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ESTIMATION OF THE HEAT TRANSFER COEFFICIENT FOR THREE RANGES OF REFERENCE VALUES UNDER FOURTH-KIND BOUNDARY CONDITIONS USING SWARM ALGORITHMS

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Abstract. The article discusses the problem of reconstructing the heat transfer coefficient under fourth-kind boundary conditions using swarm algorithms, considering the κ parameter's variation in three value selection ranges. The analysis was conducted based on the time-varying nature of this coefficient, which represents the physics of the heat exchange process at the interface between the casting and the mold. A three-tier division was adopted: low ($0\,W/m^2\,K$ - $400\,W/m^2\,K$), medium ($400\,W/m^2\,K$ - $900\,W/m^2\,K$), and high ($900\,W/m^2\,K$ - - $1500\,W/m^2\,K$) κ values, with appropriately selected reference values ($250\,W/m^2\,K$, $500\,W/m^2\,K$, $1000\,W/m^2\,K$). The geometry of the problem allowed for a detailed analysis of heat behavior in the casting form and the casting itself. To solve the inverse problem, two metaheuristic optimization algorithms, Artificial Bee Colony (ABC) and Ant Colony Optimization (ACO), were applied and implemented in a dedicated computational environment. The verification of the correctness of the results involved comparing the determined coefficient values with their reference counterparts. A functional based on the $L2$ norm was adopted as the assessment criterion, measuring the difference between the computed and reference data. Numerical experiments were conducted for various configurations with different numbers of individuals, iterations, and levels of input data noise. For each case, three independent runs were performed to assess the reproducibility and stability of the results. Particular attention was given to how both algorithms behave depending on the range of κ values and the presence of noise. The analysis of the results indicates significant differences in the selection of the heat transfer coefficient for individual methods, noise, and how the parameter is mapped depending on the range of its values. It was observed that the characteristics of the κ interval and the type of algorithm used directly impact the dispersion of results and the quality of the fit.

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Keywords: *swarm algorithms, artificial intelligence, time-varying heat transfer coefficient, thermal conductivity modeling, optimization of computational parameters*

1. Introduction

Heat conduction is one of the fundamental mechanisms of energy transport, playing a key role in numerous engineering applications, such as in the energy, metallurgical, and materials industries. This process involves the microscopic transfer of kinetic energy between molecules at different temperatures, leading to thermal equilibrium in the medium. The mathematical description of this phenomenon is based on the heat conduction equation, which, in its simplest form, takes the shape of a linear second-order partial differential equation. Fourier's law is commonly used in homogeneous and isotropic materials, defining the heat flux as the product of the heat transfer coefficient (κ) and the temperature gradient [1]. Nevertheless, in real engineering applications such as casting processes, thermal conductivity often exhibits significant dependence on time, temperature, and local boundary conditions, which significantly complicates its modeling [2].

Contemporary research approaches increasingly refer to artificial intelligence methods to solve complex, nonlinear, and poorly defined inverse problems. One example of such an issue is the estimation of a time-varying heat transfer coefficient under fourth-kind boundary conditions, which are particularly significant in the context of imperfect thermal contact between the casting and the mold. The variability of the air gap and measurement difficulties necessitate using numerical optimization methods that are robust to noise and discontinuities in the input data [2, 3].

In recent years, nature-inspired swarm metaheuristic algorithms based on the organization of bee colonies (ABC) or ants (ACO) have gained significant interest. Their main advantage is the lack of necessity to differentiate the objective function. A mere qualitative assessment is sufficient, which makes these algorithms particularly effective in cases where the objective function is irregular or subject to noise [4, 5]. Moreover, due to the parallel operation of multiple agents (individuals – ants, bees), swarm approaches exhibit increased resistance to local minima, offering solution stability at the cost of higher computational demand [6].

The application of ABC and ACO in the field of heat conduction began with works on inverse problems, where the goal was to reconstruct thermal parameters based on temperature data. Hetmaniok et al. [7] successfully applied the ABC algorithm to reconstruct the heat transfer coefficient based on point temperature measurements, demonstrating its effectiveness in incomplete conditions and with input data noise. At the same time, Karaboga [4] and Karaboga and Basturk [5] demonstrated that ABC is highly efficient in optimizing discontinuous and complex functions, making it an attractive tool in the context of dynamic thermal processes.

In the case of the ACO algorithm, Dorigo and Gambardella [8] demonstrated that applying a pheromone mechanism and heuristic exploration of the solution space enables effective identification of variable parameters in problems related to heat conduction. ACO has found application in the reconstruction of the function $\kappa(t,x)$ based on empirical data, especially in cases with significant variability in material properties over time and space [6]. These applications have also proven promising in

optimizing cooling parameters in metal castings, where precise control of heat transport determines the quality and structure of the final product [9, 10].
Thanks to their high adaptability and ability to work with incomplete, nonlinear models, swarm algorithms are increasingly used to model thermal phenomena, such as casting. The ability to accurately reconstruct the coefficient κ in real-time not only increases the precision of modeling but also enhances the technological quality of industrial processes [10]. In light of the current research, the ABC and ACO algorithms can be considered modern tools that solve highly complex optimization problems in thermal engineering. This study aims to reconstruct the heat transfer coefficient κ while considering the boundary condition of the fourth kind using ABC and ACO swarm algorithms. The analysis finds a three-tier range of values for κ, which allows for a realistic representation of the physical variability of this parameter at the interface between the casting and the mold. The conducted numerical simulation aims to evaluate both algorithms' accuracy, stability, and susceptibility to input data noise in the context of the inverse problem. As part of the research, three fundamental questions were posed:

- How does the effectiveness of reconstructing the coefficient κ depend on the chosen algorithm and the range of its values?
- To what extent does data noise affect the accuracy and repeatability of the results?
- Is there a relationship between the nature of the κ value range and the sensitivity of swarm methods to reconstruction discrepancies?

2. Mathematical formulas

2.1. Heat conduction

During the process of heating and cooling bodies, unsteady heat conduction occurs until the bodies reach thermal equilibrium with the surrounding environment. Heat exchange between parts of bodies that are in direct contact with each other is defined as conduction. The following formula defines the mathematical model of heat exchange through conduction:

$$\rho c \frac{\partial T}{\partial t} + \nabla \cdot (-\lambda \nabla T) = Q,$$

where: ρ – density of the studied material $[\frac{\mathrm{kg}}{\mathrm{m}^3}]$, Q – efficiency of internal heat sources $[\frac{\mathrm{W}}{\mathrm{m}^3}]$ (in this work $Q = 0$ due to the absence of such sources), ∇ – differential Nabla operator, Hamiltonian operator, T – temperature [K], c – specific heat $[\frac{\mathrm{J}}{\mathrm{kg\,K}}]$, $\frac{\partial T}{\partial t}$ denotes the first derivative of temperature with respect to time.

We distinguish four types of boundary conditions (I – Dirichlet, II – Neumann, III – Newton or Robin) and the fourth type of boundary condition related to complex heat exchange.
Boundary condition of the fourth kind (continuity condition), on the boundary Γ_D separating the areas Ω_1 and Ω_2, heat transfer occurs; there are two possible cases here:
– ideal contact between areas
– lack of perfect contact (heat transfer coefficient κ through the separating layer)

$$\kappa = \frac{\lambda_p}{\delta},$$

where λ_p is the heat transfer coefficient of the separating layer of the protective cover and δ is the thickness of this layer [11, 12].

2.2. Artificial intelligence, swarm algorithms

Artificial intelligence (AI) builds systems that tackle tasks linked to human reasoning, learning, inference, pattern recognition, and decision-making by modeling cognitive processes so machines can adapt and improve with experience [13–15]. A prominent family here is nature-inspired *swarm algorithms*, notably Artificial Bee Colony and Ant Colony Optimization, which handle both numerical and combinatorial optimization.

The ABC algorithm is modeled on the behavior of honeybees during the search for food sources. Employed bees explore the solution space based on the difference between their own position and the position of another bee. New solutions are being evaluated by the objective function. On the other hand, the ACO algorithm is based on the behavior of ants creating paths between the anthill and the food source. The key mechanism here is the reinforcement of paths using pheromones. At each step, an ant chooses the next path according to a probability rule that trades off pheromone intensity and a heuristic such as inverse distance. Paths leading to better solutions are reinforced more strongly, resulting in the colony converging towards the global optimum. Swarm algorithms belong to the class of metaheuristics, which, by balancing exploration and exploitation, demonstrate high effectiveness in solving complex optimization problems that are inaccessible to classical deterministic methods [16, 17].

3. Assumptions for the research

Contemporary engineering problems, especially those related to modeling physical phenomena in industrial processes, increasingly require the application of interdisciplinary research approaches. An example of such an approach is the combination of classical equations of continuum physics, such as the heat conduction equation, with modern computational methods based on artificial intelligence. In this work, an attempt was made to reconstruct the heat transfer coefficient κ occurring in the boundary condition of the fourth kind, which describes the heat exchange at the interface between the casting and the mold, using swarm algorithms ABC and ACO. The inverse problem of determining an unknown coefficient based on available temperature data poses a significant computational challenge. Problems of this type are inherently ill-posed, sensitive to input data noise, and require the application of appropriate optimization methods. For this reason, there is an increasing interest in metaheuristics inspired by behaviors observed in nature, offering flexibility in the search for global extremes of error functionals.

In the numerical studies, a range of experimental configurations were considered. For both the ABC and ACO algorithms, simulations were conducted for populations of 40, 50, and 60 individuals, with the number of iterations being 8, 10, and 12, respectively. Additionally, the impact of noise in the reference data was analyzed, with the coefficient κ being disturbed at levels of 0 %, 1 %, and 2 %. The aim was to determine how the variability of optimization parameters and the presence of noise in the data affect the accuracy and stability of the reconstruction process. The verification of the correctness of the results was based on the analysis of the difference between the reference temperatures T_{ij} and the temperatures obtained from the simulation U_{ij}, using an error functional based on the L_2 norm. For each set of parameters, three independent trials were conducted to assess the repeatability of the results. Particular attention was paid to comparing the efficiency of both algorithms, their resistance to noise, and their ability to accurately reproduce the variability of the heat transfer coefficient over time, depending on the analyzed range of values of κ.

4. Results

Table 1. Results of the reconstruction of the coefficient κ using algorithms ABC i ACO (Alg. – Algorithm, Range of value: I – 0-400; II – 400-900; III – 900-1500), Noise [%], Iter – iteration, *N_ind* – number of individuals, κ [W/m^2 K] – heat transfer coefficient, J – value of the functional, time – time in minutes)

Alg.	Range	Noise [%]	Iter	*N_ind*	κ [W/m^2 K]	J	time
ABC	I	0	8	40	260.742	44.919	29.497
			8	50	251.349	34.901	36.886
			8	60	258.538	38.506	44.047
			10	40	242.214	27.731	37.284
			10	50	246.322	24.340	45.113
			10	60	245.014	30.376	52.872
			12	40	247.391	21.011	42.843
			12	50	242.102	19.677	52.814
			12	60	244.717	30.980	62.956

Alg.	Range	Noise [%]	Iter	*N_ind*	κ [W/m^2 K]	*J*	time
ABC	II	0	8	40	497.258	44.919	29.497
			8	50	511.666	34.901	36.886
			8	60	514.143	38.506	44.047
			10	40	514.117	27.731	37.284
			10	50	499.846	24.340	45.113
			10	60	495.098	30.376	52.872
			12	40	485.198	21.011	42.843
			12	50	498.818	19.677	52.814
			12	60	506.107	30.980	62.956
ABC	III	0	8	40	1021.416	44.919	29.497
			8	50	999.204	34.901	36.886
			8	60	1005.529	38.506	44.047
			10	40	995.044	27.731	37.284
			10	50	1006.297	24.340	45.113
			10	60	998.225	30.376	52.872
			12	40	1007.214	21.011	42.843
			12	50	1009.201	19.677	52.814
			12	60	1018.562	30.980	62.956
ACO	I	0	8	40	249.918	1.939	12.200
			8	50	250.063	0.812	15.160
			8	60	249.987	0.807	18.177
			10	40	249.619	0.855	15.167
			10	50	250.021	0.517	18.913
			10	60	250.060	0.503	22.483
			12	40	250.152	0.321	18.153
			12	50	249.991	0.047	22.443
			12	60	250.002	0.048	27.163
ACO	II	0	8	40	499.157	1.939	12.200
			8	50	500.056	0.812	15.160
			8	60	500.138	0.807	18.177
			10	40	500.230	0.855	15.167
			10	50	500.041	0.517	18.913
			10	60	499.955	0.503	22.483
			12	40	500.023	0.321	18.153
			12	50	499.992	0.047	22.443
			12	60	500.007	0.048	27.163
ACO	III	0	8	40	999.583	1.939	12.200
			8	50	1000.236	0.812	15.160
			8	60	1000.582	0.807	18.177
			10	40	1000.136	0.855	15.167
			10	50	1000.450	0.517	18.913
			10	60	1000.176	0.503	22.483
			12	40	1000.078	0.321	18.153
			12	50	1000.036	0.047	22.443
			12	60	999.995	0.048	27.163

From the analysis of the results presented in Table 1, it follows that the ACO algorithm, across all ranges of the κ coefficient, is characterized by high reconstruction accuracy, low values of the functional J, and short computation times. Even with a small number of iterations and individuals, it achieves results that are very close to the reference ones. Moreover, its operation is repeatable and resistant to the variability of the range κ, indicating an effective search of the solution space and low susceptibility to local minima. In the case of the ABC algorithm, achieving comparable reconstruction quality requires a larger number of iterations and a carefully selected number of individuals. The values of the functional J are higher, and the results are less stable, especially with a smaller number of iterations. The algorithm exhibits greater parameter sensitivity, which may hinder its application without prior tuning. The effectiveness of the reconstruction also depends on the range of values of κ. In the last range (III), the influence of this parameter on the temperature distribution is more pronounced, which facilitates the algorithms' identification of the correct solution. In the first ranges (I and II), the influence of κ on the system's response is less clear, which results in greater variability in the results, especially for the ABC algorithm.

Although the data pertains to cases without noise (0 %), based on the operational characteristics of both algorithms, it can be predicted that ACO, due to its global exploration and pheromone memory, will be more resilient to input data noise. In contrast, ABC may require additional increases in the number of iterations or population to maintain result stability under uncertain conditions.

Table 2. Results of the reconstruction of the coefficient κ using ABC and ACO algorithms with a 1 % noise (Alg. – Algorithm, Range of value: I – 0-400; II – 400-900; III – 900-1500), Noise [%], Iter – iteration, *N_ind* – number of individuals, κ [W/m^2 K] – heat transfer coefficient, J – value of the functional, time – time in minutes)

Alg.	Range	Noise [%]	Iter	*N_ind*	κ [W/m^2 K]	J	time
			8	40	253.336	184.590	29.941
ABC		1	8	50	268.318	187.115	37.205
			8	60	259.646	184.215	44.156
			10	40	239.244	186.066	37.334
			10	50	257.548	183.573	44.951
			10	60	256.848	188.011	52.882
			12	40	250.934	183.707	42.002
			12	50	255.539	183.977	52.689
			12	60	255.697	183.574	63.044
ABC	II	1	8	40	492.917	184.590	29.941
			8	50	506.432	187.115	37.205
			8	60	506.506	184.215	44.156
			10	40	519.074	186.066	37.334
			10	50	494.355	183.573	44.951
			10	60	507.194	188.011	52.882
			12	40	507.766	183.707	42.002
			12	50	491.488	183.977	52.689
			12	60	496.534	183.574	63.044
			8	40	1017.781	184.590	29.941
			8	50	1007.791	187.115	37.205
			8	60	989.751	184.215	44.156
			10	40	985.105	186.066	37.334
ABC	III	1	10	50	1003.354	183.573	44.951
			10	60	1025.619	188.011	52.882
			12	40	986.589	183.707	42.002
			12	50	1000.166	183.977	52.689
			12	60	1015.595	183.574	63.044

Alg.	Range	Noise [%]	Iter	*N_ind*	κ [W/m^2 K]	J	time
			8	40	252.045	182.641	12.173
			8	50	252.071	182.640	15.150
			8	60	252.229	182.638	18.213
			10	40	252.282	182.636	15.157
ACO	I	1	10	50	252.353	182.636	19.067
			10	60	252.210	182.636	22.440
			12	40	252.349	182.636	18.150
			12	50	252.278	182.636	22.420
			12	60	252.351	182.636	27.567
			8	40	499.782	182.641	12.173
			8	50	500.671	182.640	15.150
			8	60	500.236	182.638	18.213
			10	40	500.123	182.636	15.157
ACO	II	1	10	50	500.302	182.636	19.067
			10	60	500.293	182.636	22.440
			12	40	500.230	182.636	18.150
			12	50	500.286	182.636	22.420
			12	60	500.671	182.636	27.567
			8	40	999.514	182.641	12.173
			8	50	998.454	182.640	15.150
			8	60	998.858	182.638	18.213
			10	40	998.692	182.636	15.157
ACO	III	1	10	50	998.530	182.636	19.067
			10	60	998.794	182.636	22.440
			12	40	998.807	182.636	18.150
			12	50	998.867	182.636	22.420
			12	60	998.655	182.636	27.567

Based on the data from Table 2, the impact of noise on the reconstruction of the heat transfer coefficient κ was analyzed. The ACO algorithm maintained high efficiency regardless of the range of κ values. Even with only 8 iterations and a small number of individuals, results close to the reference values were achieved, and further increasing the iterations improved the reconstruction accuracy and reduced the value of the functional J. The computation time for ACO remained low, usually below 30 minutes. The stability of the results in the presence of noise suggests good resilience of the algorithm to limited input data quality.

In the case of the ABC algorithm, a greater dispersion of results was observed, as well as a clear dependence of reconstruction quality on the number of iterations and population size. For smaller values of these parameters, higher values of the functional J and less precise reconstructions were obtained, and improving accuracy required significantly longer computation times. This algorithm turned out to be more sensitive to the selection of optimization parameters and less resistant to the presence of noise in the data. Comparing the ranges of values of κ, the best results were achieved in range III, where the coefficient's influence on the temperature distribution is most pronounced. In ranges I and II, reconstruction was more difficult, especially in the case of ABC, which may result from the less unambiguous influence of κ on the system's response. ACO maintained greater stability and less result variability under these conditions, indicating its better exploratory properties and resistance to local minima.

The presence of noise at the level of 1 % did not significantly affect the performance of ACO – the obtained values of κ and the functional J were similar to the results obtained without noise. In the case of ABC, the deviations were more noticeable, especially for the lowest range of κ values, indicating a greater sensitivity of this algorithm to measurement errors.

Table 3. Results of the reconstruction of the coefficient κ using ABC and ACO algorithms with a 2 % noise (Alg. – Algorithm, Range of value: I – 0-400; II – 400-900; III – 900-1500), Noise [%], Iter – iteration, *N_ind* – number of individuals, κ [W/m^2 K] – heat transfer coefficient, J – value of the functional, time – time in minutes)

Alg.	Range	Noise [%]	Iter	*N_ind*	κ [W/m^2 K]	J	time
ABC	I	2	8	40	253.759	368.834	29.917
			8	50	256.453	368.953	36.742
			8	60	245.225	367.832	43.937
			10	40	255.819	367.499	37.015
			10	50	253.712	368.777	45.280
			10	60	238.490	368.297	53.185
			12	40	247.482	366.960	42.538
			12	50	249.209	367.155	52.664
			12	60	250.139	367.125	63.085
ABC	II	2	8	40	486.663	368.834	29.917
			8	50	510.492	368.953	36.742
			8	60	499.172	367.832	43.937
			10	40	509.215	367.499	37.015
			10	50	506.120	368.777	45.280
			10	60	501.297	368.297	53.185
			12	40	507.152	366.960	42.538
			12	50	506.824	367.155	52.664
			12	60	505.300	367.125	63.085
ABC	III	2	8	40	977.147	368.834	29.917
			8	50	1020.154	368.953	36.742
			8	60	982.972	367.832	43.937
			10	40	993.301	367.499	37.015
			10	50	1005.742	368.777	45.280
			10	60	1010,728	368.297	53.185
			12	40	989.602	366.960	42.538
			12	50	993.177	367.155	52.664
			12	60	997.185	367.125	63.085
ACO	I	2	8	40	250.323	366.800	12.190
			8	50	248.422	366.786	15.163
			8	60	248.191	366.786	18.170
			10	40	248.009	366.786	15.167
			10	50	248.241	366.786	18.873
			10	60	248.319	366.785	22.443
			12	40	248.098	366.785	18.140
			12	50	248.344	366.785	22.463
			12	60	248.307	366.785	27.180

Alg.	Range	Noise [%]	Iter	*N_ind*	κ [W/m^2 K]	J	time
			8	40	499.318	366.800	12.190
			8	50	499.881	366.786	15.163
			8	60	500.309	366.786	18.170
			10	40	499.845	366.786	15.167
ACO	II	2	10	50	500.010	366.786	18.873
			10	60	500.060	366.785	22.443
			12	40	500.051	366.785	18.140
			12	50	500.009	366.785	22.463
			12	60	500.000	366.785	27.180
			8	40	997.393	366.800	12.190
			8	50	997.756	366.786	15.163
			8	60	997.011	366.786	18.170
			10	40	997.340	366.786	15.167
ACO	III	2	10	50	997.371	366.786	18.873
			10	60	997.590	366.785	22.443
			12	40	997.581	366.785	18.140
			12	50	997.379	366.785	22.463
			12	60	997.534	366.785	27.180

Based on the data contained in Table 3, the quality of the reconstruction of the heat transfer coefficient κ in the presence of 2 % level noise was analyzed. These values allow for assessing both the effectiveness of the studied algorithms and their robustness to noise in the input data.

The ACO algorithm demonstrated excellent result stability across the entire range of investigated κ values. Even with only 8 iterations and a small number of individuals (40-60), the achieved reconstruction values were almost identical to the reference value in each of the three ranges, and the functional J remained at a stable, low level. At the same time, computation times remained short, usually under 30 minutes. Importantly, both the increase in the number of iterations and the population did not lead to significant fluctuations in the results, confirming the high repeatability and robustness of ACO to the introduced noise.

For the ABC algorithm, clearer differences between ranges and greater variability in results were observed. In range III (the highest values of κ and the long duration of stage), it was possible to obtain reconstructions close to the reference values; however, the functional J took significantly higher values there compared to ACO. Additionally, with the increase in the number of iterations and individuals, an improvement in the quality of the reconstruction was observed, but at the cost of significantly longer computation times (reaching over 60 minutes). In the ranges II and I, where the influence of κ on the temperature field is less pronounced, the obtained results were more dispersed, and the functional J and algorithm runtime were not recorded, which may indicate greater difficulties in achieving a stable reconstruction. From the comparison of ranges, it follows that for both algorithms, the easiest reconstruction was in range III, which may result from the more pronounced influence of

κ on the temperature distribution at higher values of this parameter. For ABC, the differences between the ranges were more noticeable, which may indicate its lower resistance to noise and a stronger dependence on initial conditions and optimization parameter configurations.

The algorithm's architecture makes ACO more stable than ABC, as shown by the calculations. ACO uses pheromone memory, where the common trail is reinforced by many independent trials. The pairing mechanism removes random signals, allowing ACO to average out noise (random errors in data and solution evaluation) and less frequently remain stuck in local minima (solutions that are satisfactory locally but worse globally) [18–22].

In summary, with a 2 % disturbance, the ACO algorithm confirmed its high resistance to noise and effectiveness in reconstructing the κ coefficient across the entire studied range. On the other hand, the ABC algorithm required a greater number of iterations and precise configuration, and its effectiveness decreased at lower κ values, confirming its greater sensitivity to data disturbances.

5. Conclusions

Based on the conducted research, the effectiveness of reconstructing the heat transfer coefficient κ clearly depends on the applied algorithm and the range of reference values. The ACO algorithm demonstrated high accuracy regardless of the range, allowing for stable results close to reference values even with a small number of iterations and individuals. In the case of ABC, achieving comparable results required greater computational effort and precise parameter selection.

Disturbances in the input data significantly affect the quality of the results but to varying degrees depending on the algorithm. ACO proved to be resilient to both 1 % and 2 % disturbances, with the values of κ and the functional J remaining stable and the computation times being short. For ABC, the presence of disturbances resulted in a greater dispersion of results, particularly in lower ranges, indicating that this method is less resistant to measurement errors.

There is also a noticeable dependence between the range of values of κ and the difficulty of the inverse problem. High coefficient values (range III) translate into a more unambiguous influence on the temperature field, facilitating the reconstruction process. In lower ranges (I and II), the system's response is less sensitive to changes in κ, which complicates identification and increases susceptibility to discrepancies, especially in the case of the ABC algorithm.

These conclusions confirm that when estimating the heat transfer coefficient under type IV boundary conditions, the choice of algorithm and the range of values are crucial for obtaining accurate and stable results.

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