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KINEMATIC SYNTHESIS OF THE GOUGH-STEWART PLATFORM IN ADAMS AND MATLAB

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Abstract. In this paper, the kinematic synthesis of a parallel Gough-Stewart platform mechanism is addressed in order to achieve the motion of the working part of the mechanism along a prescribed trajectory. A virtual prototype of this platform was created in the environment of the dynamics of systems bodies program ADAMS. In the process of cosimulation of MATLAB, MATLAB/Simulink and ADAMS, the input trajectories of actuator velocities necessary to achieve the desired motion of the effector point were identified. The trajectory was defined by parametric equations for the epitrochoid, hypocycloid, epicycloid and Archimedes spiral. The velocities of the actuators were derived by deriving the position vector of the distances between the base and platform points. The solution logic using the given software is presented by a block diagram.

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Keywords: *Gough-Stewart platform, kinematics, analysis, synthesis, mechanism, ADAMS, MATLAB, Simulink*

1. Introduction

The Gough-Stewart platform belongs to the group of parallel mechanisms. It is made up of a fixed and a moving platform, which are connected by six actuators (limbs). By changing the length of the actuators, the position of the movable platform relative to the fixed one is changed [1].

Parallel architectures offer several advantages, including higher kinematic accuracy, lower weight, greater structural rigidity, efficient placement of actuators, and lower manufacturing costs. Simultaneous control of multiple members allows for more efficient use of the working member. Other advantages of these manipulators are the possibility of high speed and overall motion dynamics. They are also used due to their ability to provide precise and repetitive movements.

The most common use of parallel mechanisms today is in the following industries:

- Industrial automation – e.g. for assembly, welding, palletizing, positioning [2] and other operations.
- Medical robotics – in surgical procedures and also in rehabilitation [3, 4].
- Industrial manufacturing – for precise positioning of tools or equipment, for example in 3D printing, or when machining a semi-finished product with a CNC machine.
- Aerospace – in the assembly and testing of components for aerospace programs [5-7].
- Simulators and applications for virtual reality [8, 9].
- Industrial testing and measurement – e.g. in positioning of analyzed objects [10].
- Research and education – for various experiments in robotics, automation and advanced mechanical systems, earthquake simulation [10].

The solution of the Gough-Stewart platform has been studied by a number of authors. A historical overview of different parallel robots, their classification, research trends, and research methodology has been reviewed by [11], design and solution of non-planar links has been carried out [12]. Adams-Simulink co-simulation for platform control using controller singularities has been addressed [13]. The platform was used to analyze multidimensional motion characteristics that can well simulate the oscillatory motion of shark scales during swimming, which led to a significant reduction in drag effects [14], and also for the micro-vibration simulator, which reproduced the space six-dimensional acceleration [15].

A comprehensive solution to the related problems of kinematics, dynamics and control was presented in [16]. Our paper deals with the inverse kinematics of the platform mechanism, where based on the prescribed motion of the platform point, the velocities of the limbs of platforms need to be determined, using the ADAMS-MATLAB co-simulation. The determined velocities will be input for the dynamic analysis in terms of determining the energy resource requirements for the rate of change of the limb lengths.

2. Theoretical background, inverse kinematics

Given the robot's end-effector location, inverse kinematics equations calculate the joint angles required to move the end-effector to that location. In order to calculate inverse kinematics, the configuration shown in Figure 1 is used, and the following equations are needed to obtain a solution (1)-(11).

The following position vectors of the connection points on its corresponding coordinate system are derived:

$$GP_i = \begin{bmatrix} P_{xi} \\ P_{yi} \\ P_{zi} \end{bmatrix} = \begin{bmatrix} r_p\cos(\lambda_i) \\ r_p\sin(\lambda_i) \\ 0 \end{bmatrix}, \tag{1}$$

where

$$\lambda_i = \frac{i\pi}{3} - \frac{\theta_p}{2}, \quad \text{for } i = 1, 3, 5, \tag{2}$$

$$\lambda_i = \lambda_{i-1} + \theta_p, \quad \text{for } i = 2, 4, 6. \tag{3}$$

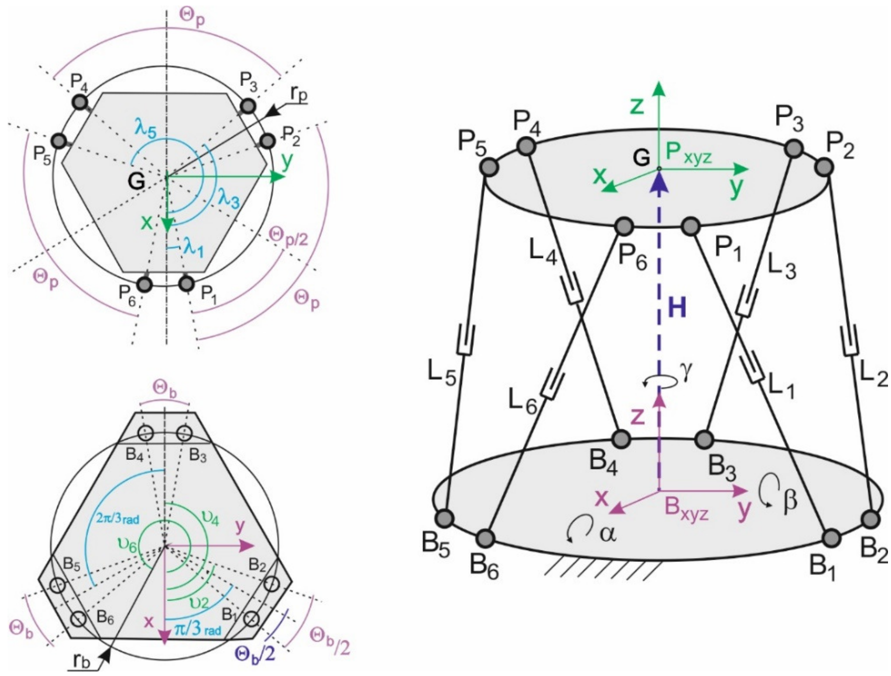


Fig. 1. Hexapod configuration, Gough-Stewart platform

For the base platform, the same procedure is done by simply adjusting the value of the corresponding parameters

$$B_i = \begin{bmatrix} B_{xi} \\ B_{yi} \\ B_{zi} \end{bmatrix} = \begin{bmatrix} r_b\cos(\upsilon_i) \\ r_b\sin(\upsilon_i) \\ 0 \end{bmatrix}. \tag{4}$$

We refer to the origin of the moving platform as G. The position vector of G w.r.t. the B_{xyz} coordinate system is denoted as $H = [H_x\ H_y\ H_z]^T$. ${}^BR_T$ is the rotation matrix defined by the roll (a), pitch (b), and yaw (g) angles. For any rotation performed on the moving platform, this matrix maps any vector represented in P_{xyz} coordinates to its corresponding B_{xyz} coordinates.

Define $R_X(\alpha)$ as the matrix that represents the rotation about the fixed x-axis, in the same manner $R_Y(\beta)$ and $R_Z(\gamma)$ represent rotations about the fixed y and z-axes. We have that

$$R_X(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}, \tag{5}$$

$$R_Y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}, \tag{6}$$

$$R_Z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

There are twelve different combinations for the order of rotation of these angles, also considering combinations of three rotations with respect to only two axes that can represent arbitary 3D rotations. The order of the angles relative to the fixed platform is α, β, γ i.e.:

$$^{B}R_T = R_Z(\gamma)R_Y(\beta)R_X(\alpha) =$$
$$= \begin{bmatrix} \cos\beta\cos\gamma & \cos\gamma\sin\alpha\sin\beta - \cos\alpha\sin\gamma & \sin\alpha\sin\gamma + \cos\alpha\cos\gamma\sin\beta \\ \cos\beta\cos\gamma & \cos\alpha\cos\gamma + \sin\alpha\sin\beta\sin\gamma & \cos\alpha\sin\beta\sin\gamma - \cos\gamma\sin\alpha \\ -\sin\beta & \cos\beta\sin\alpha & \cos\alpha\cos\beta \end{bmatrix}. \tag{8}$$

Thus, the vector L_i corresponding to the link i is obtained as follows (in BXYZ coordinates).

$$L_i = R_{XYZ}GP_i + H - B_i, \quad i = 1, \ldots, 6 \tag{9}$$

$$L_i =$$
$$\begin{bmatrix} r_p[\sin(\alpha)\sin(\beta)\cos(\gamma) - \cos(\alpha)\sin(\gamma)]\sin(\lambda_i) + r_p\cos(\beta)\cos(\gamma)\cos(\lambda_i) + x - r_b\cos(v_i) \\ r_p[\sin(\alpha)\sin(\beta)\sin(\gamma) + \cos(\alpha)\cos(\gamma)]\sin(\lambda_i) + r_p\cos(\beta)\sin(\gamma)\cos(\lambda_i) + y - r_b\sin(v_i) \\ r_p\cos(\alpha)\cos(\beta)\sin(\lambda_i) - r_p\sin(\beta)\cos(\lambda_i) + z \end{bmatrix}. \tag{10}$$

and their lengths l_i, correspond to the actuator positions

$$l_i = \|L_i\|. \tag{11}$$

3. Creation of the model

The parallel coupled mechanical system contains 6 limbs (actuators). Each actuator consists of two members that can move relative to each other. The actuators connect the upper, movable, and lower, fixed platform. The mechanism thus has 14 members, including the frame. The actuators are attached to the platforms by linkages that allow rotary movement of Figure 2. This hexapod has 6 degrees of freedom:

$$n = 6(t-1) - 4\sum U_i - 5\sum T_i - 3\sum S_i = 6(14-1) - 4.6 - 5.6 - 3.6 = 6DOF \quad (13)$$

The input kinematic parameters were defined in the sliding constraint of each actuator.

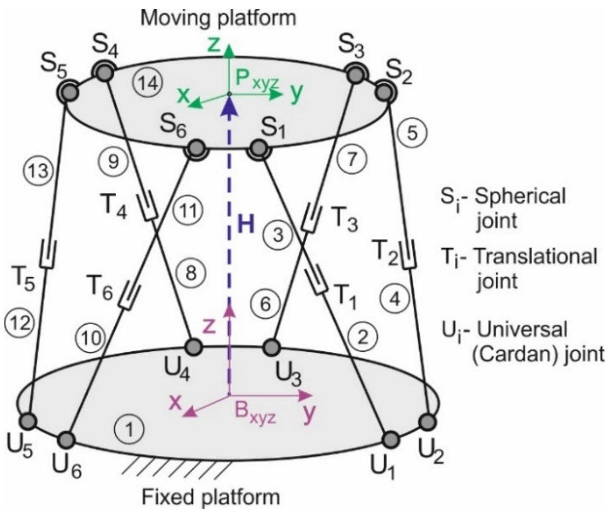


Fig. 2. Kinematic scheme of the mechanism

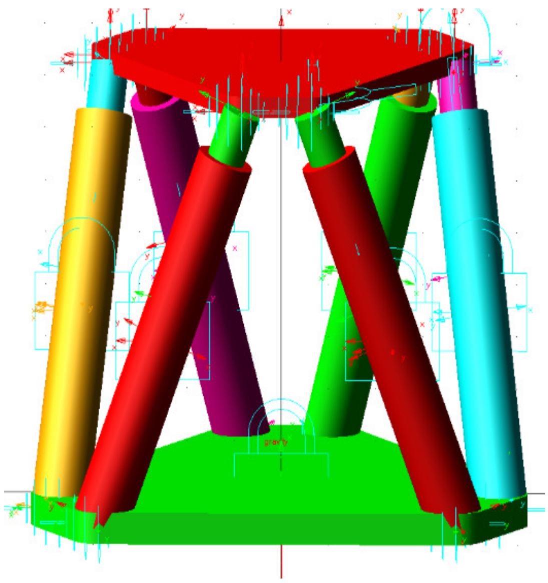


Fig. 3. Gough-Stewart platform model in ADAMS

A virtual prototype of the mechanism was created in the ADAMS/View environment, Figure 3. The basis for the creation of this virtual prototype was the "Hexapod CIDESI HxCF1" model [10].

4. Software solution

The task of the software solution is to design a solution algorithm within the ADAMS and MATLAB and MATLAB/Simulink interfacing software such that the point of interest P on the moving part of the Gough-Stewart platform moves along a prescribed trajectory. The block diagram in Figure 4 presents this interfacing with the individual steps of the solution.

In Matlab, a time series of changes in actuator lengths is created so that the point P moves from the start position to the final position in a selected number of time steps as follows:

1. The centre of the moving platform is moved to the starting point of the prescribed trajectory.
2. The centre of the platform is moved to the subsequent point of the prescribed trajectory.
3. Since the motion of the platform is controlled through the change in length of the actuators, these length changes are calculated and controlled with respect to the geometrical parameters of the mechanism, i.e. a position matrix (11) is constructed.
4. From the position matrix, the rates of change of length of the actuators are calculated. The velocity values are usable in the actual motion simulation and trajectory drawing.

Steps 2 to 4 are repeated for each point in the time sequence, with the subsequent point becoming the starting point for the next simulation step. This produces a time series of individual actuator lengths (or changes in actuator lengths), from which a time series of length change rates is constructed and entered into Simulink.

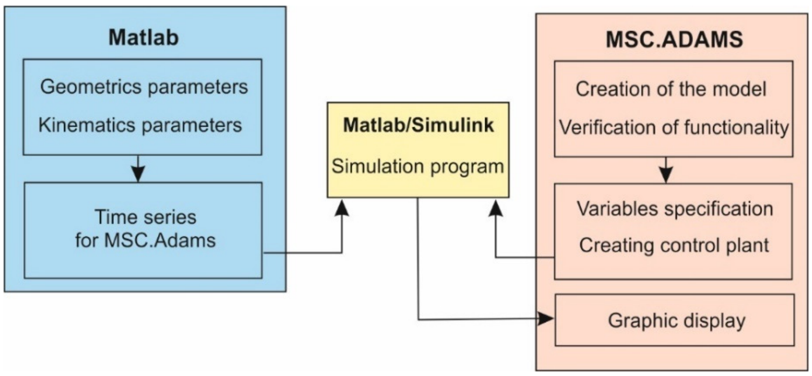


Fig. 4. Block diagram of ADAMS and MATLAB and MATLAB/Simulink cooperation

A virtual prototype of the platform was created in ADAMS, variables were defined, which in our case correspond to the actuator length change rates, and a control plant was created. The defined variables and the control plant from ADAMS were input together from the time series from MATLAB into MATLAB/Simulink. Simulink then controlled the animation of the platform motion in ADAMS (Fig. 4). A block diagram for Simulink is shown in Figure 5.

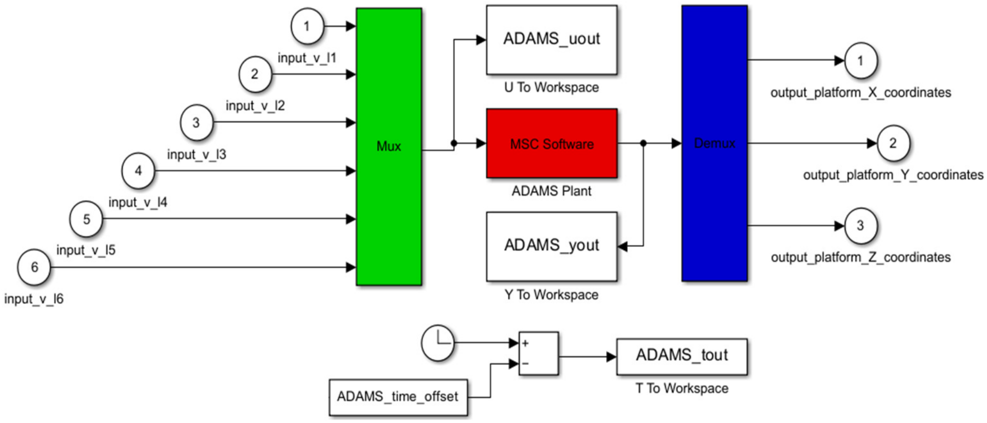


Fig. 5. MATLAB/Simulink block scheme

5. Presentation of solutions

The description of the desired trajectory was given by the parametric equations of the epitrochoid, hypocycloid, epicycloid and Archimedean spiral curves (Fig. 6).

For a given total motion time along the selected trajectory and the number of motion execution steps, the time series of the velocities of the actuators of the mechanism (for one selected position of the moving platform plane) was determined, which is shown in Figure 6, when the moving platform moved in a plane parallel to the fixed platform (roll = 0°, pitch = 0°).

Figure 6 presents the time histories of the rates of change of the length of the actuators for different prescribed types of trajectories (curve: epitrochoid, hypocycloid, epicycloid, spiral) for the center of the moving platform T.

For other selected positions of the moving platform plane (for example: roll = = 35 °, pitch = 35 °, roll = pitch = 35 °), we obtain different values for the velocities of each actuator, as shown for the spiral in Figure 7, where the variations of the velocity curves for each actuator are presented.

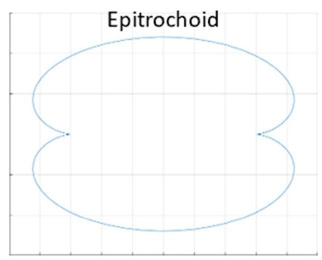
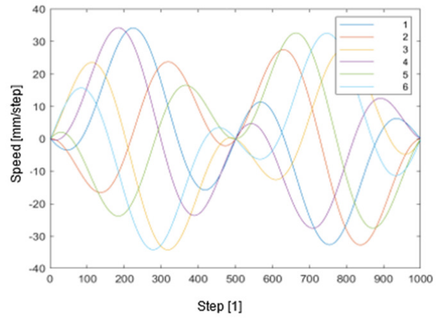
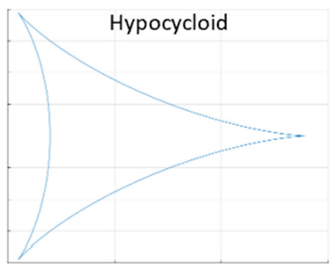
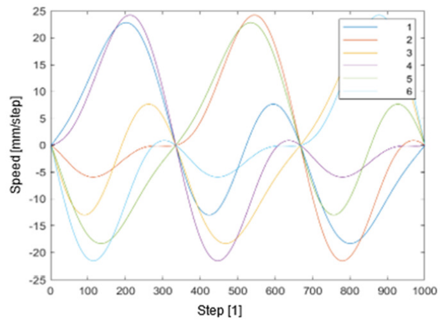
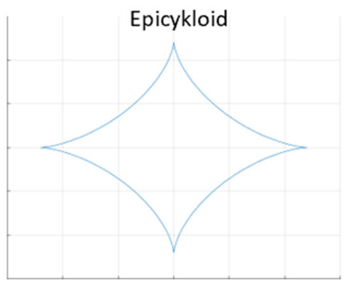
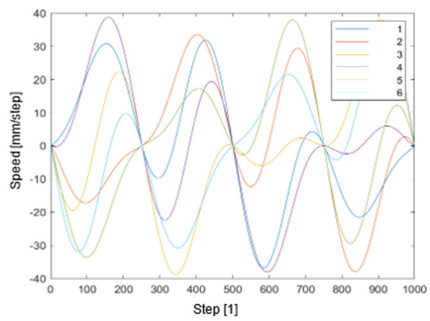
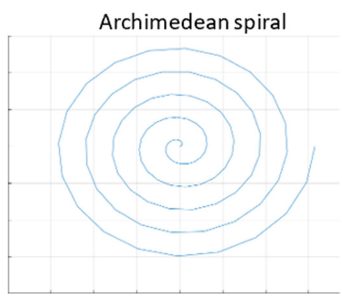
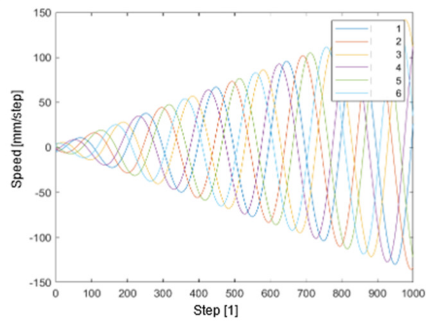
Parametric notation	The curve	The time series of actuator speeds
x = r(3*cos(t) - cos(3t)) y = r(3*sin(t) - sin(3t))		
x = r(2*cos(t) + cos(2t)) y = r(2*sin(t) - sin(2t))		
x = r(3*cos(t) + cos(3t)) y = r(3*sin(t) - sin(3t))		
x = a * ϑ * cos(ϑ) y = a * ϑ * sin(ϑ)		

Fig. 6. Parametric equations of trajectory and velocity

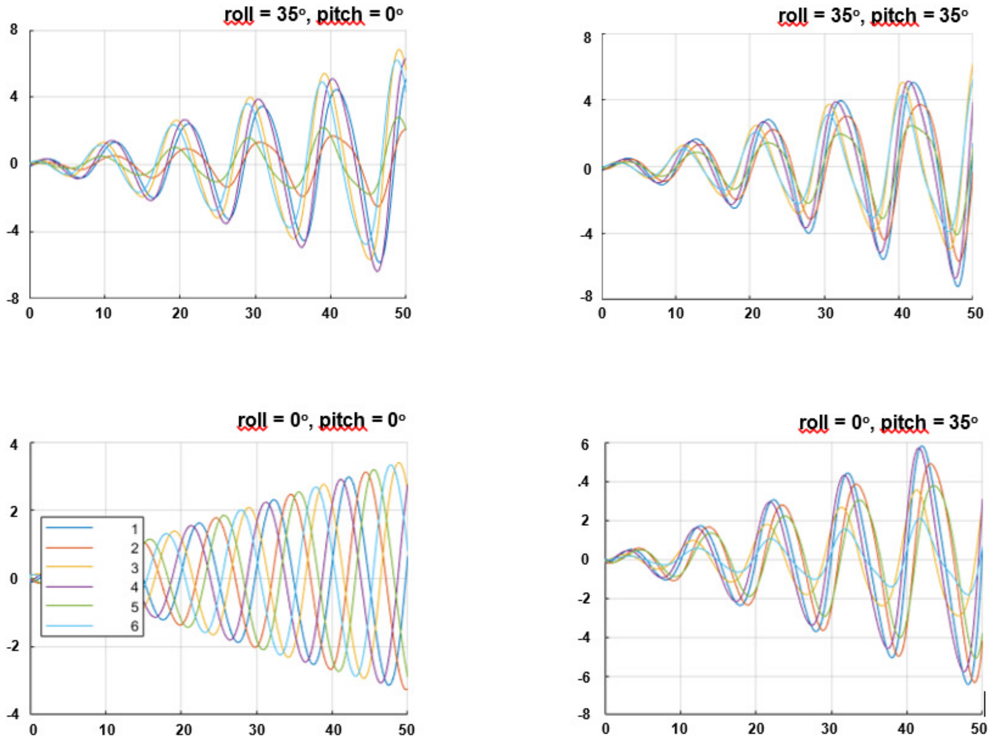


Fig. 7. Changes in velocity curves for each actuator in steps for the spiral

6. Conclusions

The paper presents the collaboration of MATLAB, MATLAB/Simulink and ADAMS in the design and simulation of the motion of the Gough-Stewart platform, in order to achieve the movement of the working part of the mechanism (more precisely, to achieve the movement of a point on the moving platform) along a prescribed trajectory. In particular, a kinematic solution was presented.

If the masses of the parts of the mechanism, external force effects, and the reactions in the linkages are taken into account, then the model is also applicable for solving the dynamics problem [17] and can be used to solve, for example, the problem of the required performance parameters of the drive units for the actuators. Using the methods of design study, design of optimization and optimization it is possible to analyse and optimize the work with the above platform, which will be the subject of further research [18].

Since this parallel mechanism is limited in the range of the map of working positions, it is in some cases advisable to place additional members on the moving platform to increase the range of movement of the working part of the device (however, these additional members may increase the demands on the drive unit of the mechanism).

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