible effect in temperature level during laser irradiation.

This paper is part of project No N N501 3667 34.

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