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PARAMETER ESTIMATION OF WEIBULL PROBABILITY DISTRIBUTION BY SEVEN METHODS – A WIND REGIME OF THE CITY OF NITRA, SLOVAKIA

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Abstract. Slovakia currently has a relatively large unused potential in the area of electricity production from solar radiation and wind as renewable sources. The conversion of the wind's mechanical energy into electrical energy depends, among other things, on the wind speed and its turbulence. Perhaps the most widely used probability distribution for a wind speed model is the Weibull distribution. In the article, we deal with the comparison of seven methods for estimating the parameters of this distribution – maximum likelihood method, method of moments, empirical method, empirical method of Lysen, power density method, least squares method and weighted least squares method – on wind speed records from the city of Nitra for the period of 2005-2021. The vicinity of this city is one of the places identified as a suitable location for the installation of wind turbines. The performance of individual estimation methods is evaluated based on the indicators – the coefficient of determination R^2 and the root mean square error *RMSE*. Based on these values, the most accurate method is the weighted least squares method, although all other methods achieved similarly good results.

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Keywords: *maximum likelihood method, moment method, empirical method, power density method, least square method*

1. Introduction

The electricity generated from wind, alongside other renewable energy sources, represents clean, available and secure energy. With the European target of producing at least 42.5 % of the produced energy from renewable sources, it is necessary to increase the installed wind capacity from 204 GW in 2022 to more than 500 GW in 2030 [1]. The total installed capacity in Europe built in 2023 reached 18.3 GW with 79 % of it being produced onshore. Among the members of the European Union, the leader in wind energy production is Denmark, with a 56 % share of it in the power mix of the country. Right on the opposite side are Slovakia and Slovenia, where the share of wind power is close to zero in their energy mixes. [2] In Slovakia, there are currently two small wind farms, located in the south-west of the country, with a total installed capacity of 3 MW/year. However, there are another seventeen wind farms under the process of approval that may bring another capacity of 750 MW in the future. All the planned wind farms are situated in the western and south-western regions, with two of them in the vicinity of the city of Nitra. Therefore, the wind conditions in this city have been chosen as an object of the study.

The growing share of electricity production from solar and wind sources increases the stochastic nature of the power system. The wind speed is one of the most important parameters to model wind energy, as it determines how much of the wind energy is converted into electric power at the rotor of the wind turbine. The random character of the wind speed requires the study of the speed distribution to determine the wind energy capacity of the site. For this purpose, the most commonly used probability distribution is the two-parameter Weibull distribution. Since the Weibull distribution parameters play a significant role in the wind speed applications, it is a necessity to find the most suitable method for their estimation. The applicability and the accuracy of the method might vary with the sample size, sample data distribution, sample data format and the goodness of fit test [3]. The authors in [4] compared seven parameter estimation methods (the graphical method, the method of moments, standard deviation method, maximum likelihood method, power density method, modified maximum likelihood method, the equivalent energy method) when fitting the Weibull distribution to the 10-min time series wind speed measured at two different heights (20 m and 30 m) in three sites in Bangladesh. Based on their performance, the method of moments ranked as the best, with the maximum likelihood method being the second best and the power density method the third best. Gungor et al. in [5] investigated the appropriateness of four methods (the least square method, the standard deviation method, the maximum likelihood method, and the energy pattern factor method) finding out that the least square method is clearly outperformed by the other three methods when applied to the wind speed data from Izmir-Aliağa, Türkiye. Wind speed data from four stations in Alberta, Canada were examined in [6] to find the best estimation method for finding the parameters of the Weibull distribution. Whilst the empirical method of Justus, the empirical method of Lysen, the energy pattern factor method and the maximum likelihood method provided a very favourable efficiency, the graphical method achieved poor results. In [7], authors compared the analytical methods (the maximum likelihood method, the moment method, the energy pattern factor method, and the empirical method) to the heuristic optimization algorithms (particle swarm, crow search, aquila optimizer, bald eagle search optimizer) to identify the most suitable estimation method for the parameters of the Weibull distribution applied to wind speed data from Egypt.

In this study, we have chosen seven methods for Weibull parameter estimation – the method of moments (MOM), the empirical method (EM), the empirical method of Lysen (EML), the power density method (PDM), the maximum likelihood method (MLM), the least squares method (LMS) and the weighted least squares method (WLMS). Their efficiency is tested based on the fit of the distribution to the wind speed data from the city of Nitra, south-western Slovakia, divided into groups representing the seasons of the year. The performance of each method is assessed by the coefficient of determination R^2 and the root mean square error *RMSE*.

2. Parameter estimation of Weibull probability distribution

The cumulative distribution function (CDF) of the Weibull probability distribution is defined as

$$F(x) = 1 - \exp\left(-\left(\frac{x}{c}\right)^k\right); \tag{1}$$

the probability density function (PDF), as the derivative of (1), is then

$$f(x) = \frac{k}{c^k} x^{k-1} \exp\left(-\left(\frac{x}{c}\right)^k\right). \tag{2}$$

Both (1) and (2) are defined for $x > 0$, $k > 0$ and $c > 0$. Variable x represents the wind speed, k is a dimensionless shape parameter and c is a scale parameter in units of the wind speed. The scale parameter c is proportional to the mean wind speed. Further, the mean $E(X)$ and the variance $D(X)$ of this distribution are given as follows

$$E(X) = c\Gamma\left(1 + \frac{1}{k}\right), \tag{3}$$

$$D(X) = c^2\left(\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right), \tag{4}$$

respectively. Here $\Gamma(a)$ denotes the gamma function

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx, \;\; a > 0. \tag{5}$$

For the majority of the wind regimes all over the world, the shape parameter k ranges from 1.5 to 3 [8]. Higher value of the shape parameter implies more stability in the wind speed, whereas the higher value of the scale parameter c implies higher wind speed.

2.1. The parameter estimation methods

Several methods have been created for estimation of the Weibull probability distribution parameters k and c. The estimates of the parameters k and c will be denoted by $\hat{k}$ and $\hat{c}$, respectively, further in the text.

Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Weibull distribution with the parameters k and c. Let $x_1, x_2, \dots, x_n$ be a realization of the random sample. Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the ordered statistics of $X_1, X_2, \dots, X_n$ and let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be ordered observations.

In the method of moments, the estimates of the parameters are obtained by equating the first and the second moment of the Weibull distribution to the first and second sample moments, respectively, as follows

$$c\Gamma\left(1+\frac{1}{k}\right) = \bar{x}, \tag{6}$$

$$c^2\Gamma\left(1+\frac{2}{k}\right) = \frac{1}{n}\sum_{i=1}^{n} x_i^2. \tag{7}$$

Here,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{8}$$

is the sample mean wind speed. Dividing equation (7) with the squared equation (6) leads to equation

$$\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i^2}{(\bar{x})^2}. \tag{9}$$

The estimate $\hat{k}$ of the shape parameter is found as a solution of (9) numerically by the Newton-Raphson method in an iterative process. The estimate $\hat{c}$ of the scale parameter is then found as a solution of (6).

The empirical method is a special case of the method of moments where the estimation of the shape parameter is calculated from the equation

$$\hat{k} = \left(\frac{\bar{x}}{s_x}\right)^{1.086} \tag{10}$$

where

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2} \tag{11}$$

is the sample standard deviation. The estimate $\hat{c}$ is found in the same way as in the method of moments by the equation (6).

In the empirical method suggested by Lysen in [9], the estimate of the shape parameter $\hat{k}$ is found by the same equation (10) as it is in the empirical method, whereas the scale parameter is estimated by

$$\hat{c} = \bar{x}\left(0.568+\frac{0.433}{k}\right)^{-\frac{1}{k}}. \tag{12}$$

The energy pattern factor is defined as the ratio of the mean of the cube of the wind speed to the cube of the mean of the wind speed [3]

$$E_{pf} = \frac{\overline{(x^3)}}{(\bar{x})^3} = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i^3}{\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^3}. \tag{13}$$

The energy pattern method employs E_{pf} to obtain the estimate of the shape parameter as

$$\hat{k} = 1 + \frac{3.69}{\left(E_{pf}\right)^2}. \tag{14}$$

The estimate of the scale parameter $\hat{c}$ is calculated from the equation (6).

The maximum likelihood method is based on the maximization of the likelihood function $L(x_1, x_2, \dots, x_n; \theta)$ or its logarithm $\ln L(x_1, x_2, \dots, x_n; \theta)$ where $\theta \in \Theta$ is the unknown parameter (in general, it is a vector parameter) and $x_1, x_2, \dots, x_n$ is a realization of the random sample $X_1, X_2, \dots, X_n$ of size n from the distribution with PDF $f(x, \theta)$. For the Weibull probability distribution, the log-likelihood function is of the form

$$\ln L(x_1, \dots, x_n; k, c) = n \ln\frac{k}{c^k} - \frac{1}{c^k}\sum_{i=1}^{n} x_i^k + (k-1)\sum_{i=1}^{n} \ln x_i. \tag{15}$$

After one sets the derivative of (15) with respect to k and c equal to zero, the following equations for the estimates of the parameters are found

$$\frac{1}{\mathrm{k}} - \frac{\sum_{i=1}^{n} x_i^k \ln x_i}{\sum_{i=1}^{n} x_i^k} + \frac{1}{n}\sum_{i=1}^{n} \ln x_i = 0, \tag{16}$$

$$c = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^k\right)^{\frac{1}{k}}. \tag{17}$$

The estimate $\hat{k}$ is found as a solution of (16), calculated numerically by the Newton--Raphson method in an iterative process. Once the estimate $\hat{k}$ is known, the estimate $\hat{c}$ is calculated from (17).
In the least square method, one uses the linearization of the CDF (1) of the form

$$\ln\left(-\ln\left(1 - F(x)\right)\right) = k \ln x - k \ln c. \tag{18}$$

Substituting $\ln\left(-ln\left(1 - F(x)\right)\right) = Y$, $\ln x = X$, $-k \ln c = a$, $k = b$, the equation (18) can be written in a simpler form

$$Y = bX + a. \tag{19}$$

The estimates of the parameters k and c are found as solutions of (19) applying the least squares method. The estimates are given as

$$\hat{k} = \frac{n\sum_{i=1}^{n}\ln(x_{(i)})\ln\left(-\ln\left(1-\hat{F}(x_{(i)})\right)\right)-\sum_{i=1}^{n}\ln(x_{(i)})\sum_{i=1}^{n}\ln\left(-\ln\left(1-\hat{F}(x_{(i)})\right)\right)}{n\sum_{i=1}^{n}\ln^2 x_{(i)}-\left(\sum_{i=1}^{n}\ln x_{(i)}\right)^2}, \quad (20)$$

$$\hat{c} = \exp\left(-\frac{\sum_{i=1}^{n}\ln\left(-\ln\left(1-\hat{F}(x_{(i)})\right)\right)-\hat{k}\sum_{i=1}^{n}\ln(x_{(i)})}{n\,\hat{k}}\right). \quad (21)$$

As we may see in equations (20), (21), the unknown values of the CDF (1) from (18) are replaced with the estimate in the form of the mean rank [10]

$$\hat{F}\left(x_{(i)}\right) = \frac{i}{n+1} \quad (22)$$

where i denotes the i-th smallest value of the ordered observations $x_{(1)}, x_{(2)}, \dots, x_{(n)}$.

In the weighted least squares method, as a modification of the least squares method, the estimates of the regression parameters a and b from equation (19) minimize the function

$$Q(a,b) = \sum_{i=1}^{n} w_i\left(Y_i - a - b\ln\left(x_{(i)}\right)\right)^2 \quad (23)$$

where the weights w_i, $i = 1,2,\dots,n$, are defined as [11]

$$w_i = \left(\left(1-\hat{F}\left(x_{(i)}\right)\right)\ln\left(1-\hat{F}\left(x_{(i)}\right)\right)\right)^2, i = 1, 2, \dots, n. \quad (24)$$

Here again, $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ denote the ordered observations and $\hat{F}\left(x_{(i)}\right)$ is defined by (22). Then, the estimates $\hat{k}$ and $\hat{c}$ of the Weibull probability distribution parameters are obtained as solutions of the equations

$$\hat{k} = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln(x_{(i)})\ln\left(-\ln\left(1-\hat{F}(x_{(i)})\right)\right)-\sum_{i=1}^{n} w_i\ln(x_{(i)})\sum_{i=1}^{n} w_i\ln\left(-\ln\left(1-\hat{F}(x_{(i)})\right)\right)}{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i\ln^2 x_{(i)}-\left(\sum_{i=1}^{n} w_i\ln x_{(i)}\right)^2}, \quad (25)$$

$$\hat{c} = \exp\left(-\frac{\sum_{i=1}^{n} w_i\ln\left(-\ln\left(1-\hat{F}(x_{(i)})\right)\right)-\hat{k}\sum_{i=1}^{n} w_i\ln(x_{(i)})}{\hat{k}\sum_{i=1}^{n} w_i}\right). \quad (26)$$

3. Data description

The wind speed data, analysed in the paper, were recorded at the Nitra – Veľké Janíkovce meteorological station (indicator 11968), GPS latitude 48°16''50' [48.28056], GPS longitude 18°08"08' [18.13556], the height of 132 meters above sea level. The station is located on the outskirts of the city of Nitra, within the grounds of a small airport. It is surrounded by fields; the general face of the surroundings is partially sheltered. The coordinates refer to a measuring plot that has the standard dimensions required by internal regulations, i.e. 20 m × 20 m. The mast for wind measurement is within the measuring plot; at the airport in Nitra, it is located on the roof of the building. The standard height for measuring wind direction and speed at monitoring stations is 10 m above the ground. To measure wind characteristics, Vaisala automatic instruments and GILL ultrasonic instruments are currently being used. The anemometers have a 2-year calibration interval.

The data were collected from the meteorological reports within the time frame of January 2005 to December 2021 included. The data were recorded at hourly intervals. Before the pre-processing, the set counts for 149,016 records; after the errors, the missing data and zero wind speeds were removed, the set contains 143,879 data points (the percentage of data removed was 3.46 %). The data were further split into four datasets corresponding to seasons of the year – spring (March, April, May), summer (June, July, August), autumn (September, October, November) and winter (December, January, February). The sizes of respective datasets: spring – 36,960, summer – 36,341, autumn – 34,374, winter – 36,204.

Table 1. The descriptive statistics of the seasonal datasets

	Mean	Standard deviation	Minimum	Maximum	Skewness	Kurtosis
Spring	4.145	2.885	0.100	19.900	0.854	3.332
Summer	3.360	2.384	0.100	17.100	0.991	3.686
Autumn	3.696	2.769	0.100	21.000	0.876	3.175
Winter	4.065	3.043	0.100	18.900	0.885	3.215

	Lower quantile	Median	Upper quantile	Coefficient of variation [%]
Spring	1.800	3.500	6.000	69.59
Summer	1.500	2.800	4.800	70.95
Autumn	1.400	2.900	5.600	74.93
Winter	1.500	3.300	6.100	74.85

According to Table 1, the highest mean wind speed was observed in spring with value of 4.145 m/s. The lowest mean wind speed was observed in summer with value of 3.360 m/s. The coefficient of variation ranges from 69.59 % to 74.93 %, which indicates a very high variability of the wind speed in this location. The coefficient of kurtosis ranges from 3.175 to 3.686 therefore, the distribution can be regarded as highly leptokurtic distribution. The coefficient of skewness ranges from 0.854 to 0.991, which indicates a highly right skewed distribution.

4. Results

The parameters of the Weibull distribution were estimated applying all seven methods. The initial value for the Newton-Raphson method was the estimate $\hat{k}$ that was obtained using the least squares method. To assess the goodness of fit of the methods, two indicators were calculated. The coefficient of determination (R^2) and the root mean square error (*RMSE*) were considered to decide on the best fitting model. The *RMSE* determines the accuracy of model by calculating the average of the square difference between the observed and the predicted probabilities of the theoretical distribution. The R^2 is used to measure the linear relationship between the observed and the predicted probabilities of the theoretical distribution. The *RMSE* and R^2 are calculated by

$$RMSE = \left(\frac{1}{n}\sum_{i=1}^{n}\left[F_n(x_i) - \hat{F}(x_i)\right]^2\right)^{\frac{1}{2}}, \quad (27)$$

$$R^2 = \frac{\sum_{i=1}^{n}[\hat{F}(x_i)-\bar{F}]^2}{\sum_{i=1}^{n}[\hat{F}(x_i)-\bar{F}]^2+\sum_{i=1}^{n}[F_n(x_i)-\hat{F}(x_i)]^2} \quad (28)$$

where $\hat{F}$ is the estimated cumulative distribution function, $\bar{F} = \frac{1}{n}\sum_{i=1}^{n}\hat{F}(x_i)$ is its mean. The closer the value of R^2 is to 1, the better the fit. Similarly, the closer the value of *RMSE* is to 0, the better the fit.

4.1. Spring dataset

As can be seen in Table 2, the best fit is obtained by the weighted least squares method with $R^2 = 0.9972$ and $RMSE = 0.0149$. The second best is the maximum likelihood method, followed by the method of moments and both empirical methods. The worst fit is achieved when the parameters are estimated by the power density method. The coefficient of determination R^2 ranges from 0.9948 to 0.9972, which means that all the methods fit the data very well.

Table 2. The estimates of the Weibull probability distribution parameters and the goodness of fit criteria for the spring dataset

	MOM	EM	EML	PDM	MLM	LSM	WLSM
$\hat{k}$	1.4605	1.4825	1.4825	1.4913	1.4505	1.4719	1.3231
$\hat{c}$	4.5762	4.5852	4.5889	4.5886	4.5748	4.5382	4.6410
R^2	0.9951	0.9951	0.9951	0.9948	0.9960	0.9948	0.9972
$RMSE$	0.0197	0.0213	0.211	0.220	0.0190	0.0218	0.0149

4.2. Summer dataset

For the summer dataset (Table 3), the best fit is achieved by the method of moments, closely followed by the maximum likelihood method and the power density method. The worst fit is obtained by the least squares method with R^2 = 0.9953 and $RMSE$ = 0.0207. Here, the weighed least squares method achieved the second worst values of the goodness of fit indicators; however, the difference between the method of moments, as the best one, and the weight least squares method is 0.0005 (R^2) and 0.0008 ($RMSE$).

Table 3. The estimates of the Weibull probability distribution parameters and the goodness of fit criteria for the summer dataset

	MOM	EM	EML	PDM	MLM	LSM	WLSM
$\hat{k}$	1.4303	1.4517	1.4517	1.4499	1.4418	1.5011	1.3378
$\hat{c}$	3.6982	3.7060	3.7089	3.7053	3.7094	3.6681	3.6993
R^2	0.9974	0.9970	0.9970	0.9971	0.9972	0.9953	0.9969
$RMSE$	0.0151	0.0162	0.0162	0.0161	0.0155	0.0207	0.0159

4.3. Autumn dataset

Similarly as for the spring dataset, the autumn dataset is fitted the best by the weighted least squares method with R^2 = 0.9948 and $RMSE$ = 0.0203 (Table 4). The second best is the maximum likelihood method, followed by the method of moments. The worst fit is achieved by the power density method. The coefficient of determination R^2 ranges from 0.9900 to 0.9948, which means that all the methods fit the data very well. The difference between the best one and the worst one is 0.0048 (R^2) and 0.0107 ($RMSE$).

Table 4. The estimates of the Weibull probability distribution parameters and the goodness of fit criteria for the autumn dataset

	MOM	EM	EML	PDM	MLM	LSM	WLSM
$\hat{k}$	1.3489	1.3681	1.3681	1.3960	1.3298	1.3574	1.1751
$\hat{c}$	4.0297	4.0396	4.0426	4.0530	4.0219	3.9790	4.0822
R^2	0.9925	0.9916	0.9916	0.9900	0.9933	0.9916	0.9948
$RMSE$	0.0264	0.0282	0.0282	0.0310	0.0248	0.0282	0.0203

4.4. Winter dataset

Also, the winter dataset is fitted the best by the weighted least squares method with R^2 = 0.9966 and $RMSE$ = 0.0166 (Table 5). The second best is the maximum likelihood method, followed by the method of moments. The worst fit is achieved by the power density method. The coefficient of determination R^2 ranges from 0.9918 to 0.9966, which means that all the methods fit the data very well.

Table 5. The estimates of the Weibull probability distribution parameters and the goodness of fit criteria for the winter dataset

	MOM	EM	EML	PDM	MLM	LSM	WLSM
$\hat{k}$	1.3504	1.3697	1.3697	1.3962	1.3270	1.3451	1.1899
$\hat{c}$	4.4332	4.4441	4.4474	4.4581	4.4202	4.3789	4.4964
R^2	0.9940	0.9932	0.9932	0.9918	0.9949	0.9937	0.9966
$RMSE$	0.0235	0.0253	0.0253	0.0280	0.0216	0.0243	0.0166

5. Conclusion

In the paper, the wind speed data from Nitra-Veľké Janíkovce were fitted by the two-parameter Weibull distribution, with seven methods used for the estimation of the probability distribution parameters. The coefficient of determination and the root mean square error were used to evaluate the performance of the considered methods. All seven methods performed well and are applicable for estimating the Weibull distribution parameters for all seasons, as indicated by high values of R^2 and low values of $RMSE$. In three seasons out of four (spring, autumn, winter), the weighed least squares method achieved the best results. The maximum likelihood method ranked as the second one, and the third one was the method of moments. On the other hand, the power density method had the worst fit; however, the value of R^2 exceeded 0.99 in all cases. In summer, the method of moment performed as the best one, the maximum likelihood method as the second best and, as opposed to the rest of the seasons, the power density distribution achieved the third best result. Further, we can conclude that the weighted least squares method performed better than the least squares method, which implies that the weight factor influences the performance of the parameter estimation. To sum it up, the weight least squares method provided the best fit in most of the modelled datasets. Furthermore, this method has several advantages – it is computationally simple, and the parameters of Weibull distribution can be estimated with closed formulas.

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