THE EFFECT OF DAMPING COEFFICIENTS
ON THE TORSIONAL VIBRATION
OF THE DAMPED MULTI-BRANCH GEARS SYSTEM

Jabbar Firouzi[1], Hassan Ghassemi[1], Karim Akbari Vakilabadi[2], Hadi Khalilnezhad[2]

[1] *Department of Maritime Engineering, Amirkabir University of Technology
Tehran, Iran*
[2] *Department of Mechanic Engineering, Imam Khomeini University of Marine Science
& Technology, Nowshahr, Iran*
gasemi@aut.ac.ir

Abstract. The effect of the damping coefficients has been considered to estimate the tor-sional vibration of the multi-branch gears systems. In this paper, the effect of the viscous damping between the gears and the fluid in which the gear is working and the effect of the structural damping of the shafts in a three-branched gear system is investigated. Initially, the governing equations were derived and then the damping effects were studied using MATLAB code. In order to validate the results which prepared by MATLAB code are compared with the other computational methods. To investigate the effects of the stiffness and damping on the shafts and gears, different gear branch systems are considered. Some of the vibration results on the damped gear branched systems with viscous damper and struc-tural damper are analyzed and discussed.

1. Introduction

In analysing the vibrations of continuous structures, identification and model-ling of damping are of particular importance and complexity. This complexity has led to the analysis of structures either without damping or simple damping with approximations of real state [1]. A torsional vibration is a phenomenon that can cause the axial cutting to be in the centre of the rotary shaft due to the twisting of the centre, so this should be considered when designing the shaft. In marine vessels, the vibration in the shaft connecting the engine to the propeller is due to the inhar-monious excitation resulting from these two parts. So, the study of the torsional vibration of the transmission shaft is vital for moving the ship. In the systems

with torsional vibration, linear values with angular values are replaced in vibrational equations.

Many engineering vibration problems can be solved with the theory of a degree of freedom (DOF) system. Complex systems may have several DOF. The standard method for solving such systems, if the DOF is not more than three degrees, is to obtain the equations of motion by Newton's law of motion, the influence coefficient method, or by the Lagrange equations. Then differential equations of motion are solved by assuming an appropriate solution. If the DOF increases, solving differential equations of motion becomes more complicated for damped system [2]. Lin et al. showed that vibration propagation in complex ship structures at low frequencies can be attenuated by imposing irregularities to the ring frame locations in ships [3].

The natural frequency analysis for gear shaft systems is of practical importance and has a high sensitivity to the optimal design of these systems [4]. The torsional vibration in the marine power transmission system is usually very dangerous for the shaft line and the crankshaft [5]. So damping always exists in marine propulsion systems. Torsional damping may come from several sources, for example, shaft material, bearings, couplings, torsional vibration dampers, aerodynamic damper, etc. Shaft material or hysteretic dampers are due to the interaction of shaft material molecules, which leads to an increase in the material temperature of the shaft. A torsional vibration damper is a part that can be used in the connections between two-shaft cross-sections. The gear aerodynamic damping occurs due to interaction of the gear with the working fluid (such as steam, gas, air, etc.), lubricant, and coolant; which results in dissipation of the energy in the form of heat.

In other studies, the finite element method (FEM) has been used to solve the torsional vibration problems of direct and gear-branch systems [6]. Wu et al. have modelled the gearbox systems by elimination the twisting angles of the gears engaged decrease them to the direct equivalent system (or direct transmitted). Then, the overall mass matrix, damping matrix, stiffness matrix, and the torque vector were obtained from the direct transmitted system by the conventional FEM, and then obtained the natural frequencies and the mode shapes of the system [7]. For torsional vibration investigating of the crankshaft, it is assumed by 2-DOF and its stiffness is infinite. Also, due to insignificant influence, damping coefficients on the natural frequencies and the mode shapes of the devices were ignored from the damping coefficients matrix in solving the governing equations [8].

Drew et al. described two new test methods that were investigated for their suitability as methods of determining the torsional damping levels of the test gearbox of the back-to-back rig. The first test method investigated was to attempt a standard multi-input multi-output (MIMO) modal analysis of the gearbox test, in which the gearbox was used as a system with two inputs (torque per shaft) and the two output (angular velocity per shaft) was considered. The second test method was to assume the shape of a model for the test gearbox to match the calculation of the reacceptances for the gearbox and adjust the damping level in the model until it matched the experimental results [9].

Equipment failures due to torsional vibration are an important problem in the industry. Low torsional damping levels cause high stresses. Gearboxes are an important part of the marine power transmission system, so measuring the natural frequency of torsional vibration and investigate the effects of damping are of particular importance [10].

The effects of the stiffness and damping on both shafts and gears are important in multi-branch systems. Zou et al. employed a modal synthesis method for analysis the torsional vibration of the system. In this method, the system is divided into several subsystems and then solved by FEM [11]. Also, Ray presented a FEM of torsional vibration in geared shafts [12].

In research carried out by Murawski and Charchalis, the torsional vibration of the power transmission system was investigated by modelling the propeller, propeller shaft, middle shaft and main engine. In their research, numerical analysis was performed based on the FEM with experimental data measured by an experienced team, which was highly accurate and had an error rate of less than 6% [8].

Study of the ship propeller shaft vibrations is of particular importance. In this regard, the added mass and damping matrices of the propeller have an influence on the vibrational characteristics of the propulsion system. Mao and Young [13] have investigated the influence of the skew angle on the added mass and damping matrices, whose obtained results show that the increase in the Skew angle effects of the sway, heave, pitch and yaw components of the added mass and damping matrices, while the influence on the surge and roll components is negligible [13].

Aureli et al. modelled a rectangular thin cross section cantilever beam using the Euler-Bernoulli beam theory and investigated bending vibration in a high-viscosity fluid. The results of this study show that increasing the frequency and amplitude of the vibration causes vortex shedding and convection phenomena [14]. Firouzi and Ghassemi have recently presented the effect of variations in shaft diameter and torsional stiffness on the whirling speed of the ship propeller shafting system for systems that directly or indirectly attached the propeller to the engine [15]. Piatkowski used the algorithm phase-amplitude interpolation to create contours charts and estimate the shaft torsional vibrations and transient operating conditions of rotary machines [16].

The following sections are organized as follows. Governing equations are described in Section 2. Validation is given in Section 3. Vibration analysis of the damped gear branched system are presented in Section 4, and Section 5 is given for the conclusions.

2. Governing equations

Power transmission systems have many elements such as shaft, gearbox, coupling, bearings and stern tube between the engine to the propeller. Their tasks are to deliver the power from the engine to the propeller to provide forward speed to the ship. Here, we consider a gear branched system as shown in Figure 1. It has

two shafts and 4 gears with stiffness and damping. All of the consideration parameters define the mass momentum of inertia of the shafts I_p and I_q, the mass momentum of inertia of the gears I_i ($i = 1 - 4$), the damping of each gear C_i ($i = 1 - 4$), torsional dampers of different segments of the shaft C_p and C_q, and torsional stiffness of shafts k_p and k_q.
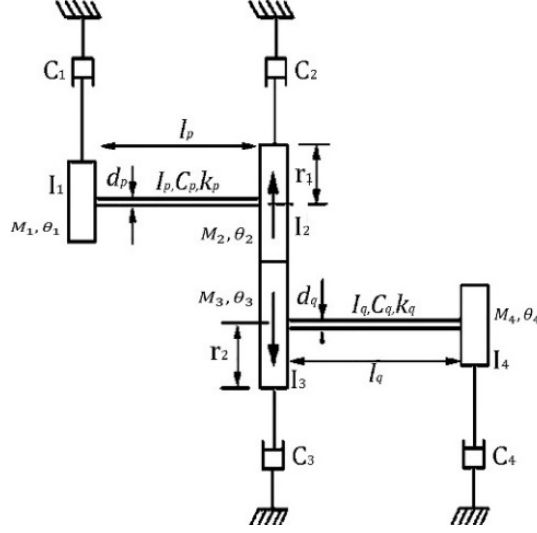

Fig. 1. Gear branched system

We write the vibrational equations of the above system as follows:

$$\left(I_1 + \frac{1}{3}I_P\right)\ddot{\theta}_1 + \frac{1}{6}I_P\ddot{\theta}_2 + (C_1 + C_P)\dot{\theta}_1 - C_P\dot{\theta}_2 + k_P(\theta_1 - \theta_2) = M_1 \quad (1)$$

$$\frac{1}{6}I_P\ddot{\theta}_1 + \left(I_2 + \frac{1}{3}I_P\right)\ddot{\theta}_2 - C_P\dot{\theta}_1 + (C_2 + C_P)\dot{\theta}_2 - k_P(\theta_1 - \theta_2) = M_2 \quad (2)$$

$$\left(I_3 + \frac{1}{3}I_q\right)\ddot{\theta}_3 + \frac{1}{6}I_q\ddot{\theta}_4 + (C_3 + C_q)\dot{\theta}_3 - C_q\dot{\theta}_4 + k_q(\theta_3 - \theta_4) = M_3 \quad (3)$$

$$\frac{1}{6}I_q\ddot{\theta}_3 + \left(I_4 + \frac{1}{3}I_q\right)\ddot{\theta}_4 - C_q\dot{\theta}_3 + (C_4 + C_q)\dot{\theta}_4 - k_q(\theta_3 - \theta_4) = M_4 \quad (4)$$

Then for each shaft we write the above equations in matrix form.

$$\begin{bmatrix} I_1 + \frac{1}{3}I_P & \frac{1}{6}I_P \\ \frac{1}{6}I_P & I_2 + \frac{1}{3}I_P \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} C_1 + C_P & -C_P \\ -C_P & C_2 + C_P \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} k_P & -k_P \\ -k_P & k_P \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} \quad (5)$$

$$\begin{bmatrix} I_3 + \dfrac{1}{3}I_q & \dfrac{1}{6}I_q \\ \dfrac{1}{6}I_q & I_4 + \dfrac{1}{3}I_q \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{Bmatrix} + \begin{bmatrix} C_3 + C_q & -C_q \\ -C_q & C_4 + C_q \end{bmatrix} \begin{Bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{Bmatrix} + \begin{bmatrix} k_q & -k_q \\ -k_q & k_q \end{bmatrix} \begin{Bmatrix} \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} M_3 \\ M_4 \end{Bmatrix} \quad (6)$$

where: $I_P = \rho(\pi d^4/32)l_P$ and $k_P = G(\pi d^4/32)/l_P$ the mass moment of inertia and torsional stiffness of the shaft p, respectively, also for shafts q the mass moment of the inertia and torsional stiffness of the shaft are obtained.

Using the ratio between gears 2 and 3:

$$r_2\theta_2 = -r_3\theta_3, \quad \theta_3 = -\frac{r_2}{r_3}\theta_2 = -R\theta_2, \quad R = \frac{r_2}{r_3} \quad (7)$$

The negative sign R is due to the opposite direction θ_3 to θ_2. Since r_2 and r_3 are the radius of the gears 2 and 3, respectively. R is the gear ratio. Relations between the shafts q and p with considering the gear ratio, the following equations are defined:

$$\{\acute{\theta}\} = [\lambda]\{\theta\}, \quad \{\acute{\theta}\} = \{\theta_3, \theta_4\}, \{\theta\} = \{\theta_2, \theta_4\}, [\lambda] = \begin{bmatrix} -R & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

By derivation of equation (8) we have:

$$\{\dot{\acute{\theta}}\} = [\lambda]\{\dot{\theta}\}, \quad \{\ddot{\acute{\theta}}\} = [\lambda]\{\ddot{\theta}\} \quad (9)$$

where:

$$\{\dot{\acute{\theta}}\} = \{\dot{\theta}_3, \dot{\theta}_4\}, \{\dot{\theta}\} = \{\dot{\theta}_2. \dot{\theta}_4\}, \{\ddot{\acute{\theta}}\} = \{\ddot{\theta}_3, \ddot{\theta}_4\}, \{\ddot{\theta}\} = \{\ddot{\theta}_2. \ddot{\theta}_4\} \quad (10)$$

In the above equations, $\{,\}$ represents the vector of the column and $[\lambda]$ represents the transformation matrix to remove the dependence of the angular displacement θ_3. By replacing the equations (8), (9) in equation (6) and multiplying the two sides of the resulting equation in $[\lambda]^T$ we will have:

$$[m_e]\{\ddot{\theta}\} + [C_e]\{\dot{\theta}\} + [k_e]\{\theta\} = \{M_e\} \quad (11)$$

$$[m_e] = \begin{bmatrix} R^2\left(I_3 + \dfrac{1}{3}I_q\right) & -R\left(\dfrac{1}{6}I_q\right) \\ -R\left(\dfrac{1}{6}I_q\right) & I_4 + \dfrac{1}{3}I_q \end{bmatrix}, \quad [C_e] = \begin{bmatrix} R^2(C_3 + C_q) & RC_q \\ RC_q & C_4 + C_q \end{bmatrix},$$

$$[k_e] = \begin{bmatrix} R^2k_q & Rk_q \\ Rk_q & k_q \end{bmatrix}, \quad \{f_e\} = \begin{Bmatrix} -RM_3 \\ M_4 \end{Bmatrix} \quad (12)$$

Thus, the mass matrix $[m_e]$, the damping matrix $[C_e]$, the hard matrix $[k_e]$ and the load vector $\{M_e\}$ of the drive shaft q are defined by equations (12). If $R = -1$,

similar relations are obtained for the drive shaft p (according to equation (1)). In other words, the equation (12) is the key word of the method.

$$
\begin{bmatrix}
I_1 + \dfrac{1}{3}I_P & \dfrac{1}{6}I_P & 0 \\[2mm]
\dfrac{1}{6}I_P & \left(I_2 + \dfrac{1}{3}I_P\right) + R^2\left(I_3 + \dfrac{1}{3}I_q\right) & -R\left(\dfrac{1}{6}I_q\right) \\[2mm]
0 & -R\left(\dfrac{1}{6}I_q\right) & I_4 + \dfrac{1}{3}I_q
\end{bmatrix}
\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_4 \end{Bmatrix}
$$
$$
+ \begin{bmatrix}
C_1 + C_P & -C_P & 0 \\
-C_P & (C_2 + C_P) + R^2(C_3 + C_q) & RC_q \\
0 & RC_q & C_4 + C_q
\end{bmatrix}
\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_4 \end{Bmatrix}
\qquad (13)
$$
$$
+ \begin{bmatrix}
k_P & -k_P & 0 \\
-k_P & k_P + R^2 k_q & Rk_q \\
0 & Rk_q & k_q
\end{bmatrix}
\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_4 \end{Bmatrix}
= \begin{Bmatrix} M_1 \\ M_2 - RM_3 \\ M_4 \end{Bmatrix}
$$

3. Validation

In this work, the governing equations of the torsional vibration of the multi-branch gears system are derived and free vibration is considered. Overall mass, damping and torsional stiffness matrices were obtained, and a MATLAB code was used for solving and obtaining natural frequencies of the system. To verify the results, the system shown in Figure 2 was considered. Our results were compared with the results shown in Table 2, and they were in good conformity.
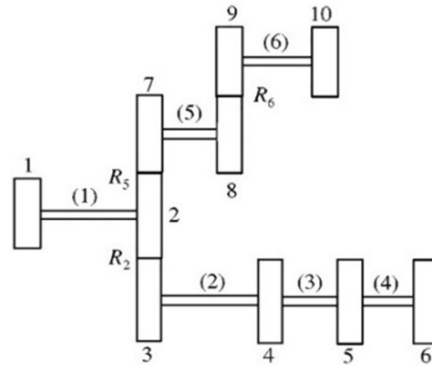

Fig. 2. The original three-branched gear system

For the original three-branched gear system, shown in Figure 2, the main dimensions and physical properties of the gears and shafts are presented in Table 1.

In fact, the given data, the mass moment of inertia for the gears, I_j ($j = 1 - 10$) and the torsional stiffness of the shafts k_i ($i = 1 - 6$) are shown in Table 1. Also,

other data such as the diameter of the gears D_j, the diameter of the shaft d_i, and the length of the shaft l_i are given in this table. For gears and shafts $\rho = 20.2063$ kg/m^3 and G $= 79.29 \cdot 10^9$ N/m^2, $R_2 = \frac{D_2}{D_3} = 2.318$, $R_5 = \frac{D_2}{D_7} = 4.8182$ and $R_6 = \frac{D_8}{D_9} = 8.4906$ is gear ratio.

Table 1

Main dimensions and physical properties of shafts and gears

Gear			Shaft			
No. j	Mass moments of inertia I_j [kg·m^2]	Diameters D_j [m]	No. i	Torsional stiffness $k_i \times 10^6$ [N·m/rad]	Diameters D_i [m]	Lengths l_i [m]
1	1098.213	1.69	1	0.6169	0.127	3.283
2	111.448	1.3	2	0.1966	0.102	4.219
3	4.067	0.56	3	3.9861	0.102	0.208
4	45.985	1.039	4	7.7621	0.102	0.107
5	26.438	0.908	5	0.2192	0.102	3.784
6	26.438	0.908	6	0.2429	0.064	0.521
7	0.407	0.27				
8	33.895	1.2192				
9	0.0407	1.431				
10	9.2196	0.698				

The natural frequencies of the torsional vibration of the above system are presented in the following table in two states with/without of the shafts mass. The first frequency obtained is zero ($\omega_0 = 0$) and other natural frequencies are given and compared in Table 2.

Table 2

Several of the lowest natural frequencies of the original three-branched gear system

	ω_1	ω_2	ω_3	ω_4	ω_5	Shaft mass
Present method	23.087	41.2902	215.2224	376.6524	710.4798	considered
	23.089	41.3231	217.6014	376.9155	712.6998	neglected
FEM [2]	23.087	41.2902	215.2224	376.6524	710.4786	considered
	23.089	41.3228	217.6013	376.9155	712.6985	neglected
FEM [10]	23.089	41.3228	217.6013	376.9155	712.6985	neglected
Holzer [9]	23.092	41.3290	217.9722	376.9155	713.3585	neglected

4. Vibration analysis of the damped gear branched system

This example is given for vibration analysis of the damped gear branched system. It includes stiffness on the shaft and damping on both shafts and gears in two states as shown in Figure 3.
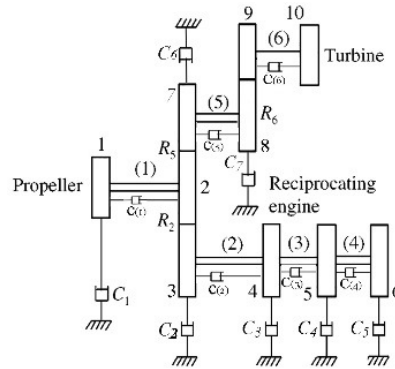

Fig. 3. The three-branched gear system with viscous damper and with structural damper

4.1. Vibration analysis of a damped gear branched system
with viscous damper

We consider the analysis of the damped gear branched system with the damper between the gears and fluid, as shown in Figure 3. In this case, the structural damper is equal to zero $C_{(i)}$ $(i = 1 - 6) = 0$.

The damping coefficients for the seven dampers shown in the figure above, C_j $(j = 1 - 7)$, were assumed to be the same. We study the influence of damping with consideration of the mass of all shafts for different cases $C_1 = 9 - 11.7$ Nms/rad and C_j $(j = 2 - 7) = 100 - 130$ Nms/rad. The real part (ω_{Rr}) of the special values of $(\alpha_r \cdot r = 1 - 6)$ is related to the damping parameter and the imaginary part (ω_{Ir}) to the damping natural frequencies, whose values are shown in Table 3.

Table 3

**Influence of damping on the natural frequencies of the damped gear branched system
with the viscous damper**

Damping coefficient C_j $(j = 1 - 7)$ [N·m·s/rad]	Eigenvalues $\alpha_r = \bar{\omega}_{Rr} + i\bar{\omega}_{Ir}$						
	α_1	α_2	α_3	α_4	α_5	α_6	Remarks
C_j	0.01	1.52	9.87	1.49	1.30	1.87	ω_{Rr}
	23.08	41.29	217.36	376.91	712.69	828.36	ω_{Ir}
1.1 C_j	0.02	1.68	10.86	1.64	1.43	2.06	ω_{Rr}
	23.08	41.28	217.31	376.91	712.69	828.36	ω_{Ir}
1.2 C_j	0.02	1.83	11.85	1.79	1.56	2.25	ω_{Rr}
	23.08	41.28	217.26	376.91	712.69	828.36	ω_{Ir}
1.3 C_j	0.02	1.98	12.84	1.93	1.69	2.43	ω_{Rr}
	23.08	41.27	217.20	376.91	712.69	828.36	ω_{Ir}

As shown in Table 3, we increase the value of C_j in the three steps by adding 10% at each step. By increasing the value of C_j, the natural frequency values of the damping (ω_{Rr}) slightly increase while (ω_{Ir}) does not almost change.
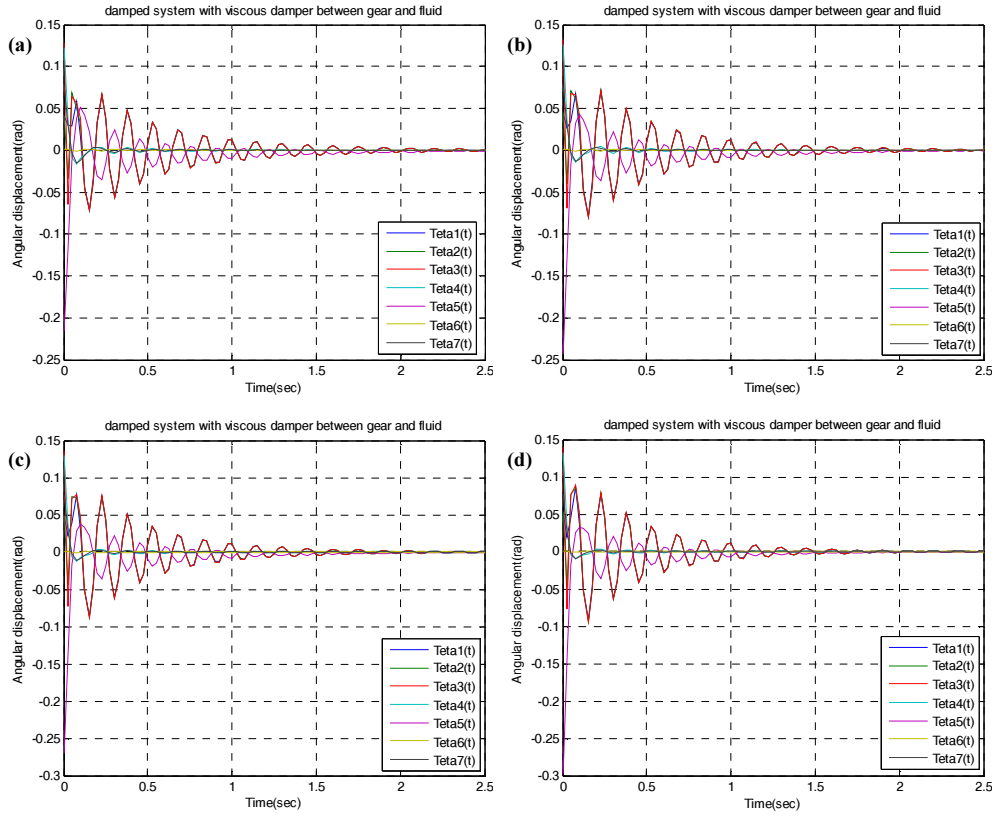

Fig. 4. Damped system with viscous damper between gears and fluid
(a) for C_j (b) for 1.1 C_j (c) for 1.2 C_j (d) for 1.3 C_j

In Figure 4, by increasing the amount of damping coefficient C_j, the torsional vibration amplitude increases at initial times and the torsional vibration in θ_3 and θ_5 will be damped later than the other parts. By increasing the amount of damping coefficient C_j, the duration of the damping is also reduced.

4.2. Vibration analysis of a damped gear branched system with viscous damper and structural damper

We consider Figure 3 for analysis of the damped gear branched system with the viscous damper and structural damper. In this case, consider the values of C_j as before and consider the structural damping coefficients for shafts equal to

$C_{(i)}$, $(i = 1 - 6) = 150 - 195$ Nms/rad. The values obtained for the damping natural frequency (ω_{Ir}) and the damping parameter (ω_{Rr}) are given in Table 4.

Table 4

Influence of damping on the natural frequencies of the damped gear branched system with viscous damper and with structural damper

| Damping coefficient $C_{(i)}$ $(i = 1 - 6)$ C_j $(j = 1 - 7)$ [N·m·s/rad]	Eigenvalues $\alpha_r = \bar{\omega}_{Rr} + i\bar{\omega}_{Ir}$						
	α_1	α_2	α_3	α_4	α_5	α_6	Remarks
$C_{(i)}$, C_j	0.09 23.08	2.16 41.26	25.62 216.10	4.97 376.83	161.48 694.16	9.84 828.30	ω_{Rr} ω_{Ir}
$1.1\ C_{(i)}$, $1.1\ C_j$	0.10 23.08	2.36 41.25	27.85 215.83	5.40 376.9182	174.17 691.08	10.65 828.29	ω_{Rr} ω_{Ir}
$1.2\ C_{(i)}$, $1.2\ C_j$	0.11 23.08	2.58 41.23	30.38 215.49	5.88 376.80	190.00 686.90	11.26 828.27	ω_{Rr} ω_{Ir}
$1.3\ C_{(i)}$, $1.3\ C_j$	0.12 23.08	2.79 41.22	32.92 215.12	6.37 376.78	205.84 682.32	12.58 828.26	ω_{Rr} ω_{Ir}
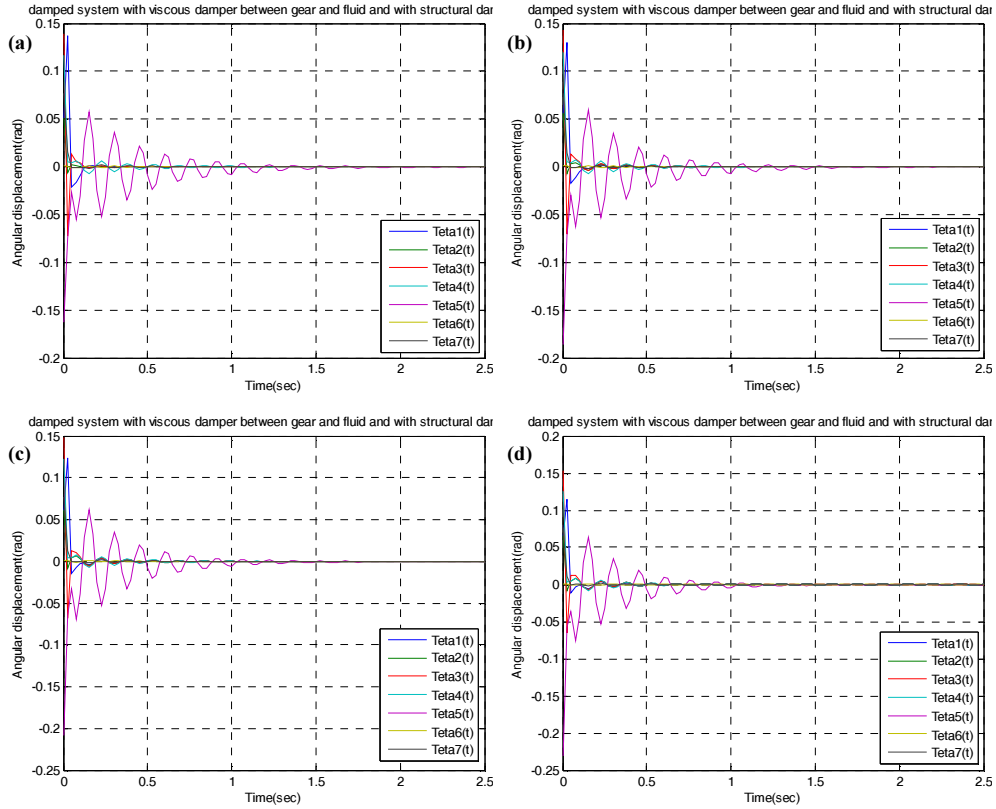
Fig. 5. Damped system with viscous damper and with structural damper (a) for $C_{(i)}$, (b) for $1.1\ C_{(i)}$, $1.1\ C_j$ (c) for $1.2\ C_{(i)}$, $1.2\ C_j$ (d) for $1.3\ C_{(i)}$

In Table 4, we increase the amount of $C_{(i)}$ and C_j in the three steps at each step by the amount of 10%. By increasing these values, the damping natural frequency (ω_{Ir}), other than for α_1, is not significantly reduced in the other cases, but in all cases the values of the damping parameter (ω_{Rr}) increase. In Figure 5, increasing the amount of damping coefficient $C_{(i)}$ and C_j, the torsional vibration amplitude increases at initial times and torsional vibration at θ_5 will be damped later than other parts. As the number of damping coefficients increases, the duration of the damping also decreases.

5. Conclusions

The effect of the damping coefficients has been achieved to estimate the torsional vibration of the multi-branch gears systems. Based on the presented computational results, the following conclusions are drawn:

1. Increased damping leads to an increase in the torsional vibration amplitude in the initial times, and this issue is very important in the vibration and should increase the damping coefficient, especially in different parts of the engine for any reason (oil reduction, erosion, etc.) is prevented.
2. With a structural damper applied to the shafts and the gears, the torsional vibration amplitude is lower than that for which only the viscous damper was considered.
3. Increased damping, in particular structural damping, leads to a reduction in the damping time of the vibration amplitude, but, on the other hand, causes energy dissipation.

It is our spirit to investigate on the stiffness of gearwheel teeth in near future. In this regard, all dynamics equations are required to be obtained. The MATLAB code should be improved and employed with new examples of the stiffness effect of the gearteeth.

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