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METHOD FOR DETERMINING THE CENTER OF GRAVITY HEIGHT IN WHEELCHAIRS

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Abstract. Wheelchairs are the primary means of movement for people with mobility disabilities, regardless of their type of drive. One of the main features of wheelchairs, besides their ability to generate drive, maneuverability, and body support, is their stability of movement. This stability largely depends on the height of the center of gravity in the human-wheelchair system. This parameter is very important from the perspective of wheelchair user safety, especially when overcoming architectural obstacles such as curbs, stairs, or slopes. Currently, many models available on the market have additional protection in the form of support wheels, which prevent the wheelchair from tipping over backwards with the user. However, the use of these support wheels is associated with an increase in the weight and dimensions of the wheelchair, as well as a decrease in ergonomics. Faced with this problem, the authors have set out to present an innovative method for calculating the height of the center of gravity and conducted a series of experimental tests on various wheelchair designs. The research analyzed the displacement of the center of gravity depending on the angle of the inclination of the ground from 0^o to 15^o. The conclusions obtained regarding stability and the calculation of the center of gravity in relation to maintaining balance can form the basis for designing new wheelchair structures.

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Keywords: *wheelchair, center of gravity, CoG, height of center of gravity, measurement of CoG, stability*

1. Introduction

Wheelchairs, regardless of their type of drive, play a key role in the everyday life of people with mobility disabilities. One of the features that determines the use of wheelchairs is the location of their center of gravity, both in terms of the wheelchair itself and in the context of the entire human-wheelchair system. The location of the center of gravity affects many aspects of use, which is why it is extremely important to take it into account when designing a wheelchair and its selection when purchasing it. For wheelchairs, as in the case of all vehicles, a relative lowering of the center of gravity translates into increased stability [1]. This is particularly important in the context of user safety. A center of gravity that is too high can increase the risk of the wheelchair and its occupant tipping over when navigating architectural obstacles, e.g. curbs or steep ramps. Currently, there are many models of wheelchairs available on the market, additionally equipped with support wheels, which are designed to prevent the wheelchair and the user from tipping over backwards. However, the use of these additional safeguards has consequences in the form of increased weight and dimensions of the wheelchair, and consequently, also reduced ergonomics of their use. It should be emphasized that, at the same time, the analysis of the position of the center of gravity on a plane parallel to the ground plane provides information about the maneuverability of the system [2]. The closer this point is to the center, passing through the plane of symmetry of the system, the more maneuverable the wheelchair is. This property is important both in terms of moving in closed spaces (apartments, narrow corridors) and on uneven surfaces (damaged pavements, unpaved surfaces). In addition, the position of the center of gravity translates directly into the distribution of the load of the individual wheel axles [3] and rolling resistance [4], which in turn translates into the adhesion of the tires to the surface and the transfer of hand force to the actual torque of the wheels. The center of gravity of wheelchairs can be determined using the ISO 7176-1:2014 standard, which focuses on measuring the tilt angle of wheelchairs for both locked and unlocked drive wheels [5]. Another method for determining stability, based on the analysis of parameterization in the CAD environment, is presented in [6]. It also draws attention to the problem of determining the position of the center of gravity solely on the basis of anthropotechnical atlases. In the state of the art, there are also known solutions consisting in measuring the COG using devices using the measurement of the moment of inertia, for which there is a correlation between the height of the center of gravity and the value of the vibration amplitude of the measuring device [7]. In the study [4], the authors conducted a series of tests aimed at assessing the correctness of the methods for testing the COG position using tilting platforms, demonstrating a tolerance of the center of gravity position of up to 5 mm.

In view of the problem posed in this way, the authors of this article proposed a new method for determining the height of the center of gravity. This study is an extension of the solution normally used for calculating this parameter for motor vehicles using measurement during tilt on a weighing scale with four support points [8]. The article presents the results of a series of experimental tests of the center of gravity position for various types of manual wheelchair structures in order to verify the developed method. The developed results can serve as a basis for future design work on wheelchair structures, as well as for modeling the human-wheelchair system.

2. Materials and methods

The proposed method was developed based on the analysis and identification of the geometric similarity between the wheelchair system and a motor vehicle with different wheel diameters for the front and rear axles. Therefore, this system can be modeled as a rigid beam with two support points [9]. The full derivation of the discussed model can be found in [3, 10], where the authors conducted an analysis of the influence of the center of gravity position on the axle load of individual wheels. Based on the state-of-the-art analysis of wheelchair construction, it was assumed that the wheelchair is a symmetrical body, with symmetry determined by the XY plane passing through the midpoint of the seat width along the Z axis, as shown in Figure 1. This assumption was supported by measurements of the tested wheelchairs, which showed that the displacement of the center of gravity along the Z axis was negligible. Therefore, the scope of research was limited, guided by the assumption of the symmetry of the wheelchair system with respect to the XY plane (Fig. 1). Figure 1 shows the distribution of the force system and the parametric dimensions of the wheelchair system's center of gravity for the horizontal position (Fig. 1a) and for the tilted position (Fig. 1b). The center of gravity position was determined in the XY plane using the distance x_2 from the rear wheel axis along the ground plane and h as the height from the ground.

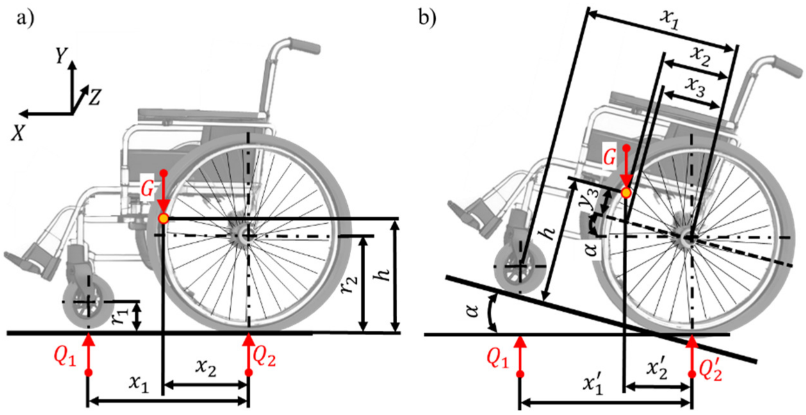

Fig. 1. The force system and dimensions of the wheelchair system in the tests of the height position of the center of gravity: a) before tilting, b) after tilting

For the system thus created, the sum of moments about the reaction point of the force Q_2 for the rear wheel can be determined as equations (1) and (2):

$$Gx_2 - Q_1x_1 = 0 \rightarrow x_2 = x_1\frac{Q_1}{G}, \tag{1}$$

$$Gx_2' - Q_1'x_1' = 0 \rightarrow x_2' = x_1'\frac{Q_1'}{G}, \tag{2}$$

where: Q_1, Q_2 are the values of the forces acting on the front (smaller) and rear (larger) wheels of the wheelchair before tilting; Q_1', Q_2' – the values of the forces acting on the front (smaller) and rear (larger) wheels of the wheelchair after tilting; G – weight of the wheelchair; h – height of the center of gravity of the wheelchair system in a perpendicular projection to the ground; x_1 – wheelbase of the wheelchair; x_2 – position of the center of gravity relative to the axis of the rear (larger) wheel of the wheelchair along the wheelchair; x_1' – horizontal projection of the wheelbase of the wheelchair after tilting; x_2' – horizontal projection of the distance of the center of gravity point from the axis of the rear wheel of the wheelchair after tilting; r_1, r_2 – radii of the front and rear wheels, respectively.

Based on the analysis of the geometry of the wheelchair system after tilting shown in Figure 1b, the following equations can be formulated:

$$\frac{x_2'}{x_3} = \cos\alpha \rightarrow \frac{x_2'}{\cos\alpha} = x_3, \tag{3}$$

$$\frac{x_2 - x_3}{y_3} = \mathrm{tg}\alpha \rightarrow y_3 = \frac{x_2 - x_3}{\mathrm{tg}\alpha}, \tag{4}$$

$$x_1' = x_1 \cos\alpha, \tag{5}$$

$$h = y_3 + r_2, \tag{6}$$

where: α – the wheelchair inclination angle; x_3 – distance of the vertical projection of the center of gravity position from the rear wheel axis; y_3 – position of the center of gravity point relative to the rear wheel axis in the perpendicular projection to the ground.

After inserting equation (3) into (4), the formula for calculating y_3 takes the form:

$$y_3 = \frac{x_2}{\mathrm{tg}\alpha} - \frac{x_2'}{\cos\alpha \cdot \mathrm{tg}\alpha} = x_2 \mathrm{ctg}\alpha - x_2' \sin\alpha. \tag{7}$$

Finally, after deriving the formulas, the sought position of the center of gravity height h takes the form:

$$\begin{aligned} h &= x_1 \frac{Q_1}{G} \mathrm{ctg}\alpha - x_1 \frac{Q_1'}{G} \cos\alpha \sin\alpha + r_2 = \\ &= \frac{x_1}{G} [Q_1 \mathrm{ctg}\alpha - Q_1' \cos\alpha \sin\alpha] + r_2. \end{aligned} \tag{8}$$

The experimental tests were performed using a dynamometer [11], which is an advanced research station enabling the kinematic and dynamic analysis of wheelchairs in laboratory conditions, enabling the simulation of various terrain conditions [11]. The measurement of the change in the position of the center of gravity is carried out by reading the loads on the weighing scale [10]. It consists of 4 tensometric transducers ($W_1 - W_4$, Fig. 3), which support the system in its corners. From the mechanical point of view, this system can be modeled as a system of four beams, the beginnings and ends of which are located at the support points, which are individual scales. A detailed description of the measurement, methodology and analysis of the results obtained from this research station is included in [10]. The diagram of forces and physical dimensions of the system lifting the wheelchair on the weighing scale is shown in Figure 2.

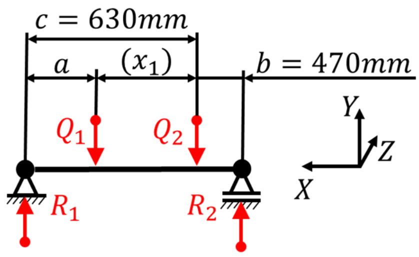

Fig. 2. Diagram of forces and physical dimensions of the weighing scale system

In this system, the sum of moments at the points of application of force Q_1 and Q_2 can be determined, resulting in the following system of equations:

$$\begin{cases} Q_2 = \dfrac{R_2(x_1+b) - R_1 a}{x_1} = \dfrac{R_2(x_1+b) - R_1(c - x_1)}{x_1} \\ Q_1 = \dfrac{R_2(x_1+a) - R_1 b}{x_1} = \dfrac{R_2 c - R_1 b}{x_1} \end{cases} \tag{9}$$

where: R_1, R_2 – the values of forces measured on the left or right side of the weighing pan, respectively; a, b, c – dimensions characterizing the weighing pan in accordance with the diagram shown in Figure 2.

Similarly to the system of equations (9), the values of forces for the analyzed system after its deflection can be written down, allowing us to formulate the following equations:

$$\begin{cases} Q_2' = \dfrac{R_2'(x_1'+b') - R_1' a'}{x_1'} = \dfrac{R_2'(x_1\cos\alpha+b') - R_1'(c - x_1\cos\alpha)}{x_1\cos\alpha} \\ Q_1' = \dfrac{R_2'(x_1'+a') - R_1' b'}{x_1'} = \dfrac{R_2' c - R_1' b'}{x_1\cos\alpha} \end{cases} \tag{10}$$

assuming that $b = b'$, because for the designed system this dimension remains unchanged during the tilting – the contact point of the rear wheelchair wheel and the dynamometer roller is the center of rotation of the tilting movement of the system. Therefore, transforming equation (8), the final form of the dependence for the height of the center of gravity h takes the form:

$$h = \frac{x_1}{G}[Q_1 \mathrm{ctg}\alpha - Q_1' \cos\alpha \sin\alpha] + r_2 =$$
$$= \frac{x_1}{G}\left[\frac{R_2 c - R_1 b}{x_1}\mathrm{ctg}\alpha - \frac{R_2'(c) - R_1' b}{x_1 \cos\alpha}\cos\alpha\sin\alpha\right] + r_2 = \quad (11)$$
$$= \frac{1}{G}[(R_2 c - R_1 b)\mathrm{ctg}\alpha - (R_2' c - R_1' b)\sin\alpha] + r_2$$

The measurement series were performed each time during the tilting of the system up to an incline of 15 °, thus simulating a steep climb of 26.8 %. It should be noted that the maximum slope of the climb for disabled people using a wheelchair is 15 % (8.5 °) [12]. The decision to use an increased tilt was also made due to the greater differences in the measured mass on the tensometric scales. Recording values on a larger scale minimizes the influence of the measurement error on the final results. Figure 3 shows the wheelchair mounted on the dynamometer during the test.

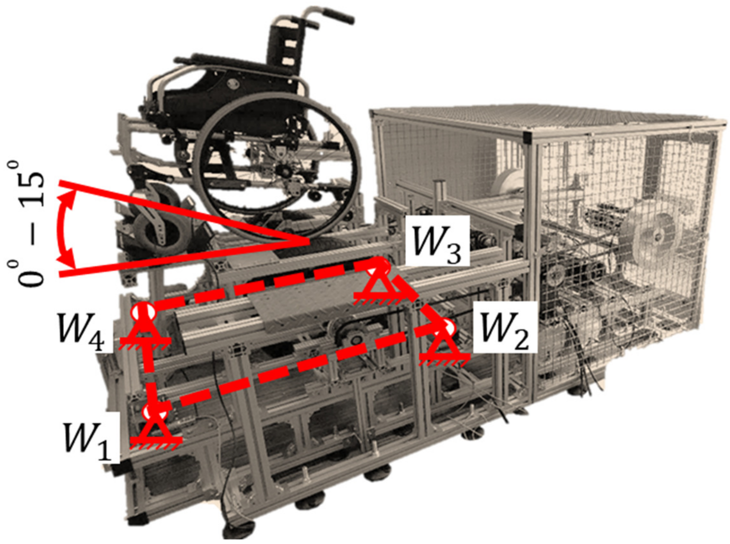

Fig. 3. Wheelchair mounted on dynamometer during testing with marker of the weighing scale system

Eleven manual wheelchairs were selected for the study, representing various models and configurations available on the market. The selection of wheelchairs took into account the frame material used, differences in geometry and construction, as well as the possibility of adjusting the position of the seat relative to the rear wheel axis. Table 1 presents the tested wheelchair models along with their technical parameters.

Table 1. Parametric table of the wheelchairs tested

Wheelchair marking	Producer	Model	Front wheel diameter* [mm]	Rear wheel diameter* [mm]	Wheelbase [mm]	Weight [kg]	Frame material	Source
A1	Sunrise Medical	Quickie Helium	127	609.6	290	6.5	Aluminum alloy	[13]
A2.1	Vermeiren	V300	200	609.6	395	20	Steel alloy	[14]
A2.2	Vermeiren	V300	200	609.6	495	20	Steel alloy	[14]
A3	Armedical	AR-405	200	609.6	400	18	Steel alloy	[15]
A4	Vermeiren	101	200	558.8	520	16.8	Steel alloy	[16]
A5	Vermeiren	Jazz S50	200	558.8	520	16.1	Steel alloy	[17]
A6	Vermeiren	708 Delight	200	609.6	415	18	Steel alloy	[18]
A7	Herdegen	700100 Living	150	300	385	11.2	Aluminum alloy	[19]
A8	Karma Medical	KM-2501-F 14	150	330	360	8	Aluminum alloy	[20]
A9	Karma Medical	KM-2512-F 20	180	609.6	375	10	Aluminum alloy	[20]
A10.1	Sunrise Medical	Breezy BasiX²	200	609.6	355	14.9	Aluminum alloy	[21]
A10.2	Sunrise Medical	Breezy BasiX²	200	609.6	430	14.9	Aluminum alloy	[21]
A11	Timago	H011 Classic-Tim	200	609.6	400	18.6	Steel alloy	[22]
A12	Karma Medical	S-Ergo 305	200	609.6	390	16	Aluminum alloy	[23]
A13	Sunrise Medical	Breezy 90	200	609.6	395	19	Steel alloy	[24]

* values were converted from inches

3. Results and discussion

Before starting to work on the results, the data collected in the experimental studies had to be processed in a mathematical modeling environment. Based on the data set of loads of the individual four strain gauge transducers, the load points of the individual four beams were determined, and the intersection points of the sections whose beginnings and ends were located at their load points were determined. These cases were considered for both a 0 ° and 15 ° incline. Figure 4 graphically presents the change in the position of the center of gravity along the wheelchair A1 in the initial and final position. The average change in position was 132 mm. As can be seen from Figure 4, the change in the position of the center of gravity along the Z axis was within ±2 mm. This value can be considered negligible.

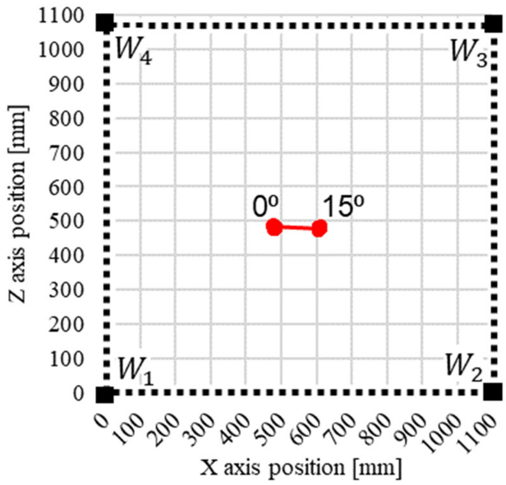

Fig. 4. Graphic representation of the change in the position of the center of gravity along the wheelchair A1 against the background of the weighing scale system (XZ plane) for a deflection from 0° to 15°; squares indicate the location of the strain gauges W_1 – W_4

Next, the reaction forces R_1 and R_2 were calculated for the inclination angles of 0° and 15°. The obtained force values, as well as the remaining characteristic parameters of the system, were substituted into the final form of equation (11). In this way, the height of the center of gravity h for the individual types of bogies was obtained. The set of developed results is presented in Figure 5.

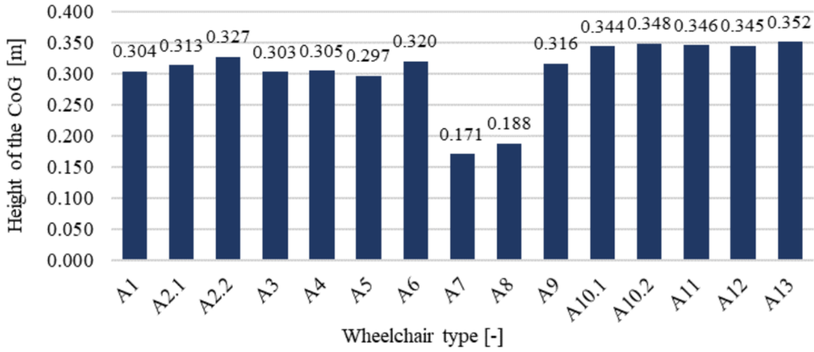

Fig. 5. Graphical representation of the height of the center of gravity for individual types of wheelchairs

Based on the obtained results, it can be concluded that the use of smaller diameter wheels, as in the case of the A7 and A8 wheelchairs, has a significant impact on lowering the position of the center of gravity, which directly affects the position of their axles. Another factor influencing the change in the height of the center of gravity is the adjustment of the user's seat position relative to the axles of the rear wheels. The closer the seat is to the rear axle (it is mounted further back), the higher the center of gravity is. This is an obvious relationship – for this very reason, the use of this type of adjustment is popular in manual wheelchairs. Additionally, this effect can be deepened by the user, because such a seat position will be more natural and ergonomic for people with a larger body posture. The higher and closer the center of gravity is to the rear axle, the less stable the user – wheelchair system will be, especially for a small and/or narrow wheelbase.

4. Summary

The position of the center of gravity largely depends on the geometry of the structure, the type of construction materials used, as well as the wheelbase. The COG is a key parameter in ensuring the safety of use and stability of wheelchairs. In this article, the authors presented an innovative method for determining the height of the center of gravity for wheelchairs. The experimental studies carried out used a variable angle of inclination. They confirmed the effectiveness of the developed method based on different wheelchair designs. The stability of a wheelchair largely depends on the position of the center of gravity, which depends on the geometry of the structure, as well as the materials used and the wheelbase. The developed results, as well as the mathematical model, can be used to improve the design of wheelchairs and develop new solutions aimed at increasing the safety of their use, as well as to better adapt them to the needs of people with mobility disabilities.

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