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EFFECTS OF ROTATION ON JEFFREY NANOFLUID FLOW SATURATED BY A POROUS MEDIUM

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Abstract. In this paper, the effects of rotation on a Jeffery nanofluid flow in a porous medium which is heated from below is studied. Darcy model is employed for porous medium and the Jeffrey fluid model is used as a base fluid. The Navier-Stokes equations of motion of fluid are modified under the influence of the Jeffrey parameter, naoparticles and rotation. The basic perturbation technique based on normal modes is applied to derive the dispersion relation for a Rayleigh number. The effects of the Taylor number, Jeffrey parameter, Lewis number, modified diffusivity ratio, nanoparticles Rayleigh number and medium porosity on the stationary convection of the physical system have been analyzed analytically and graphically. It is observed that the rotation parameter has a stabilising influence for both bottom/top-heavy configurations.

MSC 2010: *76E06, 76E07, 76S05*
Keywords: *nanofluid, Jeffrey model, convection, porous medium, rotation*

Nomenclature

a	wave number	**Greek symbols**	
$\mathbf{v}$	Darcy velocity vector [m/s]	α	coefficient of thermal expansion [K^{-1}]
d	thickness of the horizontal layer [m]	φ	nanoparticles volume fraction
D_B	diffusion coefficient [m^2/s]	κ	thermal diffusivity [$m^2\,s^{-1}$]
D_T	thermophoretic diffusion coefficient	μ	fluid viscosity [Pa s]
κ_{bm}	thermal diffusivity of the base fluid [$m^2\,s^{-1}$]	$\mathbf{\Omega}$	angular velocity [s^{-1}]
R_a	Rayleigh number	ρ_{bf}	density of base fluid [$kg\,m^{-3}$]
p	pressure [Pa]	ρ_{np}	density of nanoparticle [$kg\,m^{-3}$]
$\mathbf{g}$	gravitational acceleration vector [m/s^2]	ω	growth rate of disturbances
P_r	Prandtl number	∇_H^2	horizontal Laplacian operator
D_a	Darcy number	∇	Laplacian operator
V_a	Vadasz number	σ	thermal capacity ratio
T_a	Taylor number	ε	medium porosity

R_n nanoparticles Rayleigh number
N_B modified particle density increment
L_e Lewis number
N_A modified diffusivity ratio
R_m basic density Rayleigh number
λ_1 stress relaxation-time parameter
λ_2 strain relaxation-time parameter
λ_3 Jeffrey parameter

1. Introduction

The study of nanofluid in a porous medium has attracted various researchers for the last few years because of their applications in automotive industries, energy saving, nuclear reactors, transformers, biomedical appliances including hyperthermia, etc. Various researchers have shown that a particular type of nanofluid can be used to destroy and kill cancer cells without damaging the normal tissues. The term nanofluid was first coined by Choi [1]. Based on the Buongiorno [2] model, thermal instability of nanofluids in a porous medium has been discussed by different researchers [3-8] and applications have been found in earth's molten cores, in petroleum reservoir, the human lungs, bile duct, gall bladder with stones in blood vessels etc.

In the above discussion, all studies were presented for Newtonian nanofluid only. However, most of the industrial and biological nanofluids are non-Newtonian fluids and the standard Newton's law of viscosity cannot depict the rheological behavior of non-Newtonian nanofluids. Such types of fluids are found applications in biological sciences, geophysics and chemical and petroleum processes etc. Fluids such as engine oil, drilling muds, soap solution, sauce, foam, paints, lubricants and biological liquids such as blood are non-Newtonian nanofluids. Very Small work has been done to study the non-Newtonian nanofluids in a porous medium. There are different types of non-Newtonian nanofluids, one of such kind is the Jeffrey fluid model that has drawn attention to different researchers because Jeffrey fluid model is considered as a best model for physiological fluids [9-14]. Thermal instability problems of an elastico-viscous nanofluids have been studied Sheu [15] and Chand et al. [16]. They found that the viscoelastic nanofluids are very useful in the cooling of nuclear reactors, cooling of power plants and computers, drug delivery to kill the cancer cells and tissues, etc.

The role of rotation is very important in the study of thermal instability of a fluid layer heated from below and has various applications in fluid machinery, power plants, the automotive industry, the petroleum industry, biomechanics, mechanical engineering, geophysics, etc. Chandrasekhar [17] examined the effect of rotation on the Bénard convection which was later extended by Vadasz [18] for a porous medium layer, and Sharma and Rana [19] studied it for the case of viscoelastic fluid and others [20-23]. Various researchers have studied the effect of rotation on thermal instability by taking different types of viscoelastic fluids [24-30], and they found that rotation advances the stationary convection.

Keeping in mind the various applications mentioned above, the main aim in this paper is to study the effect of rotation on the onset of thermal instability of Jeffery nanofluid in a porous medium. To the best of my knowledge, this paper has not been published yet.

2. Mathematical model

Here, an infinite horizontal layer of Jeffrey nanofluid of thickness d in a porous medium which is heated from below bounded by thermal conducting parallel planes $z=0$ and $z=d$, as shown in Figure 1, is considered. Let the temperature and the volumetric fraction of nanoparticles at $z=0$ be T_0 and φ_0 while at $z=d$ be T_1 and φ_1 ($T_0 > T_1$ and $\varphi_1 > \varphi_0$). The gravity force $\mathbf{g}=\mathbf{g}(0,0,-g)$ is acting vertically on the physical system.

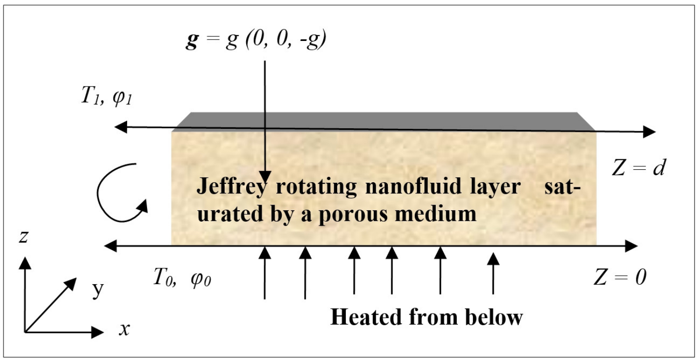

Fig. 1. Physical sketch of the problem

2.1. Assumptions

The physical model is based upon the following assumptions:

i. Thermo physical properties except for density in the buoyancy force (Boussinesq hypothesis) are constant;
ii. Nanoparticles are spherical in shape;
iii. Nanofluid is laminar, incompressible and non-Newtonian;
iv. The size of nanoparticles are very small as compared to the pore size of the matrix.

2.2. Governing equations

The governing equations for a Jeffrey nanofluid layer under rotation in a porous medium [3-5, 9-12, 21-24] are given by

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\frac{\rho_{bf}}{\varepsilon}\left(\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{v}\cdot\nabla)\mathbf{v}\right)=-\nabla p-\frac{\mu}{k_1(1+\lambda_3)}\mathbf{v}+\frac{2\rho_{bf}}{\varepsilon}(\mathbf{v}\times\mathbf{\Omega})+\left[\varphi\rho_{np}+(1-\phi)\rho_{bf}\left\{1-\alpha(T-T_1)\right\}\right]\mathbf{g}, \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\mathbf{v}\cdot\nabla}{\varepsilon}\right]\varphi=D_B\nabla^2\varphi+\frac{D_T}{T_0}\nabla^2 T, \tag{3}$$

$$\left(\rho_{nf}c\right)_{bm}\left\{\frac{\partial T}{\partial t}+\mathbf{v}.\nabla T\right\}=k_m\nabla^2 T+\varepsilon\left(\rho_{nf}c\right)_{np}\left\{D_B\nabla\varphi.\nabla T+\frac{D_T}{T_0}\nabla T.\nabla T\right\}, \tag{4}$$

where $\lambda_3=\frac{\lambda_1}{\lambda_2}c_{np},\left(\rho_{nf}c\right)_{bm}$ and $\left(\rho_{nf}c\right)_{bf}$ denote, respectively, the Jeffrey parameter (accounting for viscoelasticity), the specific heat of the material constituting the nanoparticles, the effective capacity and the heat capacity of nanofluid.

In non-dimensional form, equations (1)-(4) can be written by omitting the dashes (') for convenience as

$$\nabla\cdot\mathbf{v}=0, \tag{5}$$

$$\frac{1}{V_a}\frac{\partial \mathbf{v}}{\partial t}=-\nabla p-\frac{1}{(1+\lambda_3)}\mathbf{v}-R_m\hat{e}_z+R_aT\hat{e}_z-R_n\varphi\hat{e}_z+\sqrt{T_a}\left(v\hat{e}_x-u\hat{e}_y\right), \tag{6}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\mathbf{v}\cdot\nabla\varphi=\frac{1}{L_e}\nabla^2\varphi+\frac{N_A}{L_e}\nabla^2 T, \tag{7}$$

$$\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T=\nabla^2 T+\frac{N_A}{L_e}\nabla\varphi.\nabla T+\frac{N_AN_B}{L_e}\nabla T.\nabla T, \tag{8}$$

Here, we have used the non-dimensional variables

$$(x',y',z')=\left(\frac{x,y,z}{d}\right),\quad (u',v',w')=\left(\frac{u,v,w}{\kappa_{bm}}\right)d,\quad t'=\frac{t\kappa_{bm}}{\sigma d^2},\quad p'=\frac{pk_1}{\mu\kappa_{bm}}d^2,$$

$$\varphi'=\frac{\varphi-\varphi_0}{\varphi_1-\varphi_0},\quad T'=\frac{T-T_1}{T_0-T_1},\quad K'=\frac{K}{\gamma_0E_0\Delta Td},\quad V=\gamma_0E_0\beta dV'$$

and the non-dimensional parameters are:

thermal diffusivity of the base fluid $(\kappa_{bm})=\frac{k_m}{(\rho_{nf}c_{np})_{bf}}$, the thermal capacity ratio $(\sigma)=\frac{\left(\rho_{nf}c_{np}\right)_{bm}}{(\rho_{nf}c_{np})_{bf}}$, the Prandtl number $(P_r)=\frac{\mu}{\rho_{nf}\kappa_{bm}}$, the Darcy number

$(D_a) = \dfrac{k_1}{d^2}$, the Vadasz number $(V_a) = \dfrac{\varepsilon P_r}{D_a}$, the modified Taylor number $(T_a) = \left(\dfrac{2\Omega\rho d^2}{\varepsilon\mu}\right)^2$, the Rayleigh number $(R_a) = \dfrac{\rho_{nf} g\alpha dk\left(T_0 - T_1\right)}{\mu_{bf}\kappa_{bm}}$, the nanoparticles Rayleigh number $(R_n) = \dfrac{\left(\rho_{np} - \rho_{bf}\right)\left(\varphi_1 - \varphi_0\right) gk_1 d}{\mu\kappa_{bm}}$, the modified particle density increment $(N_B) = \dfrac{\rho_{np} c_{np}}{\left(\rho c\right)_{bf}}\left(\varphi_1 - \varphi_0\right)$, the Lewis number $(L_e) = \dfrac{\kappa_{bm}}{D_B}$, the modified diffusivity ratio $(N_A) = \dfrac{D_T\left(T_0 - T_1\right)}{D_B T_1\left(\varphi_1 - \varphi_0\right)}$, the basic density Rayleigh number $(R_m) = \dfrac{\left\{\rho_{np}\varphi_0 + \rho_{bf} + \left(1 - \varphi_0\right)\right\} gk_1 d}{\mu\kappa_{bm}}$.

The dimensionless boundary conditions are:

$$\begin{aligned} &w = 0,\ T = T_0,\ \varphi = \varphi_0 \ \text{ at } Z = 0 \\ &w = 0,\ T = T_1,\ \varphi = \varphi_1 \ \text{ at } Z = 1 \end{aligned} \tag{9}$$

2.3. Perturbation solutions

Let the physical system be slightly disturbed from the equilibrium position. We suppose that

$$v = v^*,\ p = p_b + p^*,\ T = T_b + T^*,\ \varepsilon = \varepsilon_b + \varepsilon^*,\ \varphi = \varphi_b + \varphi^*. \tag{10}$$

where $v^*, p^*, T^*, \varepsilon^*$ and φ^* are the perturbations overlapped into the physical quantities of the equilibrium state.

We describe to the quantities' dependence on x, y and t of the form $\exp(ilx + imy + \omega t)$, where l and m are the wave numbers in the x and y-direction, respectively, and ω is the growth rate of the disturbances, which is, in general, a complex constant. The perturbations quantities w, T and φ are supposed to be of the form

$$\left[w, T, \phi\left(x, y, z, t\right)\right] = \left[W\left(z\right), \Theta(z), \Phi(z)\right]\exp\left(ilx + imy + \omega t\right), \tag{11}$$

Using the perturbation equations (10) and expression (11) in the partial differential equations equations (5)-(8) and linearising the following ordinary differential equations are obtained by applying stress-free conditions for a free surface as

$$\left[\left(\frac{1}{V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda_3}\right)^2\left(D^2-a^2\right)+T_aD^2\right]W-\left(\frac{1}{V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda_3}\right)R_aa^2\Theta+\left(\frac{1}{V_a}\frac{\partial}{\partial t}+\frac{1}{1+\lambda_3}\right)R_na^2\Phi=0 \tag{12}$$

$$\frac{W}{\varepsilon}-\frac{N_A}{L_e}\left(D^2-a^2\right)\Theta-\left\{\frac{1}{L_e}\left(D^2-a^2\right)-\frac{\omega}{\sigma}\right\}\Phi=0, \tag{13}$$

$$W+\left\{\frac{N_B}{L_e}D+\left(D^2-a^2\right)-2\frac{N_AN_B}{L_e}-\omega\right\}\Theta-\frac{\tilde{N}_B}{\tilde{L}_e}D\Phi=0, \tag{14}$$

where $D=\dfrac{d}{dz}$ and $a^2=l^2+m^2$ is the dimensionless resultant wave number.
The boundary conditions for a free-free boundary are:

$$W=0,\ D^2W=0,\ \Theta=0,\ \Phi=0 \ \text{ at } \ z=0 \ \text{ and }$$
$$W=0,\ D^2W=0,\ \Theta=0,\ \Phi=0 \ \text{ at } \ z=1 \tag{15}$$

The set of differential equations (12)-(14) together with the boundary conditions (15) form an eigen value problem for the Rayleigh number R_a, and given values of the other parameters $\lambda_3, T_a, R_n, \varepsilon, L_e, N_A, N_B$, whose solutions are yet to be obtained.

We assume the solution to W, Θ and Φ is of the form

$$W=W_0\sin\pi z,\ \Theta=\Theta_0\sin\pi z,\ \varphi=\varphi_0\sin\pi z \tag{16}$$

which satisfies boundary conditions (15).
Substituting solution (16) into equations (12)-(14), integrating each equation from $z = 0$ to $z = 1$, and performing some integrations by parts, we obtain the following dispersion relations

$$R_a=\frac{\left(\dfrac{1}{1+\lambda_3}+\dfrac{\omega}{V_a}\right)\left(\pi^2+a^2\right)\left(\pi^2+a^2+\omega\right)}{a^2}+\frac{\pi^2\left(\pi^2+a^2+\omega\right)T_a}{a^2\left(\dfrac{1}{1+\lambda_3}+\dfrac{\omega}{V_a}\right)}-\frac{N_A\left(\pi^2+a^2\right)+\dfrac{L_e}{\varepsilon}\left(\pi^2+a^2+\omega\right)}{\pi^2+a^2+\dfrac{\omega L_e}{\sigma}}R_n. \tag{17}$$

Equation (17) describes the effect of the Jeffrey parameter, Lewis number, Vadasz number, Taylor number, nanoparticle Rayleigh number, modified diffusivity ratio and medium porosity on the system.

3. Stationary convection

For the case of steady state (i. e., principle of exchange of stability), we put $\omega = 0$ in equation (17), we obtain

$$R_a^S = \frac{1}{1+\lambda_3}\frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\pi^2\left(1+\lambda_3\right)\left(a^2+\pi^2\right)T_a}{a^2} - \left(N_A + \frac{L_e}{\varepsilon}\right)R_n. \quad (18)$$

Equation (18) expresses the Rayleigh number as a function of the dimensionless resultant wave number a, the Jeffrey parameter (λ_3), the Taylor number (T_a), the medium porosity (ε), the nanoparticle Rayleigh number (R_n), the Lewis number (L_e) and modified-diffusivity ratio (N_A).

Since equation (18) does not contain the particle increment parameter N_B but contains the diffusivity ratio parameter N_A in corporation with the nanoparticle Rayleigh number R_n, this indicates that the nanofluid cross-diffusion terms approach is dominated by the regular cross-diffusion term.

In the absence of rotation, that is, $T_a = 0$, equation (18) becomes

$$R_a^S = \frac{1}{1+\lambda_3}\frac{\left(a^2+\pi^2\right)^2}{a^2} - \left(N_A + \frac{L_e}{\varepsilon}\right)R_n. \quad (19)$$

In the absence of Jeffrey parameter and rotation, equation (18) becomes

$$R_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} - \left(N_A + \frac{L_e}{\varepsilon}\right)R_n. \quad (20)$$

which is in good agreement with the previous result of Nield and Kuznetsov [4].

In the absence of nanoparticles, that is, $R_n = 0$ and $N_A = 0$ equation (18) reduces to

$$R_a^S = \frac{\left(a^2+\pi^2\right)^2}{a^2} + \frac{\pi^2\left(a^2+\pi^2\right)T_a}{a^2}. \quad (21)$$

which is in good agreement with the earlier result of Chandrasekhar [17] for ordinary regular fluids.

4. Results and discussion

We analyze the influence of the Jeffrey parameter, Taylor number, Lewis number, modified diffusivity ratio of nanoparticles, nanoparticles Rayleigh number and medium porosity on stationary Rayleigh number analytically and graphically. We analyze the behavior of $\frac{\partial R_a^S}{\partial \lambda_3}, \frac{\partial R_a^S}{\partial T_a}, \frac{\partial R_a^S}{\partial L_e}, \frac{\partial R_a^S}{\partial N_A}, \frac{\partial R_a^S}{\partial R_n}$ and $\frac{\partial R_a^S}{\partial \varepsilon}$ analytically.

From equation (18), we obtain

$$\frac{\partial R_a^S}{\partial \lambda_3} = -\frac{1}{\left(1+\lambda_3\right)^2}\frac{\left(\pi^2+a^2\right)^2}{a^2} + \frac{\pi^2\left(a^2+\pi^2\right)T_a}{a^2}, \tag{22}$$

$$\frac{\partial R_a^S}{\partial T_a} = \frac{\pi^2\left(1+\lambda_3\right)\left(a^2+\pi^2\right)}{a^2}, \tag{23}$$

$$\frac{\partial R_a^S}{\partial L_e} = -\frac{R_n}{\varepsilon}, \tag{24}$$

$$\frac{\partial R_a^S}{\partial N_A} = -R_n, \tag{25}$$

$$\frac{\partial R_a^S}{\partial R_n} = -\left(N_A + \frac{L_e}{\varepsilon}\right), \tag{26}$$

$$\frac{\partial R_a^S}{\partial \varepsilon} = \frac{L_e R_n}{\varepsilon^2}. \tag{27}$$

From equation (22), $\frac{\partial R_a^S}{\partial \lambda_3} > 0,$ if $T_a > \frac{1}{\left(1+\lambda_3\right)^2}\frac{\left(\pi^2+a^2\right)}{\pi^2},$ therefore, the Jeffrey parameter has a stabilizing effect on the system for both bottom/top-heavy nanoparticles distribution which is in an agreement with the result derived by Chand et al. [28]. However, in the absence of rotation, the Jeffrey parameter has a destabilizing effect.

From equation (23), we notice that the right hand side is positive which implies that the Taylor number which is accounting for rotation has a stabilizing influence on the stationary convection of the system.

The right hand sides of equations (25) and (26), if $R_n < 0,$ therefore, the Lewis numer $\left(L_e\right)$ and modified diffusivity ratio $\left(N_A\right)$ enhance the stationary convection and postpone the stationary convection if $R_n > 0.$ Hence, Lewis number and modified diffusivity havea stabilizing/destabilizing influence for bottom/top-heavy configuration on the stationary convection which is in an agreement with the result obtained by Sheu [15], Chand and Rana [25] and Yadav et al. [5]. Equation (26) indicates that nanoparticles Rayleigh number postpones the stationary convection for both bottom/top-heavy nanoparticles distribution. The right-hand side of equation (27) is negative if $R_n < 0$, this shows the the medium porosity delays the stationary convection for bottom-heavy configuration and if $R_n > 0$, the medium porosity advances the stationary convection. which is in an agreement with the results derived by Nield and Kuznetsov [4], Chand and Rana [25], Chand et al. [28], Rana et al. [29] and Ahmad et al. [30].

We now analyze the results obtained from equations (22)-(27) numerically by plotting graphs by giving some numerical values to the parameters to depict the stability characteristics.

Figures 2-5 show that R_a^S increases with the increase in λ_3 and T_a which imply that the Jeffrey parameter and Taylor number (accounting for rotation) have stabilizing influence on the stationary convection for both bottom/top-heavy configuration. Thus, both Jeffery parameter and rotation enhance the stationary convection which clearly verify the results obtained analytically.

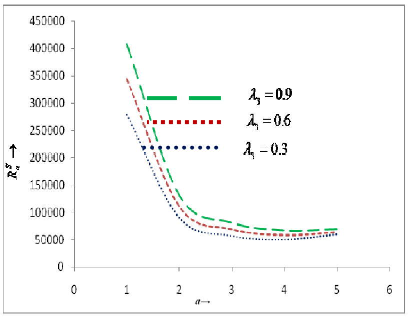

Fig. 2. Effect of λ_3 on R_a^S for bottom-heavy configuration

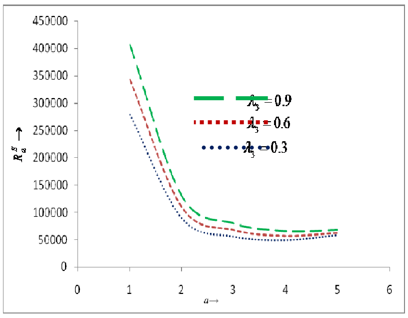

Fig. 3. Effect of λ_3 on R_a^S for top-heavy configuration

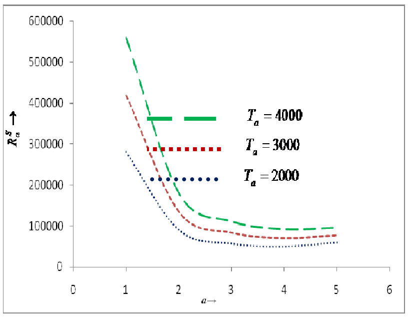

Fig. 4. Effect of T_a on R_a^S for bottom-heavy configuration

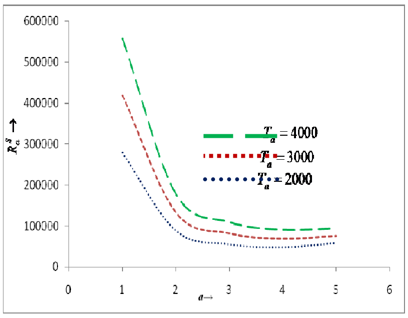

Fig. 5. Effect of T_a on R_a^S for top-heavy configuration

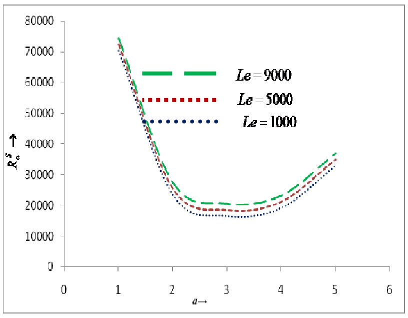

Fig. 6. Effect of Le on R_a^S for bottom-heavy configuration

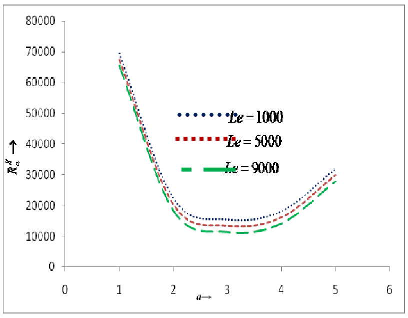

Fig. 7. Effect of Le on R_a^S for top-heavy configuration

From Figure 6, we notice that the R_a^S increases with the increase in L_e for bottom-heavy configuration whereas R_a^S decreases with the increase in L_e for top-heavy configuration, as shown in Figure 7. Hence, the Lewis number advances the stationary convection for bottom-heavy configuration, whereas it postpones the stationary convection for top-heavy configuration.

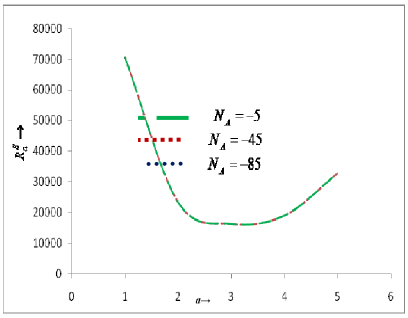

Fig. 8. Effect of N_A on R_a^S for bottom-heavy configuration

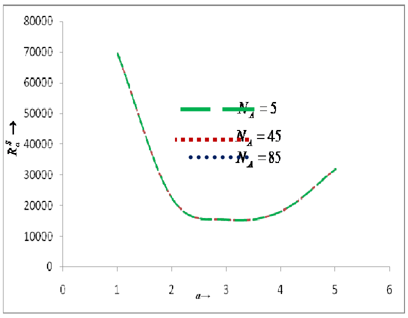

Fig. 9. Effect of N_A on R_a^S for top-heavy configuration

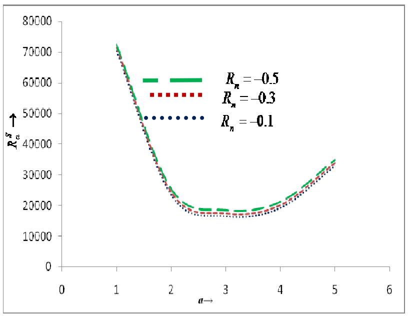

Fig. 10. Effect of R_n on R_a^S for bottom-heavy configuration

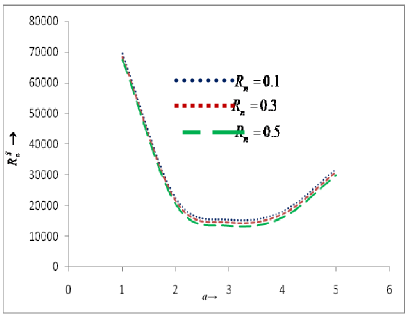

Fig. 11. Effect of R_n on R_a^S for top-heavy configuration

In Figure 8, it is found that the R_a^S decreases very slightly with the decrease in modified diffusivity ratio N_A, this indicates that the modified diffusivity ratio has slightly stabilizing effect on the system, while its reverse effect is seen in Figure 9. Figures 10 and 11 show that R_a^S increases with the decrease in R_n for both bottom/ /top-heavy configuration. Thus, the nanoparticle Rayleigh number postpones the stationary convection of the system.

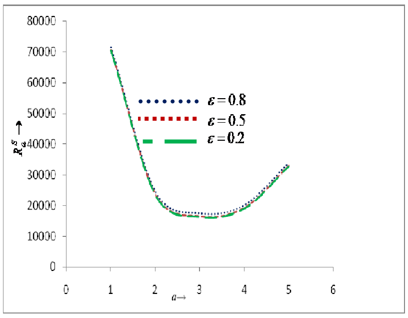

Fig. 12. Effect of ε on R_a^S for bottom-heavy configuration

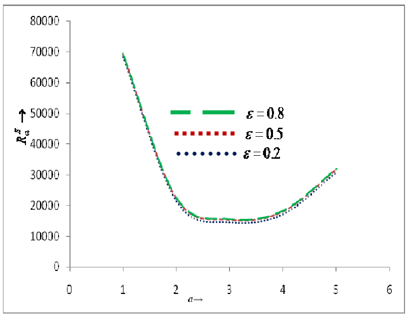

Figure 13. Effect of ε on R_a^S for top-heavy configuration

The variations of R_a^S with the wave number a for three different values of ε is plotted in Figures 12 and 13, and it is noticed that Rayleigh number decreases with the increase in medium porosity for bottom-heavy configuration, while a reverse effect is found for top-heavy configuration implying thereby medium porosity postpone/enhance the stationary convection for bottom/top-heavy configuration.

5. Conclusion

The effect of rotation on thermal instability in a Jeffrey nanofluid layer of saturated by a porous medium has been investigated. The main conclusion is as follows:

- The Jeffrey parameter advances the stationary convection if $T_a > \frac{1}{(1+\lambda_3)^2}\frac{(\pi^2+a^2)}{\pi^2}$ for both bottom/top-heavy configurations.
- Rotation advances the stationary convection for both bottom/top-heavy configurations.
- The Lewis number and modified diffusivity have stabilizing/destabilizing influence for bottom/top-heavy configuration on the stationary convection.
- The nanoparticles Rayleigh number postpones the stationary convection for both bottom/top-heavy nanoparticles distribution.
- Medium porosity delays the stationary convection for bottom-heavy configuration and advances the stationary convection for top-heavy configuration.

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References

[1] Choi, S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. In: Siginer D.A., Wang, H.P. (eds.) Developments and Applications of Non-Newtonian Flows. ASME FED-Vol. 231/MD, 66: 99-105.

[2] Buongiorno, J. (2006). Convective transport in nanofluids. *ASME J. of Heat Trans.*, *128*, 240-250.

[3] Kumar, R.N., Gowda, R.J.P., Gireesha, B.J., & Prasannakumara, B.C. (2021). Non-Newtonian hybrid nanofluid flow over vertically upward/downward moving rotating disk in a Darcy-Forchheimer porous medium. *Eur. Phys. J. Spec. Top.*, *230*, 1227-1237.

[4] Nield, D.A., & Kuznetsov, A.V. (2009). Thermal instability in a porous medium layer saturated by a nanofluid. *Int. J. Heat Mass Transfer*, 52, 5796-5801.

[5] Yadav, D., Agrawal, G.S., & Bhargava, R. (2011). Thermal instability of rotating nanofluid layer. *Int J. Eng. Science*, *49*, 1171-1184.

[6] Sowmya, G., Gireesha B.J., Sindhu, S., & Prasannakumara, B.C. (2020). Investigation of Ti_6Al_4V and AA7075 alloy embedded nanofluid flow over longitudinal porous fin in the presence of internal heat generation and convective condition. *Commun. Theor. Phys.* 72, 025004

[7] Madhukesh, J.K., Kumar, R.N., Gowda R.J. Punith, Prasannakumara, B.C. Ramesh, G.K. Khan M. Ijaz, Khan, S.U., & Chu, Yu-Ming. (2021). Numerical simulation of AA7072-AA7075/ /water-based hybrid nanofluid flow over a curved stretching sheet with Newtonian heating: A non-Fourier heat flux model approach. *J. of Molecular Liquids*, *335*, 116103.

[8] Nield, D.A., & Bejan, A. (2006). *Convection in Porous Medium*. New York: Springer.

[9] Jeffreys, H. (1926). The stability of a layer of fluid heated below. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, *2*, 833-844.

[10] Gowda, R.J. Punith, Kumar, R. Naveen, Prasannakumara, B.C., Nagaraja, B., & Gireesha, B.J. (2021). Exploring magnetic dipole contribution on ferromagnetic nanofluid flow over a stretching sheet: An application of Stefan blowing. *J. of Molecular Liquids*, *335*, 116215.

[11] Nadeem, S., & Akbar, N.S. (2009). Peristaltic flow of a Jeffrey fluid with variable viscosity in an asymmetric cannel. *Z. Naturforsch.*, *64a*, 713-722.

[12] Sushma, K., Sreenadh, S., & Dhanalakshmi, P. (2017). Mixed convection flow of a Jeffrey nanofluid in a vertical cannel. *Middle-East Journal of Scientific Research*, *25*, 950-959.

[13] Hayat, T., Ullah, H., Ahmad, B., & Alhodaly, M.S. (2021). Heat transfer analysis in convective flow of Jeffrey nanofluid by vertical stretchable cylinder. *Int. Comm. in Heat and Mass Transfer*, *120*, 104965.

[14] Ullah, H., Hayat, T., Ahmad, S., & Alhodaly, M.S. (2021). Entropy generation and heat transfer analysis in power-law fluid flow: Finite difference method. *Int. Comm. in Heat and Mass Transfer*, *122*, 105111.

[15] Sheu, L.J. (2011). Thermal instability in a porous medium layer saturated with a viscoelastic nanofluid. *Transp. Porous Media*, *88*, 461-477.

[16] Chand, R., Rana, G.C., & Puigjaner, D. (2018). Thermal instability analysis of an elastico-viscous nanofluid layer. *Engineering Transactions*, 66, 301-324.

[17] Chandrasekhar, S. (1961). *Hydrodynamic and Hydromagnetic Stability*. New York: Dover Publication.

[18] Vadasz, P. (1998). Coriolis effect on gravity-driven convection in a rotating porous layer heated from below. *Journal of Fluid Mechanics*, 376, 351-375.

[19] Sharma, V., & Rana, G.C. (2001). Thermal instability of a Walters' (Model B') elastico-viscous fluid in the presence of variable gravity field and rotation in porous medium. *J. Non-Equilib. Thermodyn.*, *26*, 31-40.

[20] Ullah, H., Khan, M.I. & Hayat, T., Khan, M.I. (2020). Modeling and analysis of megneto-Carreau fluid with radiative heat flux: Dual solutions about critical point. *Advances in Mech. Eng.*, *12*, 1-10.

[21] Ahmad, S., Ullah, H., Hayat, T., & Alsaedi, A. (2020). Computational analysis of time-dependent viscous fluid flow and heat transfer. *Int. J. of Modern Physics B*, *34*, 2050141.

[22] Ullah, H., Hayat, T., Ahmad S., Alhodaly, M.S., & Momani, S. (2021). Numerical simulation of MHD hybrid nanofluid flow by a stretchable surface. *Chinese J. of Physics*, *71*, 597-609.

[23] Ahmad S., Ullah, H., Hayat, T., & Alhodaly, M.S. (2021). Time-dependent power-law nanofluid with entropy generation. *Phys. Scr.*, *96*, 025208.

[24] Kang, J., Fu, C., & Tan, W. (2011). Thermal convective instability of viscoelastic fluid in a rotating porous layer heated from below. *J Non-Newton Mech*., 166, 93-101.

[25] Chand, R., & Rana, G.C. (2012). On the onset of thermal convection in rotating nanofluid layer saturating a Darcy-Brinkman porous medium. *Int. J. of Heat and Mass Transfer*, *55*, 5417-5424.

[26] Rana, G.C., Chand, R., & Jamwal, H.S. (2014). The onset of thermal instability of viscoelastic rotating fluid permeated with suspended particles in porous medium. *Structural Integrity and Life*, *14*, 193-198.

[27] Chand, R., Rana, G.C., & Kango, S.K. (2015). Effect of variable gravity on thermal instability of rotating nanofluid in porous medium. *FME Transactions*, *43*, 62-69.

[28] Chand, R., Rana, G.C., & Yadav, D. (2017). Thermal instability of couple-stress nanofluid with vertical rotation in a porous medium. *Journal of Porous Media*, 20, 635-648.

[29] Rana, G.C., Gautam, P.K., & Saxena, H. (2019). Electrohydrodynamic thermal instability in a Walters' (model b') rotating nanofluid saturating a porous médium. *J. of the Serbian Soc. Comp. Mech.*, *13*, 19-35.

[30] Ahmad, S., Hayat, T., Alsaedi, A., Ullah, H., Alsaedi, A., & Shah, F. (2021). Computational modeling and analysis for the effect of magnetic field on rotating stretched disk flow with heat transfer. *Propulsion and Power Research*, *10*, 48-57.