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# DEVELOPMENT OF A DYNAMIC VIBRATION ABSORBER IN THE FORM OF A VARIABLE INERTIA PENDULUM

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**Abstract.** Nowadays, to maximize space utilization, multi-story buildings are designed to occupy as little surface area as possible. However, the high slenderness of these structures makes them susceptible to forces caused by factors such as wind and tectonic plate movements. Prolonged exposure to vibrations reduces the building's lifespan and leads to its degradation. To minimize the impact of vibrations while maintaining slenderness and relatively low structural rigidity, a solution in the form of a dynamic vibration absorber can be applied. Structures already use absorbers in the form of pendulums, as seen in the Taipei 101 skyscraper, for example. The subject of study is an absorber in the form of a pendulum with a variable moment of inertia, achieved by extending the cable, which allows the structure to adapt to current forces. This significantly reduces the time of exposure to harmful influences. A skyscraper model with a variable-length pendulum was developed and analyzed through simulations, confirming the hypothesis of a significant reduction in exposure.

***MSC 2010:*** *37M05, 65L05, 70E55*
***Keywords:*** *damping, pendulum, pendulum with variable inertia, vibrations, dynamic vibration absorber, vibration control, adaptive damping system, vibration damping in skyscrapers, diagnostics, diagnostics in civil engineering*

## 1. Introduction

Modern urban centers are characterized by high-rise developments that maximize spatial utilization through expansive and functionally efficient structures with large usable areas. Tall and narrow buildings are significantly susceptible to external forces, such as wind loads, necessitating the use of materials with enhanced mechanical strength. An alternative approach to alleviating these structural challenges involves increasing the building's mass.

To address this issue, innovative solutions such as tuned mass dampers (TMDs) with adaptable inertia have been implemented. The application of dynamic vibration absorbers has been successfully utilized in the Taipei 101 in Taiwan to damp the wind-induced oscillations. Such a solution not only attenuates minor building movements, reducing discomfort for occupants, but also suppresses severe oscillations caused by earthquakes or strong winds, while maximizing fatigue strength. However, in this skyscraper, the pendulum cord length remains fixed, thereby limiting the full potential of the suspended mass in vibration suppression. This paper aims to apply an advanced dynamic model of the elastic vibration absorber that allows for the tuning of pendulum cable length, enabling precise control of oscillation damping. The concept assumes a modelling of the variable-length of pendulum by an additional degree that describes cable elongation. This approach is examined using both a physical model (test stand) and a computational model implemented within the `DynPy` environment – an analytical software system developed by Authors. This approach enables both the testing of an innovative dynamic vibration absorber and the comparison of simulation results with real-world measurements, thereby allowing for the verification of the device's actual performance as well as the accuracy of the simulation. Such an approach allows to state that extra degrees of freedom in the model lead to improved performance in modelling of the problem of adaptable TMD's and justifies simplification of rigid cable in the experimental study. The solution also facilitates building diagnostics by quantifying changes in the building's parameters dependent on the internal mass distribution and degradation, with stiffness serving as the key parameter. Prolonged exposure of skyscrapers to external excitations leads to gradual structural degradation, which may ultimately result in building collapse. Another significant issue associated with tall and slender structures is the high amplitude of vibrations, which can cause discomfort for occupants on the upper floors. The presented damper, designed as a pendulum with variable inertia, effectively mitigates oscillations, addressing these challenges regardless of prevailing conditions. The ability to fine-tune the pendulum's parameters is a considerable advantage, particularly when the building is subjected to varying external forces.

Vibration dampers are widely used in modern skyscrapers [1, 2], as they enable the design of taller and more slender structures while simultaneously enhancing their safety and user comfort [3–5]. This article analyzes the application of a pendulum with variable inertia as an effective method of vibration damping in slender and high-rise structures [6–8]. Such a solution allows for the reduction of vibration amplitudes [9, 10], which translates into an extended service life of the structure and improved operational conditions [9, 11, 12].

The results obtained from the physical model were compared with computer simulations performed in the DynPy system in the conducted research. This approach made it possible to determine the optimal parameters of the damping system, such as the length of the suspension cable, the mass of the pendulum, and its elasticity, while taking into account the specific characteristics of the designed building. These findings can provide valuable insights not only for engineers involved in skyscraper design but also in other fields where vibration damping plays a crucial role in increasing structural durability and user comfort, such as the construction of luxury yachts [13], factory buildings [14], or large machines [15, 16].

Enhancement strategies for passive PTMDs primarily involve either the optimization of dissipative elements, like viscous dampers, or the introduction of system nonlinearity through mechanisms such as additional stoppers [17, 18]. Alternatively, for systems with very low natural frequencies where traditional dampers may be impractical, Tuned Liquid Dampers (TLDs) utilizing fluid sloshing have also been explored as a compact and effective solution [19].

It is worth noting that tuned mass dampers are not the only available solution; an alternative approach involves U-shaped liquid column dampers [20, 21]. Another example of active vibration damping is the implementation of a double pendulum with variable mass elements [22], as well as smart-structures [23–25].

## 2. Numerical model implementation

This model was developed using the DynPy library and is simulated within it as DampedTrolleyWithPendulumVariableInertia. The system is modeled as a trolley on spring with damper and a concentrated mass attached to the trolley via a cord of variable length (Fig. 1).

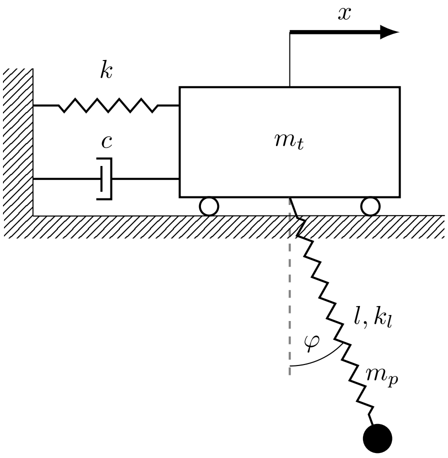

Fig. 1. Idealized model of the investigated object

The presented model accurately captures the behavior of a skyscraper and facilitates the derivation of equations of motion. The equations of motion were derived based on Lagrange's formulation, assuming small oscillations and neglecting damping in the first approximation.

The linearised model describes the dynamics in the neighbourhood of this equilibrium. Such a linearisation is adequate for tall and slender structures, providing a sufficient approximation of their dynamic response:

$$kx+(m_p+m_t)\ddot{x}-F\sin(\Omega t)+m_p\left(\frac{gl}{k_l}+l\right)\ddot{\varphi}=0 \tag{1}$$

$$m_p\left(\frac{gl}{k_l}+l\right)^2\ddot{\varphi}+m_p\left(\frac{gl}{k_l}+l\right)\ddot{x}+gm_p\left(\frac{gl}{k_l}+l\right)\varphi=0 \tag{2}$$

$$k_lu+m_p\ddot{u}-gm_p=0 \tag{3}$$

The linearized equations enabled the simulations to be performed and the numerical data required for further analysis to be obtained. The third linear equation indicates that this elongation is independent; hence, it may be regarded as a controllable factor, analogous to an actuator.

The class DampedTrolleyWithPendulumVariableInertia has been implemented as an extension of the base model TrolleyWithElasticPendulum in order to account for damping effects and the variability of pendulum inertia. Within the class, the components method specifies the individual subsystems that compose the full model. By structuring the model in this modular manner, the class facilitates the derivation of equations of motion and their subsequent linearisation.

The model provides a sufficiently accurate representation of the dynamic response of tall and slender structures, such as high-rise buildings. Furthermore, it enables the analysis of vibration control strategies, where the variable-inertia pendulum can be treated as a controllable device analogous to a tuned mass damper.

## 3. Physical model

The physical model consists of rigid elements representing the equivalent (reduced) mass of the vibrating object, connected by elastic connectors in the form of flat springs with defined stiffness. Such a configuration allows for the reproduction of excitations induced by strong wind gusts through the application of force to its upper section.

It sufficiently corresponds to the actual vibrations of a skyscraper subjected to wind-induced excitations. The vibration absorber consists of a cord fixed at the upper part of the structure, with a mounting mechanism that allows for adjusting its length, thereby altering the pendulum's moment of inertia. To replicate a point mass, a metal weight has been used. An accelerometer mounted on the upper beam of the model was used for horizontal acceleration measurements (Fig. 2).

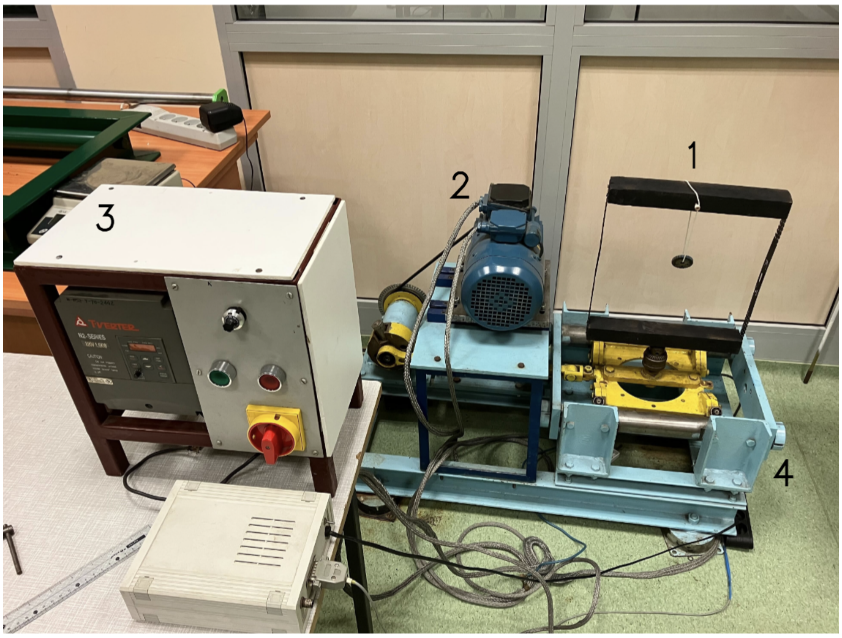

Fig. 2. Physical model of a skyscraper utilizing a dynamic vibration absorber with a variable pendulum moment of inertia. Legend: 1 – model of a skyscraper with a pendulum, 2 – electric motor, 3 – frequency inverter, 4 – shaking table

The experimental setup comprised an excitation mechanism, a structural model, and a measurement system integrating sensors with data acquisition hardware. The recorded signals were processed using acquisition cards, providing reliable input for the analysis of the structural response under controlled excitation (Fig. 3).

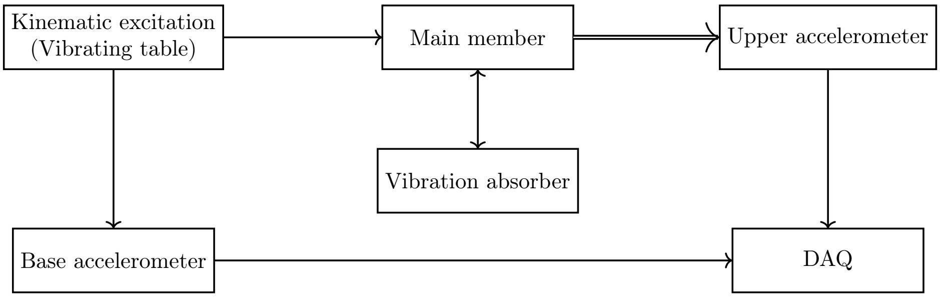

Fig. 3. Flow diagram of measuring procedure

## 4. Experimental measurements and results

The results obtained during measurements on the physical model with the application of excitation by setting initial conditions. The excitation was introduced by imposing successive measured values of the initial displacement of the structure.

The vibrations exhibit characteristics typical of an underdamped system in the absence of the pendulum. The acceleration amplitude decreases gradually, leading to prolonged exposure to vibration amplitudes that may be detrimental to the structural integrity (Fig. 4). When a pendulum of 25 cm in length is implemented with an initial displacement of 8 cm, an increased initial acceleration amplitude is observed, which directly results from the initial conditions. Despite the higher initial amplitude, a significant improvement in vibration damping is evident. The damping effect occurs much faster compared to the model without a vibration absorber, even though the initial displacement is larger.

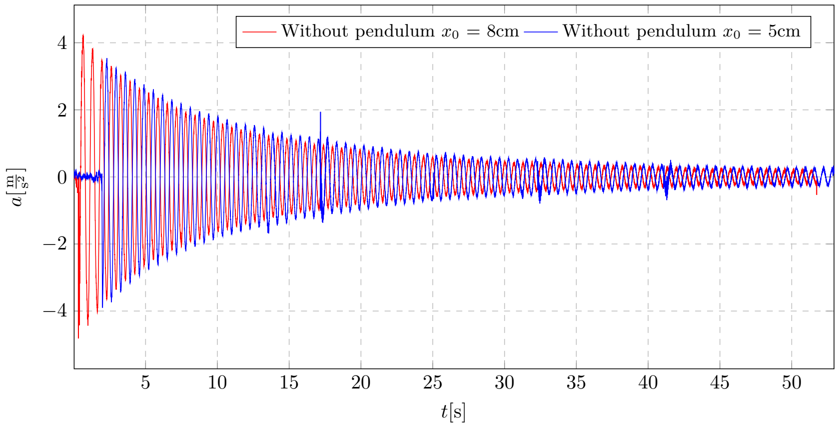

Fig. 4. Without pendulum, comparison of 8 cm and 5 cm initial displacements

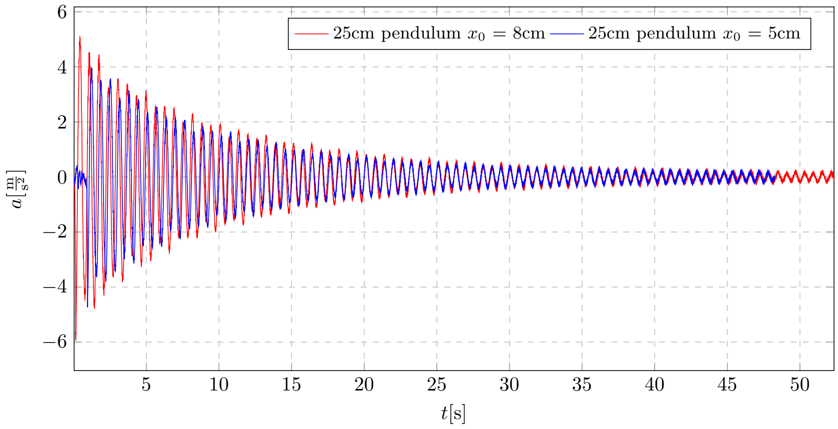

Fig. 5. 25 cm pendulum, comparison of 8 cm and 5 cm initial displacements

The vibrations for an initial displacement of 5 cm are similar to those observed for an 8 cm displacement. A notable improvement is also evident compared to the structure without a vibration damping system (Fig. 5).

The pendulum length of 15 cm demonstrates a considerable enhancement in the damping characteristics of the oscillations. The acceleration values exhibit irregular behavior, with significant reductions occurring approximately every three oscillation periods. The natural period of the structure has also been reduced (Fig. 6).

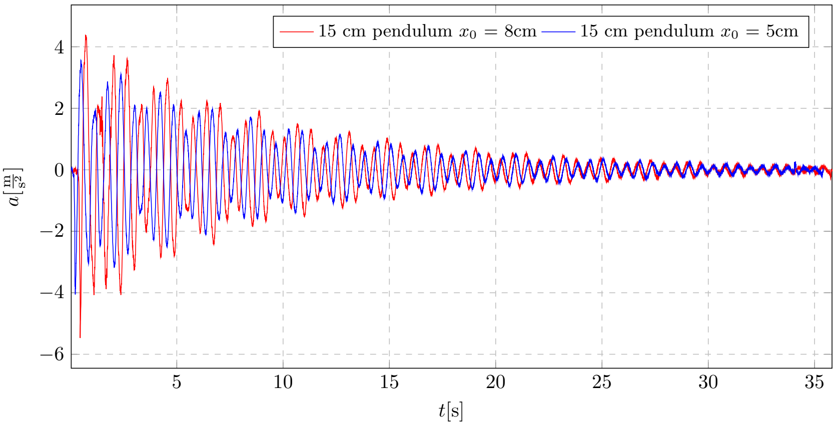

Fig. 6. 15 cm pendulum, comparison of 8 cm and 5 cm initial displacements

Initial displacement of 5 cm does not introduce significant changes in the acceleration characteristics. The vibration absorber continues to operate in a similar manner.

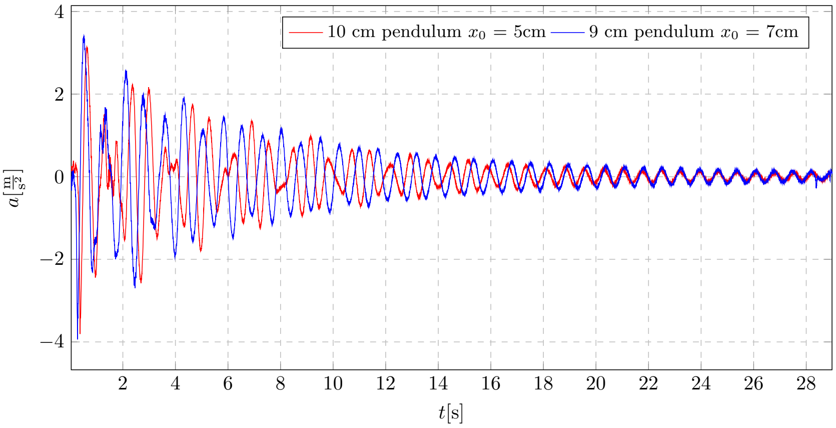

Fig. 7. 10 cm and 9 cm pendulum, comparison of different initial displacements

A pendulum length of 10 cm proved to be the optimal solution for the presented high-rise building model. The acceleration characteristics indicate not only a substantial reduction in acceleration amplitude for approximately half of the oscillations but also a further increase in the oscillation period compared to the 15 cm pendulum length (Fig. 7). Further reduction of the pendulum length led to a decrease in the effectiveness of the vibration absorber. The shortening of the suspension cable was also constrained by the mounting method and the size of the attached pendulum mass.

## 5. Data selection and parameterization for the model

The data were initially selected experimentally, guided by the known physical representation of the model and an understanding of the influence of each parameter on its behavior. Another criterion for parameter selection was to illustrate characteristic cases: overdamping, underdamping and critical damping. The selection of data for the simulation was conducted to replicate the experimentally obtained results. Fundamental relationships for vibration frequencies were utilized to reproduce the system's characteristics. From the characteristics obtained during the conducted tests, the oscillation period of the skyscraper model can be easily determined. Knowing the oscillation period, the stiffness for the simulation can be calculated using the following equations:

$$\omega^2 = \frac{k}{m} \tag{4}$$

$$k = m\omega^2 \tag{5}$$

$$k = \frac{4\pi^2 m_{trolley}}{T^2} \tag{6}$$

The system's decay time to approximately 5 percent of the initial amplitude was used to determine the damping value. This time corresponds to three times the relative damping coefficient.

$$T = \frac{t_{05}}{3} \tag{7}$$

Using the equation of mechanical vibrations described by the following formula:

$$x = e^{-\frac{cmt}{2}} \left(A\cos\left(\omega t\right) + B\sin\left(\omega t\right)\right) \tag{8}$$

and a relationship:

$$\frac{1}{T} = \frac{cm}{2} \tag{9}$$

we obtain the replicated damping value of the physical model:

$$c = \frac{2m_{trolley}}{T} \tag{10}$$

The obtained values were adjusted to achieve a more accurate representation of the results. By utilizing the previously described methodology, the parameter values were determined in Table 1.

Table 1. Vibration damping parameters selected to correspond with the physical model

| Parameter | $c\,[\frac{\text{Ns}}{\text{m}}]$ | $m_t$ [kg] | $m_p$ [kg] | $l$ [m] | $F$ [N] | $\Omega\,[\frac{\text{rad}}{\text{s}}]$ | $k\,[\frac{\text{N}}{\text{m}}]$ | $g\,[\frac{\text{m}}{\text{s}^2}]$ | $k_l\,[\frac{\text{N}}{\text{m}}]$ |
|---|---|---|---|---|---|---|---|---|---|
| Value | 0.02432 | 0.7 | 1 | $\frac{0.7848}{\pi^2}$ | 0 | 0 | 63.6889 | 9.81 | 999999 |

After substituting the selected data, the system equation takes the form:

$$63\ddot{x} + 75\dot{x} + 1.8\ddot{\varphi} + 56.875\pi^2 x = 0 \tag{11}$$

$$75\dot{\varphi} + 1.08\ddot{\varphi} + 17.658\varphi + 1.8\ddot{x} = 0 \tag{12}$$

Shown equations are TDOF, which are $x$, $\varphi$, $u$, equations of motion. On the basis of the obtained data and the selected parameters, the characteristics of damped vibrations in the physical model and in the simulation model can be compared (Fig. 8).

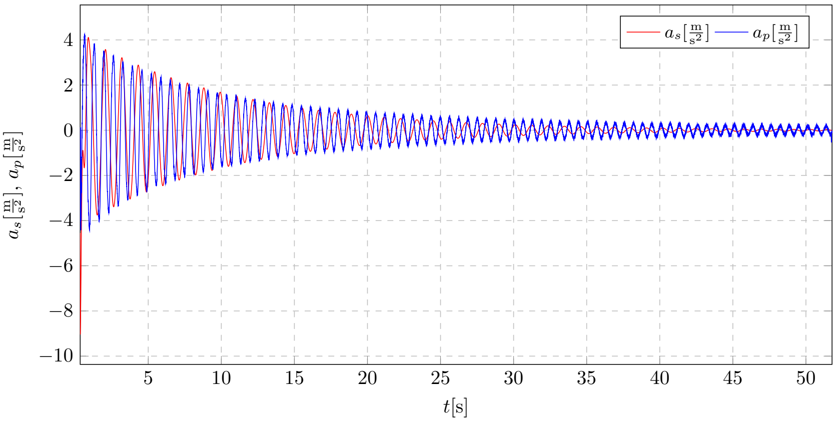

Fig. 8. Vibration response for the simulation model and the physical model

Where $a_p$ represents the acceleration response of the physical model and $a_s$ represents the acceleration response of the simulation model.

Tables 2 and 3 present three types of vibration damping, including the case of using a pendulum with a variable inertia. This analysis allowed for the assessment of the system's influence. Abbreviations: OD – overdamped system; UD – underdamped system; CD – critical damping; OPT D – optimal damping, where VM refers to variable mass variant.

Table 2. Parameters selected for the models of the system with TMD and with variable moment of inertia pendulum

| | $c\,[\frac{\mathrm{Ns}}{\mathrm{m}}]$ | $m_t$ [kg] | $m_p$ [kg] | $l$ [m] | $F$ [N] | $\Omega\,[\frac{\mathrm{rad}}{\mathrm{s}}]$ | $k\,[\frac{\mathrm{N}}{\mathrm{m}}]$ | $g\,[\frac{\mathrm{m}}{\mathrm{s}^2}]$ | $k_l\,[\frac{\mathrm{N}}{\mathrm{m}}]$ |
|---|---|---|---|---|---|---|---|---|---|
| OD | 0.1 | 0.7 | $\frac{\pi}{30}$ | 0.35 | 0 | 0 | $8.75\pi^2$ | 9.81 | 999999 |
| UD | 400 | 60 | 3 | 0.6 | 0 | 0 | $43.75\pi^2$ | 9.81 | 999999 |
| CD | 290 | 60 | 3 | 0.6 | 0 | 0 | $56.875\pi^2$ | 9.81 | 999999 |

Table 3. Parameters selected for the models of the system with TMD and with variable moment of inertia pendulum

| Parameter | OD VM | UD VM | CD VM | OPT D VM |
|---|---|---|---|---|
| $c\,[\frac{\mathrm{Ns}}{\mathrm{m}}]$ | 0.1 | 400 | 290 | 75 |
| $m_t$ [kg] | $-\frac{\pi\,\mathrm{atan}\,(t)}{30}+0.7$ | $-\pi\,\mathrm{atan}\,(t)+60$ | $-\pi\,\mathrm{atan}\,(t)+60$ | 60 |
| $m_p$ [kg] | $\frac{\pi\,\mathrm{atan}\,(t)}{30}$ | $\pi\,\mathrm{atan}\,(t)$ | $\pi\,\mathrm{atan}\,(t)$ | 3 |
| $l$ [m] | 0.35 | 0.6 | 0.6 | 0.6 |
| $F$ [N] | 0 | 0 | 0 | 0 |
| $\Omega\,[\frac{\mathrm{rad}}{\mathrm{s}}]$ | 0 | 0 | 0 | 0 |
| $k\,[\frac{\mathrm{N}}{\mathrm{m}}]$ | $8.75\pi^2$ | $43.75\pi^2$ | $56.875\pi^2$ | $56.875\pi^2$ |
| $g\,[\frac{\mathrm{m}}{\mathrm{s}^2}]$ | 9.81 | 9.81 | 9.81 | 9.81 |
| $k_l\,[\frac{\mathrm{N}}{\mathrm{m}}]$ | 99 | 999999 | 999999 | 999999 |

## 6. Simulation results

Worse effects than with the pendulum can be observed (Fig. 9).

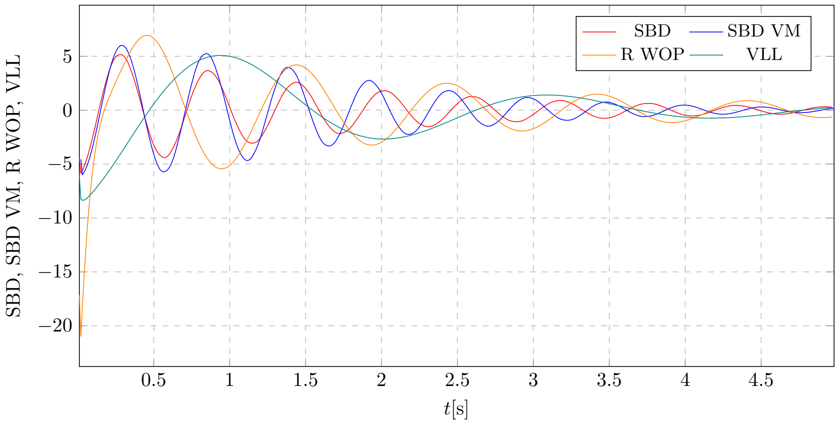

Fig. 9. Subcritical damping with and without variable mass pendulum (SBD, SBD VM) and reference and variable link length graph (R WOP, VLL)

It can be observed that consecutive zeros are spaced at equal time intervals, which is characteristic of this type of damping. It progresses slowly and vibrations cease after many oscillations, resulting in a long stabilization time for the system and a high risk of vibration interference, which can be dangerous for buildings and uncomfortable for people inside. In Figure 10 is the characteristic of the same system when using a pendulum of variable mass, which increases with the duration of the simulation. In subcritical damping with the use of a variable mass damper, faster decay of vibrations and a greater frequency of natural oscillations can be observed. Below is another graph, this time the phenomenon of supercritical damping occurs (Fig. 10).

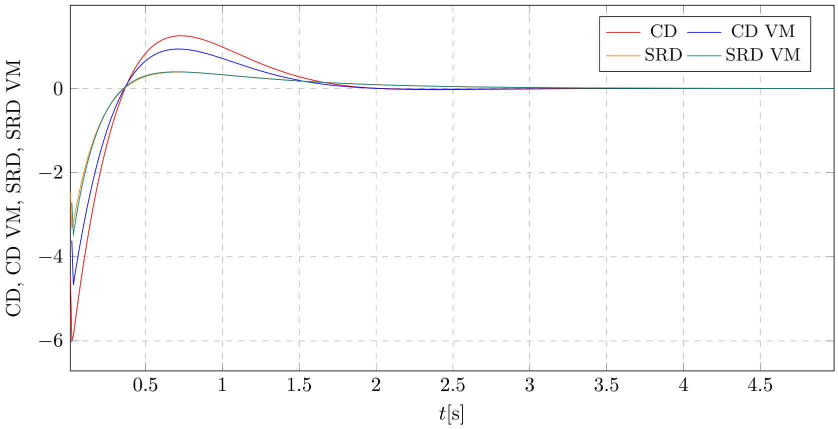

Fig. 10. Critical damping with and without variable mass pendulum (CD, CD VM) and supercritical damping with and without variable mass pendulum (SRD, SRD VM)

The absence of oscillations can be observerd in Figure 10 for supercritical damping. In the presented case, the curve does not cross the *Ox* axis. Damping is even stronger than in critical damping, which prolongs the stabilization time of the system, but the observed accelerations are smaller. Figure 10 shows the characteristic of the same system using a variable-mass pendulum, where the mass increases over the duration of the simulation. In the case of supercritical damping, the effect of the variable mass pendulum is less than in the system with subcritical damping.

One zero crossing of the profile $x(t)$ can be observed in critical damping (Fig. 10). This damping occurs the fastest and is optimal for most solutions. The short time required to damp vibrations prevents fatigue degradation of the structure and effectively avoids interference. In Figure 10 is the characteristic of the same system when using a pendulum of variable mass, which increases with the duration of the simulation.

The effect of the pendulum with variable mass is relatively small in the case of critical damping. Damping with varying length of the link.

This type of solution allows adjusting the length of the link to the prevailing conditions, making it possible to adjust parameters according to current excitations.

## 7. Conclusion

The analysis of the examined model demonstrated significant advantages resulting from the implementation of a vibration damper based on a pendulum with variable inertia. The proposed system offers enhanced adaptability to a range of excitation conditions compared to conventional solutions using a fixed-mass pendulum, such as those employed in the Taipei 101 skyscraper. Although this approach promises a higher damping efficiency, its practical application faces challenges due to limitations in material strength and the requirement for high-power electric motors dictated by the mass of the pendulum. Nonetheless, the developed model introduces new possibilities through the dynamic adjustment of the pendulum's parameters. This system may facilitate even more effective vibration reduction, thereby enabling the design of lighter and more slender structures with improved stability, vibration isolation and, prolonged service life.

The following conclusion can be drawn from performed investigation:

- selection of four times greater damping results in a five-fold reduction in amplitude,
- the adopted dynamic model accurately represents the problem under study and the amplitude convergence of the results is very good (amplitudes in the first phase of vibration do not differ by more than 5%),
- optimal operating conditions for the system can be determined for a pendulum length of 10 cm and dynamic changes of the cable length allows for controlling the vibration level.

Furthermore, this solution could prove especially beneficial in earthquake-prone areas, where the capacity to mitigate shock effects while maintaining a low mass is crucial. Another important aspect is the implementation of a robust diagnostic procedure to evaluate the degree of structural degradation. This assessment enables an accurate estimation of the building's remaining service life, thereby enhancing user safety and streamlining maintenance operations. The incorporation of variable inertia introduces a nonlinearity into the system, which influences its dynamic characteristics. This nonlinearity enables adaptive modification of the system's natural frequency, allowing it to effectively avoid resonance and maintain stable vibration behavior under varying excitation conditions. Under conditions of intense seismic activity, a damper with a variable cord length or mass can effectively reduce displacement amplitudes, preventing excessive tilting that might otherwise lead to irreversible structural damage.

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