Journal of Applied Mathematics and Computational Mechanics 2024, 23(2), 54-65
www.amcm.pcz.pl p-ISSN 2299-9965
DOI: 10.17512/jamcm.2024.2.05 e-ISSN 2353-0588

SLOW FLOW OF MICROPOLAR FLUID PAST AN IMMISCIBLE MICROPOLAR FLUID SPHERE

Krishna Prasad Madasu, Nidhi Goyal

Department of Mathematics, National Institute of Technology, Raipur-492010
Chhattisgarh, India
madaspra.maths@nitrr.ac.in, kpm973@gmail.com, goyalnidhi.1408@gmail.com

Received: 14 March 2024; Accepted: 2 June 2024

Abstract. The Stokes axisymmetric flow of an incompressible micropolar fluid past an another immiscible micropolar fluid sphere is studied analytically under small Reynolds numbers. A spherical coordinate system is used to solve the Stokes equations for the fluid velocities, pressures and microrotation vectors inside and outside the micropolar fluid drop. The boundary conditions on the micropolar fluid drop surface are satisfied by vanishing of a normal component of velocity inside and outside the micropolar fluid sphere, tangential components of velocities are continuous, tangential components of stresses are continuous, and the microrotation vector inside and outside the micropolar fluid sphere vanishes. Numerical results for the drag force acting on the micropolar fluid drop are obtained for various values of the relative viscosity of the fluid drop, micropolar parameters (vortex viscosity parameters), and shear spin viscosity parameters. It is found that the drag force exerted on the viscous drop in a micropolar fluid and the micropolar fluid drop in a viscous fluid increase with an increase in the viscosity ratio. Additionally, the findings demonstrate that the drag force acting on the micropolar drop in a micropolar fluid increases as the viscosity ratio increases, and the drag force on the gaseous bubble is less than that of a solid sphere. Well-known results are reduced, and comparisons are made with a classical viscous-viscous droplet, a micropolar-viscous droplet and a viscous-micropolar droplet. The present study has significant applications in natural, biological, and industrial processes, such as sedimentation phenomena, liquid-liquid extraction, the study of blood flow, and the rheology of emulsions.

MSC 2010: *76A05, 76D07, 76S05*
Keywords: *micropolar fluid, Stokes flow, drop, immiscible, drag force*

1. Introduction

The motion of a fluid droplet in a second immiscible fluid through a continuous medium at low Reynolds numbers is of much interest in the fields of chemical, biomedical, and environmental engineering and science. This study plays an important role in natural and industrial processes such as raindrop formation, the mechanics and rheology of emulsions, liquid-liquid extraction, motion of blood cells in an artery or vein, extraction of crude oil from petroleum products and sedimentation phenomena. The steady state solution of slow viscous flow outside and inside a fluid sphere is investigated analytically by Hadamard and Rybczynski [1, 2].

A well-accepted theory that accounts for an internal microstructure is the micropolar fluid theory initiated by Eringen [3]. Rao and Rao [4] examined the slow flow of a micropolar fluid past a rigid sphere. They found the drag force on the sphere is more in the polar fluid than that of viscous fluid. Ramkissoon and Majumdar [5] derived a general expression for the drag force using axisymmetric point force.

The cases where one of the fluids is non-Newtonian and other is Newtonian fluid are as follows. Using the spin vorticity relation, Niefer and Kaloni [6] studied the flow of a viscous fluid past a micropolar fluid sphere and vice-versa. Ramkissoon [7] obtained the exact solution for the translation of a Newtonian fluid sphere through a micropolar fluid using a no-spin condition on the microrotation. Saad [8] extended the work [6] by considering the bounded medium. Faltas and Saad [9] investigated the motion of a viscous fluid sphere in a micropolar fluid perpendicular to a plane wall and vice-versa using the boundary collocation technique. Ramkissoon and Majumdar [10] considered the Stokes flow of a micropolar fluid past a viscous fluid spheroid whose shape deviates slightly from that of a sphere. Later, Madasu and Kaur [11, 12] continued the work by considering the non-zero microrotation condition. Gomathy et al. [13] investigated the creeping flow of micropolar fluid past a micropolar fluid using continuity of velocity and pressure. The following articles are the cases when the two fluids are non-Newtonian. Srinivasacharya and Rajyalakshmi [14] investigated the slow flow of an incompressible micropolar fluid past a porous sphere.

Recently, Khanukaeva [15] discussed the flow of a micropolar fluid through a spherical cell, consisting of a solid core, porous layer and liquid envelope that is modeled using coupled micropolar and Brinkman-type equations. Yadav et al. [16] studied the Poiseuille flow of micropolar-Newtonian fluid through concentric pipes filled with a porous medium. Selvi et al. [17] examined the flow around a Reiner-Rivlin liquid sphere placed in an aqueous medium. El-Sapa [18] examined the effect of a magnetic field on the slow motion of a microstretch fluid droplet in a microstretch fluid. El-Sapa [19] studied cell models for the axisymmetric creeping flow of micropolar fluid past a porous sphere filled with micropolar fluid. Yadav et al. [20] investigated entropy production for the immiscible nature of micropolar and Newtonian viscous fluid within a channel. Salem et al. [22] examined the migration of a spherical viscous droplet along the axis of cylindrical tube filled by micropolar fluid and the related problem of spherical micropolar droplet in a viscous fluid-filled cylindrical tube. The effect of the spherical slip cavity on a spherical viscous droplet in a micropolar fluid, and spherical micropolar droplet in a viscous fluid is investigated respectively in [23, 24]. Alharbi and Salem [25] studied the effect of a tangential slip and spin slip conditions on steady motion of a micropolar drop within a concentric spherical cavity containing a micropolar fluid.

The aim of this paper is to extend to the previous study [6] the case where the interior and exterior of the fluid sphere is filled with micropolar fluid. Furthermore, calculation of the resistant force exerted on a micropolar fluid sphere in an unbounded micropolar fluid, is the key value to be determined in the current investigation. To the best of the author's knowledge, the idea of the current investigation was not done until now. The variation of the drag force versus the viscosity ratio and micropolarity parameters are presented graphically and discussed. Some previously published well-known results are also deduced from the present analysis. The knowledge of this type of phenomena is important in understanding liquid-liquid, solid-liquid, gas-liquid systems.

2. Problem formulation

Consider the steady axisymmetric flow of an incompressible micropolar fluid past an immiscible micropolor fluid sphere that is held fixed in a uniform stream of velocity U (Fig. 1). The external region and the internal region are denoted by regions I and II, respectively. The following assumptions considered the fluid inside the sphere as a micropolar fluid, and the fluid in the surrounding medium was considered to be a micropolar fluid. The flow is steady, axisymmetric, there is no interfacial mass transfer (the radial velocity is zero at interface), there are no surface-active materials, and the shape of the fluid sphere is permanently spherical.

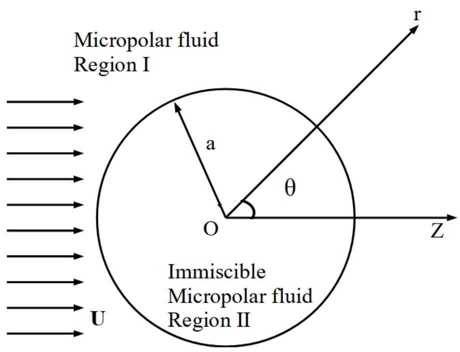


Fig. 1. Physical situation of the problem

The equations of motion for the exterior and interior regions of the fluid sphere are the equations governing the steady flow of an incompressible micropolar fluid under the Stokesian assumption with the absence of body force and body couple [3].

$$\nabla\cdot\vec{q}^{\,(i)}=0, \tag{1a}$$

$$\nabla p^{(i)}+(\mu_i+\kappa_i)\,\nabla\times\nabla\times\vec{q}^{\,(i)}-\kappa_i\,\nabla\times\vec{v}^{\,(i)}=0, \tag{1b}$$

$$\kappa_i\,\nabla\times\vec{q}^{\,(i)}-2\,\kappa_i\,\vec{v}^{\,(i)}-\gamma_i\,\nabla\times\nabla\times\vec{v}^{\,(i)}+(\alpha_i+\beta_i+\gamma_i)\nabla\left(\nabla\cdot\vec{v}^{\,(i)}\right)=0,\quad i=1,2, \tag{1c}$$

where $\vec{q}^{(i)}$, $\vec{v}^{(i)}$ and $p^{(i)}$ are velocity vector, microrotation vector and pressure, respectively. μ_i is the viscosity coefficient of the classical viscous fluid and κ_i, α_i, β_i and γ_i

are the new viscosity coefficients for the micropolar fluids. These constants conform to the usual inequalities:
$\kappa_i \geq 0; \quad 2\mu_i + \kappa_i \geq 0; \quad \gamma_i \geq 0; \quad |\beta_i| \leq \gamma_i; \quad 3\alpha_i + \beta_i + \gamma_i \geq 0, \quad i = 1,2.$

The equations for the stress tensor $t^{(i)}_{i_1 j_1}$ and the couple stress tensor $m^{(i)}_{i_1 j_1}$ are

$$t^{(i)}_{i_1 j_1} = -p^{(i)}\,\delta_{i_1 j_1} + \mu_i\,(q^{(i)}_{i_1,j_1} + q^{(i)}_{j_1,i_1}) + \kappa_i\,(q^{(i)}_{j_1,i_1} - \varepsilon_{i_1 j_1 m_1}\,\nu^{(i)}_{m_1}), \quad i = 1,2, \tag{2}$$

$$m^{(i)}_{i_1 j_1} = \alpha_i\,\nu^{(i)}_{m_1,m_1}\,\delta_{i_1 j_1} + \beta_i\,\nu^{(i)}_{i_1,j_1} + \gamma_i\,\nu^{(i)}_{j_1,i_1}, \quad i = 1,2, \tag{3}$$

where the comma denotes the partial differentiation, $\delta_{i_1 j_1}$ and $\varepsilon_{i_1 j_1 m_1}$ are the Kronecker delta and the alternating tensor, respectively.

Let (r,θ,ϕ) denote a spherical polar co-ordinate system. Since the flow of the fluid is in the meridian plane and the flow is axially symmetric, all the quantities are independent of ϕ. Hence, we assume the velocity and microrotation vectors as

$$\vec{q}^{\,(i)} = q^{(i)}_r(r,\theta)\,\vec{e}_r + q^{(i)}_\theta(r,\theta)\,\vec{e}_\theta, \quad i = 1, 2. \tag{4}$$

$$\vec{\nu}^{\,(i)} = \nu^{(i)}_\phi(r,\theta), \quad i = 1, 2. \tag{5}$$

Let $\psi^{(i)}, i = 1,2$ denote the Stokes stream functions of the exterior and interior regions of the fluid sphere. Then the velocity components in terms of stream functions are

$$q^{(i)}_r = \frac{1}{r^2}\frac{\partial\psi^{(i)}}{\partial\zeta}, \quad q^{(i)}_\theta = \frac{1}{r\sqrt{1-\zeta^2}}\frac{\partial\psi^{(i)}}{\partial r}, \quad i = 1, 2. \tag{6}$$

where $\zeta = \cos\theta$.
Introducing the following nondimensional variables [14]
$r = a\tilde{r},\ \psi^{(i)} = Ua^2\tilde{\psi}^{(i)},\ p^{(i)} = \dfrac{\mu_1 U}{a}\tilde{p}^{(i)},\ \nu^{(i)}_\phi = \dfrac{U}{a}\tilde{\nu}^{(i)}_\phi.$
After elimination of the pressure $p^{(i)}$ and the microrotation vector $\nu^{(i)}_\phi$ from Eq. (2), we obtain linear partial differential equations for the stream functions as

$$E^4\left(E^2 - l_i^2\right)\psi^{(i)} = 0, \quad i = 1, 2. \tag{7}$$

and microrotation vectors are given by

$$\nu^{(i)}_\phi = \frac{1}{2r\sqrt{1-\zeta^2}}\left(E^2\psi^{(i)} + \frac{(2+\chi_i)}{\chi_i}l_i^{-2}E^4\psi^{(i)}\right), \quad i = 1, 2. \tag{8}$$

where

$$l_i^2 = \frac{a^2\kappa_i(2+\chi_i)}{\gamma_i(1+\chi_i)}, \quad \chi_i = \frac{\kappa_i}{\mu_i} \quad \text{are the micropolar parameters}, \quad i = 1, 2,$$

and

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\zeta^2}{r^2}\frac{\partial^2}{\partial \zeta^2} \quad \text{is the Stokesian operator.}$$

3. Boundary conditions

In order to get the exact solution to the problem, we need to first determine the stream functions $\psi^{(1)}$ and $\psi^{(2)}$. Then, the velocity and microrotation vectors outside and inside the fluid sphere can be determined. On the liquid-liquid interface, we assume the mutual impenetrability, continuity of tangential velocity and tangential stresses, and no-spin condition on the microrotation [6, 12]. Therefore, the boundary conditions are as follows:

$$q_r^{(1)} = 0, \tag{9}$$

$$q_r^{(2)} = 0, \tag{10}$$

$$q_\theta^{(1)} = q_\theta^{(2)}, \tag{11}$$

$$t_{r\theta}^{(1)} = t_{r\theta}^{(2)}, \tag{12}$$

$$\nu_\phi^{(1)} = 0, \tag{13}$$

$$\nu_\phi^{(2)} = 0. \tag{14}$$

4. Solution of the problem

The solution of equation Eqs. (7) are given by [6–8]

$$\psi^{(1)} = \left[r^2 + A\,r^{-1} + B\,r + C\sqrt{r}K_{3/2}(l_1\,r)\right]\vartheta_2(\zeta), \tag{15}$$

$$\psi^{(2)} = \left[E\,r^2 + F\,r^4 + G\sqrt{r}I_{3/2}(l_2\,r)\right]\vartheta_2(\zeta), \tag{16}$$

Substituting Eq. (15) and Eq. (16) in Eq. (8), we get microrotation components as

$$\nu_\phi^{(1)} = \frac{1}{r\sqrt{1-\zeta^2}}\left[-B\,r^{-1} + C\,l_1^2\frac{(1+\chi_1)}{\chi_1}\sqrt{r}K_{3/2}(l_1\,r)\right]\vartheta_2(\zeta), \tag{17}$$

$$v_\phi^{(2)} = \frac{1}{r\sqrt{1-\zeta^2}}\left[5F\,r^2 + G\,l_2^2\frac{(1+\chi_2)}{\chi_2}\sqrt{r}\,I_{3/2}(l_2\,r)\right]\vartheta_2(\zeta). \tag{18}$$

The boundary conditions in terms of stream function $\psi^{(i)}$, $i = 1,2$ on the surface of sphere $r = 1$ lead to the following:

$$\frac{\partial\,\psi^{(1)}}{\partial\zeta} = 0, \tag{19}$$

$$\frac{\partial\,\psi^{(2)}}{\partial\zeta} = 0, \tag{20}$$

$$\frac{\partial\,\psi^{(1)}}{\partial r} = \frac{\partial\,\psi^{(2)}}{\partial r}, \tag{21}$$

$$(2+\chi_1)\left[2r\frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial\psi^{(1)}}{\partial r}\right)\right] - E^2\psi^{(1)} - l_1^{-2}E^4\psi^{(1)}\right] = (2+\chi_2)\sigma\left[2r\frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial\psi^{(2)}}{\partial r}\right)\right] - E^2\psi^{(2)} - l_2^{-2}E^4\psi^{(2)}\right], \tag{22}$$

where $\sigma = \dfrac{\mu_2}{\mu_1}$ is the classical ratio of viscosities between the internal and external fluids.

The case of flow past a solid sphere in a micropolar fluid is obtained when the viscosity of the drop becomes infinity ($\sigma \to \infty$), the case of micropolar fluid past a micropolar fluid sphere with the same viscosity as the surrounding medium (coalescence) ($\sigma = 1$), and as for the case of motion of a spherical gas bubble rising slowly in a micropolar fluid when the viscosity approaches zero ($\sigma = 0$).

Applying the boundary conditions Eqs. (19)-(22), Eq. (17), and Eq. (18), we get

$$A + B + C\,K_{3/2}(l_1) = -1, \tag{23}$$

$$E + F + G\,I_{3/2}(l_2) = 0, \tag{24}$$

$$-A + B - C\left(K_{3/2}(l_1) + l_1K_{1/2}(l_1)\right) - 2E - 4F + G\left(I_{3/2}(l_2) - l_2I_{1/2}(l_2)\right) = -2, \tag{25}$$

$$(2+\chi_1)(3A + l_1K_{5/2}(l_1)\,C) - \sigma(2+\chi_2)(3F - l_2K_{5/2}(l_2)\,G) = 0, \tag{26}$$

$$-B + C\,l_1^2\,\frac{(1+\chi_1)}{\chi_1}K_{3/2}(l_1) = 0, \tag{27}$$

$$-5F - G l_2^2 \frac{(1+\chi_2)}{\chi_2} I_{3/2}(l_2) = 0. \tag{28}$$

Equations (23)-(28) are solved to get the six arbitrary constants A, B, C, E, F and G. The expressions of arbitrary constants are cumbersome and lengthy, so they are not presented here.

5. Drag force on the micropolar-micropolar fluid drop

The drag force $\mathscr{F}$ acting on the micropolar fluid drop in an unbounded immiscible micropolar fluid can be evaluated by using the formula

$$\mathscr{F} = 2\pi a^2 \int_0^{\pi} r^2 \left(t_{rr}^{(1)} \cos\theta - t_{r\theta}^{(1)} \sin\theta \right) |_{r=1} \sin\theta d\theta, \tag{29}$$

where

$$t_{rr}^{(1)} = (2+\chi_1) \left[3r^{-4}A + \frac{3}{2} r^{-2} B + r^{-3/2} l_1 K_{5/2}(l_1 r) C \right] \zeta,$$

$$t_{r\theta}^{(1)} = \frac{(2+\chi_1)}{\sqrt{1-\zeta^2}} \left[3r^{-4}A + r^{-3/2} l_1 K_{5/2}(l_1 r) C \right] \vartheta_2(\zeta),$$

$$\mathscr{F} = 2\pi a \mu_1 U (2+\chi_1) B, \tag{30}$$

$$\mathscr{F} = -2\pi a \mu_1 U (2+\chi_1) \left[3T_1 l_1 (\chi_1+1) \left(\xi_1 \sigma \chi_2^2 + (\xi_2 (\chi_1+2) + \xi_3 \sigma) \chi_2 + 2T_2 l_2^2 (\chi_1 + 3\sigma + 2) \right) \right] \Delta^{-1}, \tag{31}$$

where

$$\Delta = \xi_1 \sigma \left((\xi_2 \chi_1 + 2T_1 l_1) \chi_2^2 + \left(\xi_2 \xi_5 \chi_1^2 + (\xi_3 \xi_4 \sigma + \xi_2 \xi_6) \chi_1 + 2T_1 l_1 (\xi_3 \sigma + 3\xi_2) \right) \chi_2 + 2T_2 l_2^2 (\xi_5 \chi_1^2 + (3\xi_4 \sigma + \xi_6) \chi_6 + 6T_1 l_1 (\sigma+1) \right),$$

$$\begin{aligned}
\xi_1 &= 3T_2 l_2^2 + 5T_4 l_2 - 15T_2, & \xi_2 &= 2T_2 l_2^2 - 5T_4 l_2 + 15T_2,\\
\xi_3 &= 9T_2 l_2^2 + 10T_4 l_2 - 30T_2, & \xi_4 &= 2T_1 l_1 - T_3,\\
\xi_5 &= 3T_1 l_1 - T_3, & \xi_6 &= 9T_1 l_1 - 2T_3,\\
T_1 &= K_{3/2}(l_1), & T_2 &= I_{3/2}(l_2),\\
T_3 &= K_{1/2}(l_1), & T_4 &= I_{1/2}(l_2),\\
T_5 &= K_{5/2}(l_1), & T_6 &= I_{5/2}(l_2).
\end{aligned}$$

Case I: *Viscous drop in a micropolar fluid*
If $\chi_2 \to 0$ and $\gamma_2 \to 0$ in Eq. (31), we get the drag force acting on the viscous fluid drop in an infinite micropolar fluid as

$$\mathscr{F} = -\frac{6\pi a\mu_1 U(2+\chi_1)(1+l_1)(1+\chi_1)(2+3\sigma+\chi_1)}{3l_1(1+\chi_1)(2+2\sigma+\chi_1)+(2+\chi_1)(3+3\sigma+2\chi_1)}, \tag{32}$$

This agrees with the result of Niefer and Kaloni [6] and Ramkissoon [7].

Case II: *Micropolar fluid drop in a viscous fluid*
If $\chi_1 \to 0$ and $\gamma_1 \to 0$ in Eq. (31), we get the drag force acting on the micropolar fluid drop in an unbounded viscous fluid as

$$\mathscr{F} = 4\pi a\mu_1 U\frac{\Delta_1}{\Delta_2}, \tag{33}$$

where

$$\Delta_1 = 3(l_2^2\Sigma_1+15\Sigma_2)\sigma\chi_2^2 + \big(3(l_2^2\Sigma_3+30\Sigma_2)\sigma - 6(l_2^2\Sigma_4+15\Sigma_2)\big)\chi_2 - 6l_2^2\Sigma_2(3\sigma+2),$$

$$\Delta_2 = 2(l_2^2\Sigma_1+15\Sigma_2)\sigma\chi_2^2 + \big(2(l_2^2\Sigma_3+30\Sigma_2)\sigma - 6(l_2^2\Sigma_4+15\Sigma_2)\big)\chi_2 - 6l_2^2\Sigma_2(\sigma+1),$$

$\Sigma_1 = 2\sinh(l_2)+3l_2\cosh(l_2)$, $\Sigma_2 = \sinh(l_2)-l_2\cosh(l_2)$,
$\Sigma_3 = \sinh(l_2)+9l_2\cosh(l_2)$, $\Sigma_4 = 7\sinh(l_2)-2l_2\cosh(l_2)$,
which agrees with the published result of Saad [8].

Case III: *Solid sphere*
If $\sigma \to \infty$ in Eq. (33), we get the drag force acting on the solid sphere through a micropolar fluid

$$\mathscr{F} = -\frac{6\pi a\mu_1 U(2+\chi_1)(1+\chi_1)(1+l_1)}{2l_1(1+\chi_1)+2+\chi_1}, \tag{34}$$

which agrees with the result obtained by Rao and Rao [4], and Ramkissoon and Majumdar [5].

Case IV: *Fluids in both the regions are Newtonian*
If $\chi_1 \to 0$, $\gamma_1 \to 0$, $\chi_2 \to 0$, and $\gamma_2 \to 0$ in Eq. (31), the fluid inside and outside the sphere is Newtonian. The drag force exerted on the viscous fluid sphere by the surrounding viscous fluid is

$$\mathscr{F} = -\frac{2\pi aU\mu_1(3\sigma+2)}{\sigma+1}, \tag{35}$$

which agrees with the well-known result derived by Hadamard [1] and Rybczynski [2]. Equation (35) degenerates to the case of motion of a no-slip sphere (Stokes' law) when the viscosity of the drop is infinite and as for the case of motion of a perfect-slip gas bubble when the viscosity approaches zero.

Case V: *Fluid sphere of vanishing viscosity*
In this case, $\mu_2 = 0$ in Eq. (32), and it reduces to

$$\mathscr{F} = -\frac{6\pi a \mu_1 U (2+\chi_1)(1+\chi_1)(1+l_1)}{3l_1(1+\chi_1)+3+2\chi_1}, \tag{36}$$

It represents the drag of a gaseous bubble rising slowly through a micropolar fluid. In addition, if $\chi_1 \to 0$, we get the gaseous bubble rising slowly through a viscous fluid [18]

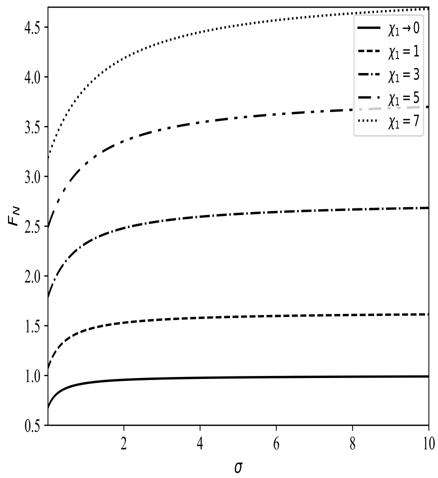
$$\mathscr{F} = -4\pi a \mu_1 U. \tag{37}$$

6. Results and discussion

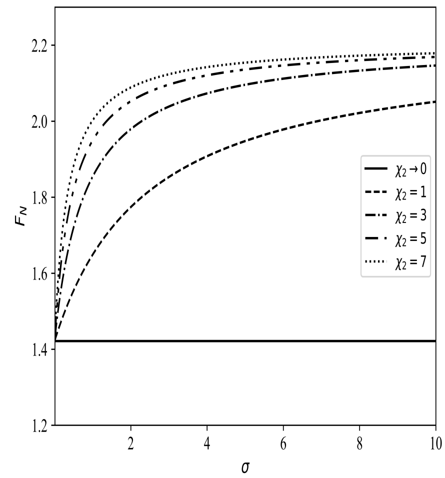
The drag coefficient $F_N = \dfrac{\mathscr{F}}{-6\pi a \mu_1 U}$ is a function of various parameters:
(i) Relative viscosity of the fluid drop $\sigma = \dfrac{\mu_2}{\mu_1}$ $(0 \le \sigma < \infty)$ [6,21].
(ii) Micropolar parameters (vortex viscosity parameters) $\chi_1 = \dfrac{\kappa_1}{\mu_1}$, $\chi_2 = \dfrac{\kappa_2}{\mu_2}$ $(0 \le \chi_1, \chi_2 < \infty)$ [3–6].
(iii) Shear spin viscosity parameters γ_1 and γ_2 $(0 \le \gamma_1, \gamma_2 < \infty)$ [3–6].
The variation of the drag coefficient F_N with the classical ratio σ of viscosities between the interior and exterior of the fluid sphere are shown in Figures 2 and 3 for various values of micropolarity parameters and shear spin viscosity parameters.

It is observed that the drag force acting on the micropolar fluid sphere in a micropolar fluid increases as the viscosity ratio increases. As expected, the drag force exerted on the viscous drop in a micropolar fluid and micropolar fluid in a viscous drop increases with an increase in the viscosity ratio. When $\sigma \to 0$, F_N represents the drag of a gaseous bubble rising slowly in a micropolar fluid. When $\sigma \to \infty$, F_N represents the drag of a solid sphere in a micropolar fluid. Our numerical result for the case of solid sphere agrees with the result obtained by Rao and Rao [4]. Also, the drag on a gaseous bubble is less than the drag on a solid sphere [18].

In Tables 1 and 2, the variation of the drag coefficient F_N for different values of micropolarity parameters χ_1 and χ_2, taking viscosity ratio for four different cases when $\sigma = 0$ (fluid sphere of vanishing viscosity), $\sigma = 1$ (fluid sphere with viscosity equal to that of the surrounding medium), $\sigma = 10$ (fluid sphere with viscosity unequal to that of the surrounding medium), and $\sigma \to \infty$ (solid sphere), respectively. If the micropolarity parameters $\chi_1 \to 0$, and $\chi_2 \to 0$ i.e., both the fluids of interior and exterior of the fluid sphere are Newtonian, it is seen that the drag coefficient increases as the viscosity ratio increases.

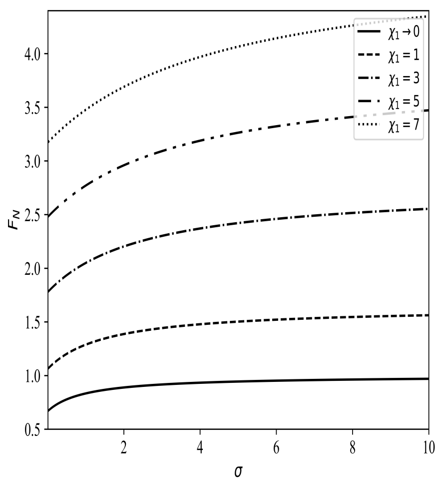


(a) for varying micropolarity parameter χ_1 outside the fluid sphere, other parameters $\dfrac{\gamma_1}{\mu_1 a^2} = 0.4$, $\dfrac{\gamma_2}{\mu_2 a^2} = 0.4$, $\chi_2 = 2$ are fixed

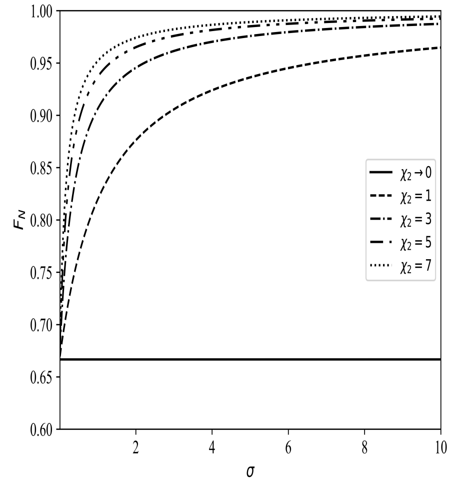


(b) for varying micropolarity parameter χ_2 inside the fluid sphere, other parameters $\dfrac{\gamma_1}{\mu_1 a^2} = 0.4$, $\dfrac{\gamma_2}{\mu_2 a^2} = 0.4$, $\chi_1 = 2$ are fixed

Fig. 2. Variation of drag coefficient F_N versus viscosity ratio σ



(a) for varying micropolarity parameter χ_1, other parameters $\dfrac{\gamma_1}{\mu_1 a^2} = 0.4$, $\gamma_2 \to 0$, $\chi_2 \to 0$ are fixed



(b) for varying micropolarity parameter χ_2, other parameters $\dfrac{\gamma_2}{\mu_2 a^2} = 0.4$, $\gamma_1 \to 0$, $\chi_1 \to 0$ are fixed

Fig. 3. Variation of drag coefficient F_N versus viscosity ratio σ

Table 1. Drag coefficient F_N for the cases of micropolar-viscous droplet $\left(\dfrac{\gamma_1}{\mu_1 a^2} = 0.4, \chi_2 \to 0, \gamma_2 \to 0\right)$ and viscous-micropolar droplet $\left(\dfrac{\gamma_2}{\mu_2 a^2} = 0.4, \chi_1 \to 0, \gamma_1 \to 0\right)$

	F_N					F_N			
χ_1	$\sigma \to 0$	$\sigma = 1$	$\sigma = 10$	$\sigma \to \infty$	χ_2	$\sigma \to 0$	$\sigma = 1$	$\sigma = 10$	$\sigma \to \infty$
0	0.666667	0.833333	0.969697	1.0	0	0.666667	0.833333	0.969697	1.0
1	1.06017	1.2877	1.56198	1.63959	1	0.666667	0.896933	0.985719	1.0
3	1.77597	2.04888	2.55335	2.75428	3	0.666667	0.93729	0.992451	1.0
5	2.47607	2.77001	3.47178	3.83128	5	0.666667	0.953152	0.994637	1.0
7	3.17006	3.47616	4.34817	4.89378	7	0.666667	0.961942	0.995758	1.0

Table 2. Drag coefficient F_N for the cases of micropolar-micropolar droplet $\left(\dfrac{\gamma_1}{\mu_1 a^2} = \dfrac{\gamma_2}{\mu_2 a^2} = 0.4, \chi_2 = 3\right)$ and micropolar-micropolar droplet $\left(\dfrac{\gamma_1}{\mu_1 a^2} = \dfrac{\gamma_2}{\mu_2 a^2} = 0.4, \chi_1 = 3\right)$

	F_N					F_N			
χ_1	$\sigma \to 0$	$\sigma = 1$	$\sigma = 10$	$\sigma \to \infty$	χ_2	$\sigma \to 0$	$\sigma = 1$	$\sigma = 10$	$\sigma \to \infty$
0	0.666668	0.93729	0.992451	1.0	0	1.77597	2.04888	2.55335	2.75428
1	1.06017	1.48672	1.61954	1.63959	1	1.77597	2.22954	2.65283	2.75428
3	1.77598	2.38782	2.699	2.75428	3	1.77597	2.38782	2.699	2.75428
5	2.47607	3.21395	3.72665	3.83128	5	1.77597	2.46363	2.71461	2.75428
7	3.17006	4.0015	4.72673	4.89378	7	1.77597	2.50981	2.72274	2.75428

7. Conclusions

An analytical solution for the problem of Stokes flow of a micropolar fluid past a micropolar fluid sphere is obtained. The problem is focused on the case when two fluid phases have a microstructure nature. On the micropolar-micropolar liquid interface, negilible mass transfer, continuity of tangential stresses and velocities, vanishing of microrotation vectors are used. The drag force is calculated, and the dependence of the drag force on the micropolarity parameters χ_1, χ_2 and the classical viscosity ratio σ is depicted graphically. The drag force exerted on the viscous drop in a micropolar fluid and micropolar fluid drop in a viscous fluid increases with an increase in the viscosity ratio. The drag force acting on the micropolar drop in a micropolar fluid increases as the viscosity ratio increases. The current work has significant applications in natural, biological, and industrial processes, such as sedimentation phenomena, liquid-liquid extraction, the study of blood flow, and the rheology of emulsions. Finally, the possible prospects for future studies of the subject may be proposed by considering spheroidal geometry [11, 12] and cell models [8].

References

[1] Hadamard, J.S. (1911). M'ecanique-mouvement permanent lent d'une sph'ere liquide et visqueuse dansun liquide visqueux. *CR Acad. Sci.*, *152*, 1735-1738.

[2] Rybczynski, W. (1911). On the translatory motion of a fluid sphere in a viscous medium. *Bull. Acad. Sci. Cracovie. Ser. A*, *40*, 40-46.

[3] Eringen, A.C. (2001). *Microcontinuum Field Theories II: Fluent Media*. New York: Springer.

[4] Rao, S.K.L., & Rao, P.B. (1970). The slow stationary flow of a micropolar liquid past a sphere. *Journal of Engineering Mathemathics*, *4*, 209-217.

[5] Ramkissoon, H., & Majumdar, S.R. (1976). Drag on axially symmetric body in the Stokes flow of micropolar fluid. *Physics of Fluids*, *19*, 16-21.

[6] Niefer, R., & Kaloni, P.N. (1980). On the motion of a micropolar fluid drop in a viscous fluid. *Journal of Engineering Mathematics*, *14*, 107-116.

[7] Ramkissoon, H. (1985). Flow of a micropolar fluid past a Newtonian fluid sphere. *ZAMM – Journal of Applied Mathematics and Mechanics*, *65*, 635-637.

[8] Saad, E.I. (2012). Cell models for micropolar flow past a viscous fluid sphere. *Meccanica*, *47*, 2055-2068.

[9] Faltas, M.S., & Saad, E.I. (2014). Slow motion of spherical droplet in a micropolar fluid flow perpendicular to a planar solid surface. *European Journal of Mechanics – B/Fluids*, *48*, 266-276.

[10] Ramkissoon, H., & Majumdar, S.R. (1988). Micropolar flow past a slightly deformed fluid sphere. *ZAMM – Journal of Applied Mathematics and Mechanics*, *68*, 155-160.

[11] Madasu, K.P., & Kaur, M. (2016). Stokes flow of micropolar fluid past a viscous fluid spheroid with non-zero boundary condition for microrotation. *Sādhanā*, *41*, 1463-1472.

[12] Madasu, K.P., & Kaur, M. (2017). Stokes flow of viscous fluid past a micropolar fluid spheroid. *Advances in Applied Mathematics and Mechanics*, *9*(5), 1076-1093.

[13] Gomathy, G., Sabarmathi, A., & Shukla, P. (2020). Creeping flow of non-Newtonian fluid past a fluid sphere with non-zero spin boundary condition. *Advances in Mathematics: Scientific Journal*, *9*, 5979-5986.

[14] Srinivasacharya, D., & Rajyalakshmi, I. (2004). Creeping flow of micropolar fluid past a porous sphere. *Applied Mathematics and Computation*, *153*, 843-854.

[15] Khanukaeva, D. (2020). Filtration of micropolar liquid through a membrane composed of spherical cells with porous layer. *Theoretical Computational Fluid Dynamics*, *34*, 215-229.

[16] Yadav, P.K., Jaiswal, S., Puchakatla, J.Y., & Filippov, A.N. (2020). Poiseuille flow of micropolar--Newtonian fluid through concentric pipes filled with porous medium. *Colloid Journal*, *82*, 333-341.

[17] Selvi, R., Shukla, P., & Filippov, A.N. (2020). Flow around a liquid sphere filled with a non-Newtonian liquid and placed into a porous medium. *Colloid Journal*, *82*, 152-160.

[18] El-Sapa, S. (2021). Effect of magnetic field on a microstretch fluid drop embedded in an unbounded another microstretch fluid. *European Journal of Mechanics – B/Fluids*, *85*, 169-180.

[19] El-Sapa, S. (2022). Cell models for micropolar fluid past a porous micropolar fluid sphere with stress jump condition. *Physics of Fluids*, *34*, 082014.

[20] Yadav, P.K., Kumar, A., & Filippov, A.N. (2023). Analysis of entropy production of immiscible micropolar and Newtonian fluids flow through a channel: Effect of thermal radiation and magnetic field. *Colloid Journal*, *85*, 95-113.

[21] Happel, J., & Brenner, H. (2012). Low Reynolds Number Hydrodynamics with Special Applications to Particulate Media. Springer Science & Business Media.

[22] Salem, A.G., Faltas, M.S., & Sherief, H.H. (2022). Migration of nondeformable droplets in a circular tube filled with micropolar fluids. *Chinese Journal of Physics*, *79*, 287-305.

[23] Salem, A.G. (2023). Effects of a spherical slip cavity filled with micropolar fluid on a spherical viscous droplet. *Chinese Journal of Physics*, *86*, 98-114.

[24] Salem, A.G. (2023). Effects of a spherical slip cavity filled with micropolar fluid on a spherical micropolar droplet. *Fluid Dynamics Research*, *55*(6), 065502.

[25] Alharbi, A.H., & Salem, A.G. (2024). Analytical and numerical investigation of viscous fluid--filled spherical slip cavity in a spherical micropolar droplet. *AIMS Mathematics*, *9*(6), 15097--15118.