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APPLICATION OF MATHEMATICAL MODELS FOR THE ANALYSIS OF THERMAL PHENOMENA IN THE WELDING PROCESS USING ABAQUS SOFTWARE

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Abstract. Numerical solutions in the field of modelling of the welding process constitute significant support for the production process and are one of the most difficult to perform in terms of the complexity of physical phenomena in the welding process. This is especially true when commercial software such as Abaqus, Ansys, etc. is used in the analysis, where welding conditions are not directly reflected in the modules of the software. This work is focused on the development of mathematical models of a moveable heating source taking into account various welding techniques. The simulations are carried out in Abaqus software, which, in its basic form, does not allow simulations of welding process. The presented work contains the developed mathematical and numerical models necessary for conducting numerical studies in the field of the analysis of the welding process. The presented DFLUX subroutine allows the implementation of any mathematical model of the heating source and modelling of the movement of the source along any trajectory. As a part of the research, mathematical models are developed for three completely different welding techniques: fillet welding, circumferential welding and spiral welding. Each of these three methods requires the use of a completely different approach. Based on the developed mathematical and numerical models, testing calculations are performed. Selected calculations are compared with experimental results presented in the literature. The presented results of calculations allow for the confirmation of the correctness of the developed mathematical and numerical models of heat source power distribution.

MSC 2010: *65Z99, 81T80*
Keywords: *heat source, mathematical model, FEM simulation, welding process*

1. Introduction

The important element of conducting numerical calculations for the analysis of the phenomena occurring in the welding process is the development of appropriate mathematical and numerical models of thermal, structural and mechanical phenomena [1-4]. Performing simulations in the field of welding in commercial engineering software, i.e. Abaqus, Ansys, Adina is difficult to perform [2, 5, 6]. These programs do not take into account the specific conditions that occur during the process.

Currently, there are many papers in the available literature related to the numerical modelling of welded joints. These works are based on various welding methods, such as traditional arc welding [7, 8], laser welding or hybrid welding [2, 9]. In most cases, experimental studies are used to verify the developed numerical models [10-12]. A large part of the available numerical modelling of welding concerns butt welding or surfacing [7, 13]. The remaining techniques included in this material are described, but in a very simple form without delving into the mathematical model itself [14-16].

In the presented work, computer calculations are performed in Abaqus software. The main advantage of Abaqus is the wide range of analyses that can be performed and the ability to adapt the solver to perform complex numerical analyses of the welding process in terms of thermal and mechanical phenomena. The basic version of this solver does not allow performing simulations of the welding process because it is not possible to take into account the distribution of volume heat source power distribution and its movement [5, 6]. Due to its modular design, Abaqus allows the user to add independently built subroutines, necessary for modelling such issues. The scheme of the model in Abaqus FEA is presented in Figure 1.

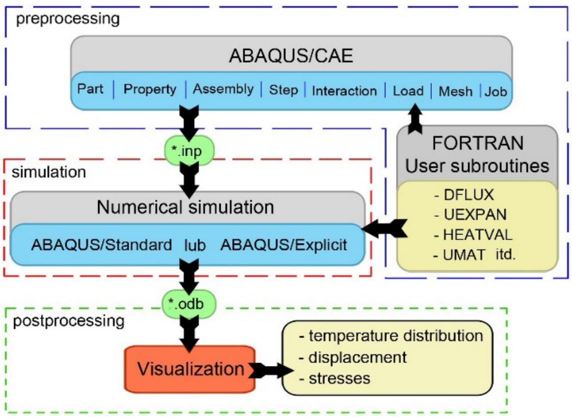

Fig. 1. Scheme of numerical model developed in Abaqus FEA

2. Heat transfer analysis

The numerical analysis of the welding process in Abaqus is performed in Lagrange coordinates using conservation of energy with Fourier law [5, 6, 8]:

$$\int_V \rho \frac{\partial U}{\partial t} \delta T \, dV + \int_V \frac{\partial \delta T}{\partial x_\alpha} \cdot \left(\lambda \frac{\partial T}{\partial x_\alpha} \right) dV = \int_V \delta T \, q_V \, dV + \int_S \delta T \, q_S \, dS \qquad (1)$$

where: λ is a thermal conductivity [W/m K], $U = U(T)$ is an internal energy [J/kg], q_V is a laser beam heat source [W/m^3], $T = T(x_\alpha,t)$ is a temperature [K], q_S is a boundary heat flux [W/m^2], δT is a variational function, ρ is a density [kg/m^3].

The position of the heat source center x_o is determined for each time t [s] depending on the adopted speed v [m/s].

$$x_o = v \cdot t \ [m] \tag{2}$$

The energy conservation equation (1) is supplemented by initial conditions and boundary conditions of Dirichlet, Neumann and Newton types. The heat loss is taken into account due to convection, radiation and evaporation [2, 5].

3. Mathematical modelling of a moveable heating source

Modelling the motion of the heating source in Abaqus requires the development of appropriate algorithms, which will be implemented in the DFLUX subroutine. The developed numerical subroutine DFLUX allows for modelling of: heat source power distribution, welding direction, welding line and source travel speed. Figure 2 shows a diagram of the DFLUX subroutine, this diagram is provided by the distributor of Abaqus software [16].

```
SUBROUTINE DFLUX(FLUX,SOL,KSTEP,KINC,TIME,NOEL,NPT,COORDS,
     1 JLTYP,TEMP,PRESS,SNAME)
C
      INCLUDE 'ABA_PARAM.INC'
C
      DIMENSION FLUX(2), TIME(2), COORDS(3),
      CHARACTER*80 SNAME


      user coding to define FLUX(1) and FLUX(2)


      RETURN
      END
```

Fig. 2. Scheme of interface of the DFLUX subroutine [16]

The implementation of this procedure allows for the analysis of thermal phenomena in the welding process for various welding techniques and various joint types. The procedure additionally allows for taking into account the change of the heat source position, the transformation of the coordinate system in the case of numerical modelling of fillet joints, and the analysis of the heating source movement in the polar system (the case of circumferential and spiral welded axially symmetric elements). All transformations are taken into account in the DFLUX subroutine according to the positioning and transformation rules.

Depending on the analyzed welding method, volumetric mathematical models are most often used to model the power distribution of the heating source: Gauss, C-I-N (cylindrical-involution-normal) and Goldak [17, 18]. The first two models are most often used to model the welding process using laser techniques, while the Goldak model is used to describe the power distribution of the electric arc [19, 20]. The basic form of these models can be effectively used to model the welding process of butt welds. In the case of fillet welds, the inclination of the heat source axis should be taken into account. Therefore, it is necessary to introduce appropriate transformations of the axis system by a given angle of inclination into the mathematical model of the source (Fig. 3) [2, 21].

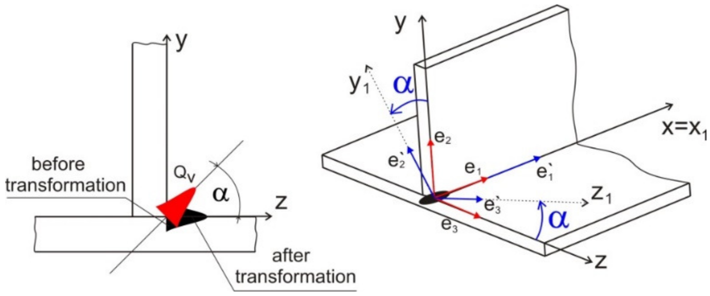

Fig. 3. Transformation of the axis system by angle α

Transformation of laser beam heat source distribution is performed using the following transformation equation:

$$A_{i'} = \gamma_{i'j} A_j \ \text{ where } \ \gamma_{i'j} = \mathbf{e}_{i'} \cdot \mathbf{e}_j \tag{3}$$

After rotation of the considered system, the transformation matrix is obtained:

$$\gamma_{i'j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \tag{4}$$

The solution the transformation matrix is obtained by changing the equation from the basic system to the rotated system:

$$\begin{cases} x = x_o \\ y = \cos\alpha \cdot y_1 + \sin\alpha \cdot z_1 \\ z = -\sin\alpha \cdot y_1 + \cos\alpha \cdot z_1 \end{cases} \tag{5}$$

Figure 4 shows an example of transformation of the Gaussian power distribution by an angle of $\alpha = 45\,°$.

Numerical calculations in the Abaqus program are performed exclusively in the Cartesian coordinate system. The DFLUX numerical subroutine allows changing the direction of the heat source movement in the case of circumferential welding (Fig. 5). It is necessary to develop the equations of transition from the source movement in the polar system to the Cartesian system [14]. The transition equations are presented as follows:

$$\begin{cases} x = R_Z \sin(\phi_0 + \omega \cdot t) \\ y = R_Z \cos(\phi_0 + \omega \cdot t) \\ z = z_0 \end{cases} \tag{6}$$

where R_Z is the beam radius, t is the time, ϕ_0 is the initial position the axis of the beam, $\omega = v/R_Z$ is the angular speed, in which v = const, is the linear speed along the perimeter of the pipe, and z_0 is the initial position on the z-axis.

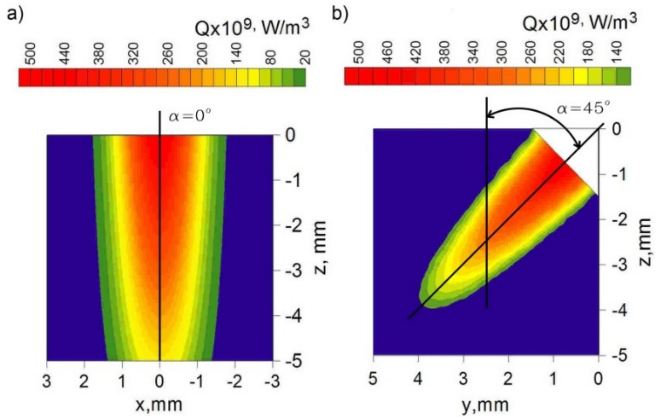

Fig. 4. Transformation of the source power distribution from the Cartesian coordinate system to the coordinate system rotated by an angle α

The spiral welding of pipes is much more difficult to model due to the need to include in the model the movement of the heating heat source along the axis of the element (Fig. 6) [14, 22].

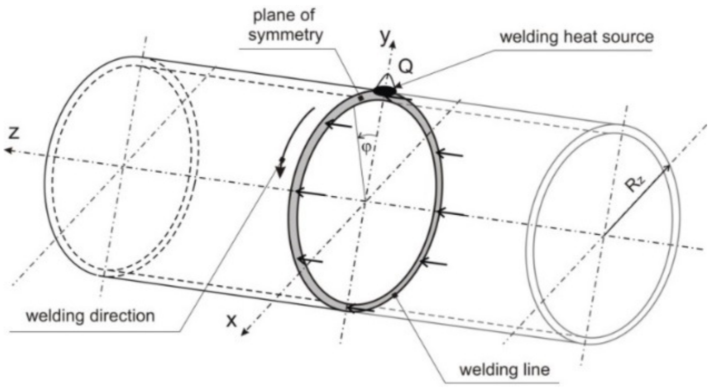

Fig. 5. Schema of circumferential welding

In such a case, the equation for transition to the Cartesian coordinate system takes the following form:

$$\begin{cases} x = R_Z \sin(\phi_0 + \omega \cdot t) \\ y = R_Z \cos(\phi_0 + \omega \cdot t) \\ z = z_0 + v_2 \cdot t \end{cases} \tag{7}$$

where R_Z is outer radius of the pipe [m], t is time [s], ϕ_0 is the angle of the initial position of heat source on the outer shell, $\omega = v_1 t$ is the angular speed in which v_1 = const is the peripheral speed, z_0 is the initial position on the axis z, and v_2 is the axial speed along z axis. Welding speed v is the result of the peripheral v_1 and axial v_2 speed $v = \sqrt{v_1 + v_2}$.

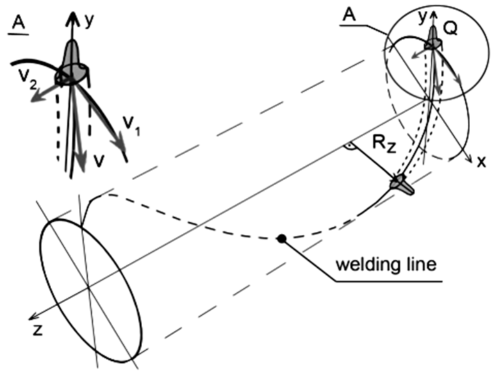

Fig. 6. Schema of spiral welding

The spiral welding model presented in Figure 6 is correct only for models symmetrical with respect to two planes (e.g. the Gaussian model, C-I-N). In the case of using a source symmetrical with respect to only one plane, e.g. the Goldak model, additional transformation equations must be introduced. It is necessary that for each time period, the source position is tangent to the weld line (Fig. 7). Therefore, the transformation matrix has the following form:

$$\gamma_{i'j} = \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \tag{8}$$

Cartesian coordinates of the heat source are defined as follows:

$$\begin{cases} x = \cos\alpha \cdot x_1 - \sin\alpha \cdot z_1 \\ y = y_1 \\ z = \sin\alpha \cdot x_1 + \cos\alpha \cdot z_1 \end{cases} \tag{9}$$

Based on the developed mathematical models of moving heating sources, testing calculations are carried out to confirm the correctness of the numerical models.

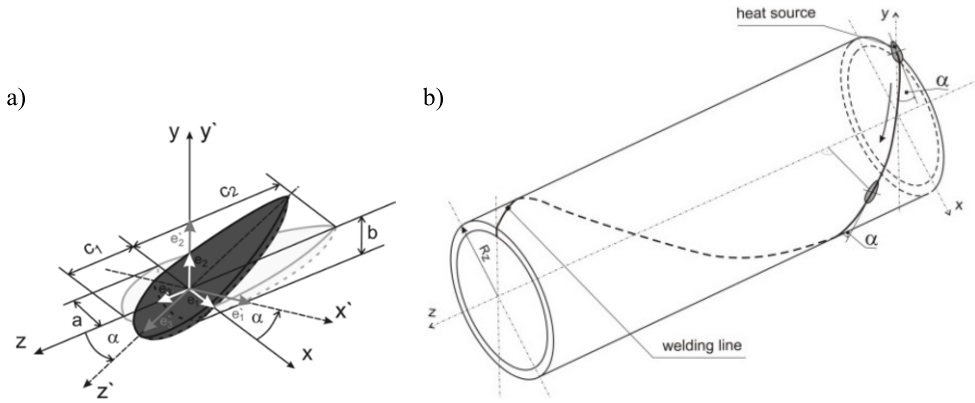

Fig. 7. Schema of the transformation of Goldak's heat source power distribution (a), schema of spiral welding using Goldak's heat source (b)

4. Numerical modelling

The Abaqus 6.14 version is used for the calculations. The coordinate system transformations and changes in the heat source path described in point 3 are developed mainly for the simulation of non-standard welded joints. The material is heated using a moving heat source with a specific power intensity distribution. In all examples, a three-dimensional cuboid finite element mesh is used. In the calculations adopted, DC3D8 elements are used in thermal analysis. The mesh density in each of the models is selected to ensure good quality of the obtained simulation results. It is assumed that the material of each of the analyzed structures is austenitic steel 304 [2, 6]. In the contact plane of the joined elements, perfect contact between surfaces is assumed. Appropriate DFLUX numerical subroutines are implemented into the computational solver of the Abaqus/Standard module. Geometric dimensions of the developed discrete models of fillet welding, circumferential welding and spiral welding are given separately for each presented welding technique.

4.1. Fillet welding

The scheme of the considered filled welding of T-joint is presented in Figure 8. The discrete model of analyzed welded T-join is made of flat with dimensions 30 mm x 100 mm x 3 mm and 30 mm x 100 mm x 1 mm. Technological parameters of the welding process are: beam power Q = 2200 W, welding speed v = 3 m/min, and the angle of inclination of the laser beam relative to the connected elements α = 16 °. The technological parameters are taken from the literature [23]. The calculations also assume: the efficiency of the welding process η = 75 % (literature efficiency for laser welding), beam radius r = 0.35 mm and penetration depth h = 4 mm.

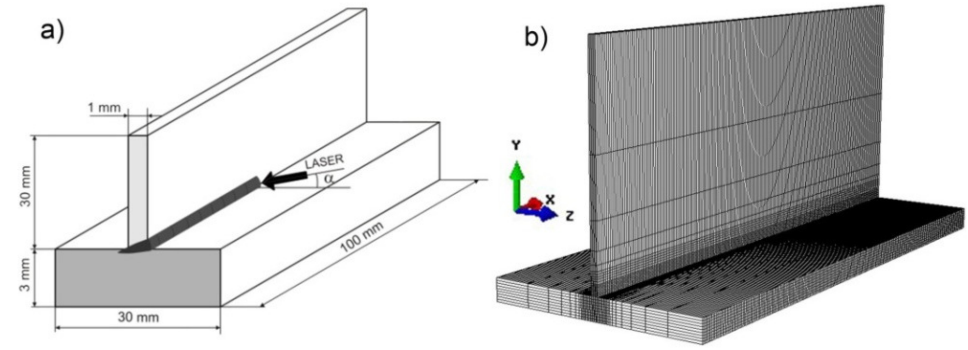

Fig. 8. Scheme of the considered system (a), discretization of the analyzed domain (b)

4.2. Peripheral welding

The numerical simulations of peripheral laser beam welding are made for pipes. The scheme of analyzed system is shown in Figure 9. The power distribution of the laser beam is described by the Gaussian model of the volumetric heat source. The following process parameters are assumed: heat source power $Q = 1500$ W, beam radius $r_0 = 1$ mm, penetration depth $h = 2$ mm, and the welding speed $v = 1.8$ m/min. Temperature distribution in circumferentially welded pipe is presented in Figure 11.

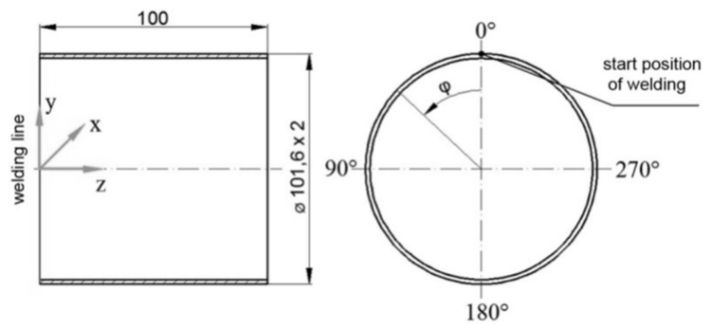

Fig. 9. Discretization scheme of the circumferentially welded element

4.3. Spiral welding

Spiral laser beam welding simulations are made for pipes. The discrete model is developed for spirally welded pipe with dimensions $R_Z = 30$ mm, $R_w = 28.4$ mm, $L = 200$ mm. Figure 10 shows finite element mesh used in calculations. The Gaussian model is used to describe distribution of a laser beam power. Technological parameters of the welding process are: beam power $Q = 1000$ W, and welding speed $v = 1.8$ m/min. The calculations also assume: beam radius $r = 0.9$ mm and penetration depth $h = 2$ mm.

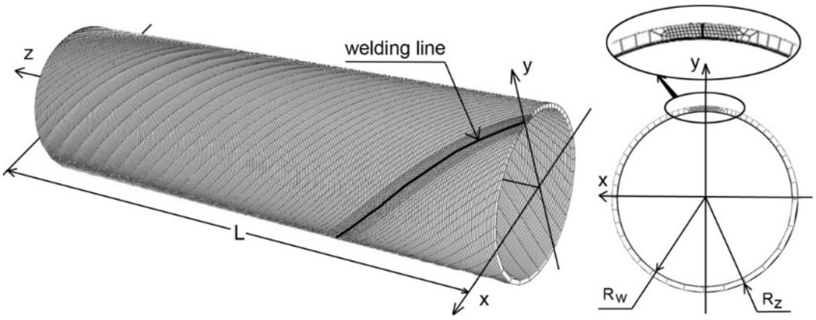

Fig. 10. Finite element mesh used in calculations

The presented numerical models and developed mathematical models were used to perform verification calculations.

5. Results and discussion

The presented simulation results show specific applications of developed models. For all analyzed welding cases using different techniques, the shape and size of the melted zone is numerically determined. In the figures, the melted zone boundary is marked with a solid line ($T_L \approx 1455$ °C). The first analyzed case is an example of a fillet weld analysis (Fig. 11). This figure shows the obtained simulation result for the parameters presented in Section 4.1.

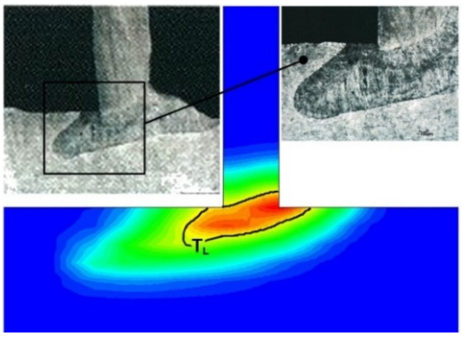

Fig. 11. Comparison of the predicted shape of the melted zone with the experiment

Figure 11 also shows a comparison of the numerically obtained result with a macroscopic image of a fillet joint taken from the literature [23]. It can be seen that the simulation results are in quite good agreement. The estimated penetration depth is almost identical to the real model.

The second model considered in the work is the model of circumferential welding of axisymmetric elements. For the process parameters presented in Section 4.2, the temperature distribution shown in Figure 12 was obtained. A very narrow melt zone is obtained. For the element with a wall thickness of 2 mm assumed in the analysis, the process parameters are selected to achieve melting without the effect of burning through the material.

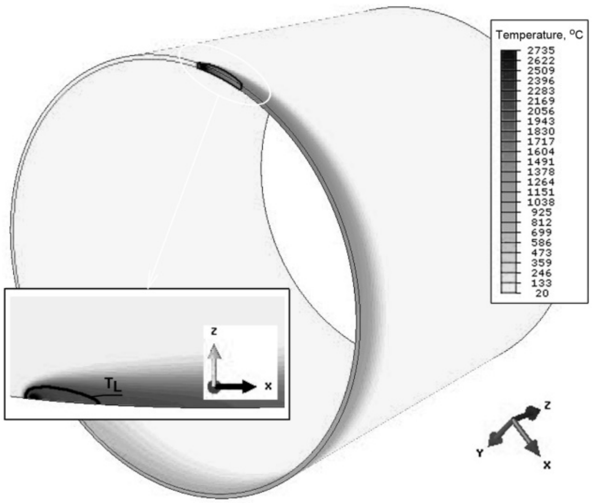

Fig. 12. Temperature distribution in circumferentially welded pipe

The last case is the spiral welding technique of pipes (Fig. 13). In this case, the numerical analysis is carried out in Lagrangian coordinates. A source with a specific power distribution moves in a spiral (see Section 4.3). The temperature profile shown in Figure 13 confirms the correctness of the developed model. For the assumed parameters, the material was melted, and the width of the melted zone was about 2 mm.

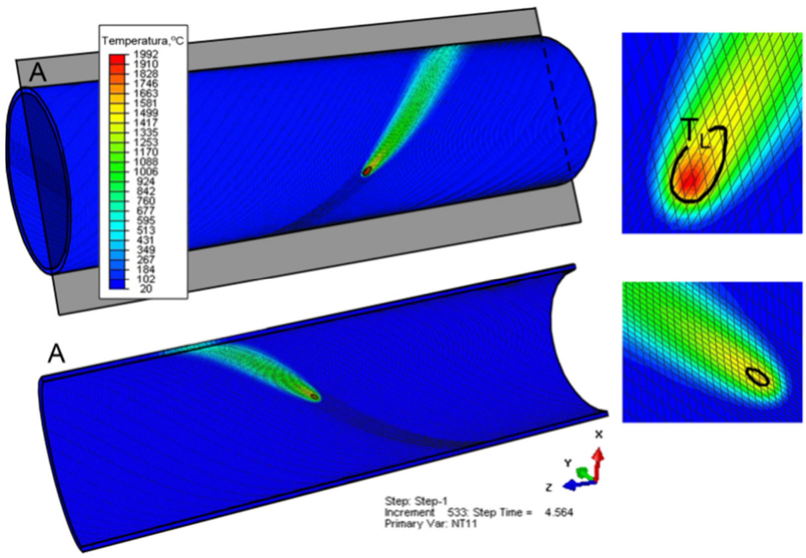

Fig. 13. Temperature profile of laser welded pipe

The simulation results presented in Section 3 confirm the correctness of the developed mathematical models. The developed mathematical models can be effectively used to conduct simulations using various welding techniques.

6. Conclusions

Developed mathematical models included in this work are necessary for conducting numerical analyses of the welding process in Abaqus software. The DFLUX subroutine is universal for the modelling of a moveable heating source traveling along any trajectory. The mathematical models presented can be effectively used for any welding technique. The welding cases considered in the paper are engineering problems commonly encountered in the industry. The numerical modelling of welding aims to accelerate the process and obtain the best possible strength of welded joints. The obtained results of numerical calculations confirm the correctness of the developed mathematical and numerical models.

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