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# DRAG ON A PERMEABLE SPHERE PLACED IN A MICROPOLAR FLUID WITH NON-ZERO BOUNDARY CONDITION FOR MICROROTATIONS

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**Abstract.** This paper concerns an analytical study of an infinite expanse of uniform flow of steady axisymmetric creeping flow of an incompressible micropolar fluid around the permeable sphere assuming a nonhomogeneous boundary condition for microrotation vector. It is assumed that microrotation vector is proportional to the rotation rate of velocity vector. The stream function solutions for the flow fields are obtained in the terms of modified Bessel's functions and Gegenbauer functions. Continuity of normal velocity, no-slip condition, non-zero microrotation vector on the sphere, uniform velocity at infinity are the different boundary conditions used to determine the flow fields explicitly. The microrotation component, pressure field, bounds of permeability parameter and drag force experienced by the permeable sphere are calculated. Dependence of the drag force on different fluid parameters is presented graphically and discussed. It is found that drag force decreases with increasing spin parameter. Several cases of interest are deduced from the present analysis.

**Keywords:** *drag force, modified Bessel's function, micropolar fluid, permeable sphere, stream function*

## 1. Introduction

The investigation of viscous flow through permeable media has fascinated substantial practical and theoretical interest in science, designing and innovation. In our day to day life, we observe that the permeable objects and permeability nature correspond to various types of viscous and non viscous fluids passing through the objects viz. cloth, sand, paper etc. Many different theoretical and experimental models have been proposed, which explain the viscous flow past and within porous bodies. Some more examples of such flow are a flow past a meshed spherical surface, soil etc.

Darcy [1] proposed the initial study of the fluid flow in a permeable medium and stated that the flow rate in porous media is proportional to the pressure gradient.

Many different approaches [2-4] implemented to explain the creeping flow of an incompressible viscous fluid about permeable sphere. Joseph and Tao [5] investigated that the slow motion of a viscous fluid past a permeable sphere in an uniform stream can be calculated in an analogous analytic form if the fluid in the permeable sphere obeys the Darcy's law. Birikh and Rudakoh [6] investigated the problem of slow motion of a permeable sphere in a viscous fluid and evaluated the drag and the flow rate of the fluid. Padmavathi et al. [8] solved the problem of stokes flow past a permeable sphere for the non-axisymmetric case and gave a general method for calculating non-axisymmetric flow both outside and inside the permeable spherical boundary and expressions for drag and torque on the sphere. Usha [9] solved the problem of creeping flow over concentric permeable spheres in relative motion. The problem on slow viscous flow past a spinning sphere with permeable surface was solved by Vasudeviah and Malathi [10], and they derived the expression for drag coefficient on the body which can be used as a formula for the determination of the permeability of the sphere.

Eringer [11] introduced a subclass of viscous fluids, which he named micropolar fluids, that ignores the deformation of the microelement but still allows for the particle micromotion to take place. Apart from the classical field of velocity, in the micropolar fluid theory there are two additional field variables, viz. the microrotation vector $v$ and the gyration parameter $J$; introduced to explain the kinematic of microrotation. Ramkisoon and Majumdar [12] derived a formula to evaluate drag on axially symmetric bodies for the case of micropolar fluid and they observed that the drag in the micropolar fluid is greater than that in the classical fluid. The slow stationary flow of a micropolar fluid past a sphere was studied by Rao and Rao [13]. Srinivasacharya and Rajyalakshmi [14] solved the problem of the creeping flow of micropolar fluid past a porous sphere and observed that the drag on the porous sphere, when the fluid is micropolar, is more than that of the Newtonian fluid. Ramkissoon [15] has obtained the solution for the problem of a micropolar fluid flow around a Newtonian fluid sphere and evaluated the drag force exerted on the sphere. The resistance force exerted on a solid sphere moving with constant velocity in micropolar fluid with a non-homogeneous boundary condition for microrotation vector was tackled by Haffmann et al. [16]. Gupta and Deo [17] have studied Stokes flow of micropolar fluid past a porous sphere with non-zero boundary condition for microrotations. Drag on Reiner-Rivlin liquid sphere placed in a micropolar fluid with non-zero boundary condition for microrotation has been solved by Jaiswal and Gupta [18]. Gupta and Deo [19] have solved the problem of axisymmetric creeping flow of micropolar fluid over a sphere coated with a thin fluid film. It is found that a sphere without coating experiences greater resistance in comparison to coated fluid. Slow steady rotation of a permeable sphere in an incompressible couple stress fluid is considered by Aparna and Murthy [20] and they also studied the problem on uniform flow of an incompressible micropolar fluid past a permeable sphere [21].

The aim of this paper is to extend the work of Aparna and Murthy [21] by assuming a non-zero boundary condition instead of zero boundary condition for microrotation vector. The microrotation on the boundary of the sphere is assumed to be proportional to the rotational rate of the velocity field on the boundary. The stream function and pressure for the flow outside and inside of the sphere are calculated. The drag experienced by a permeable sphere is evaluated and the graphical representation of the drag force with respect to the different fluid parameters is displayed. The effect of fluid parameters on velocities and stream functions are discussed.

## 2. Mathematical formulas

Here we have considered a permeable sphere of radius $a$ in an unbounded medium with the origin at the centre $O$ of the sphere (Fig. 1). We assume that the permeable sphere is stationary and a steady axisymmetric creeping flow of a micropolar fluid has been established around it by a uniform flow with velocity of magnitude U directed along the $Z$-axis far away from the sphere.

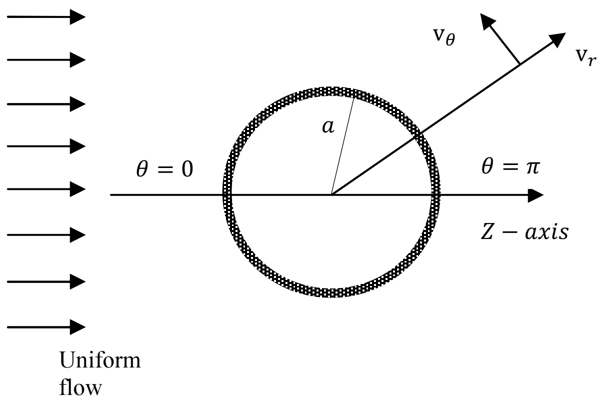


Fig. 1. Graphical representation of the model

The general form of governing equations for the slow steady motion of micropolar fluid under stokes approximation can be written as

$$\nabla \cdot \bar{v} = 0, \tag{1}$$

$$-\nabla p + \kappa \nabla \times \bar{\omega} - (\mu + \kappa) \nabla \times (\nabla \times \bar{v}) = 0, \tag{2}$$

$$-2\kappa \bar{\omega} + \kappa \nabla \times \bar{v} - \gamma \nabla \times (\nabla \times \bar{\omega}) + (\alpha + \beta + \gamma) \nabla (\nabla \cdot \bar{\omega}) = 0, \tag{3}$$

where $\bar{v}$ being the velocity vector, p the pressure, $\bar{\omega} = v_\varphi(r, \theta)\hat{e}_\varphi$ the microrotation vector, $\mu$ the classical viscosity coefficient of the fluid, $\kappa$ the vortex viscosity coefficients, α, β, γ are gyro viscosity coefficients satisfying the following inequalities

$$3\alpha + \beta + \gamma \geq 0,\ \ 2\mu + \kappa \geq 0,\ \ \gamma \geq |\beta|,\ \ \kappa \geq 0,\ \ \gamma \geq 0. \tag{4}$$

To non-dimensionalize the equations and variables, we put

$$r = a\tilde{r},\ \ \psi = Ua^2\tilde{\psi},\ \ p = \frac{\mu U}{a}\tilde{p},\ \ v_\varphi = \frac{U}{a}\tilde{v}_\varphi, \tag{5}$$

and drop tildes subsequently in further analysis. Since the flow field is axisymmetric, we can introduce stream functions $\psi(r,\theta)$ which is related to the velocity in spherical coordinate system $(r,\theta,\varphi)$ by Happel and Brenner [7]

$$\bar{\mathrm{v}} = \mathrm{v}_r(r,\theta)\hat{e}_r + \mathrm{v}_\theta(r,\theta)\hat{e}_\theta = -curl\left[\frac{\psi}{r\sin\theta}\hat{e}_\varphi\right], \tag{6}$$

and we obtain two velocity components of the flow as

$$\mathrm{v}_r = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta},\ \ \mathrm{v}_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}. \tag{7}$$

Eliminating the pressure from equation (2) and using (3) we get the differential equation

$$E^4\left(E^2 - \lambda^2\right)\psi = 0, \tag{8}$$

where $E^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{(1-\zeta^2)}{r^2}\dfrac{\partial^2}{\partial\zeta^2}$, $\zeta = \cos\theta$ and micropolar parameter

$$\lambda^2 = \frac{\kappa(2\mu+\kappa)a^2}{\gamma(\mu+\kappa)} \tag{9}$$

Using the equation (3) we get the micro-rotation component

$$v_\varphi = \frac{1}{2r\sin\theta}\left\{E^2\psi + \frac{\gamma(\mu+\kappa)}{\kappa^2}E^4\psi\right\}. \tag{10}$$

## 3. Method of solution

The solution of Eq. (8) can be obtained by superimposing the solution of

$$E^2\psi_0 = 0,\ \ E^4\psi_1 = 0,\ \ \left(E^2 - \lambda^2\right)\psi_2 = 0, \tag{11}$$

in the form,

$$\psi = \psi_0 + \psi_1 + \psi_2. \tag{12}$$

Suppose $\psi_e$ and $\psi_i$ denote the stream function solution for the external flow $(r \geq a)$ and for the internal flow $(r \leq a)$ respectively. Using the method of separation of variables, the stream function solution for external flow and internal flow are obtained as

$$\psi_e = \left[r^2 + \frac{A_1}{r} + B_1 r + C_1\sqrt{r}K_{3/2}(\lambda r)\right] G_2(\zeta) \tag{13}$$

and

$$\psi_i = \left[A_2 r^2 + B_2 r^4 + C_2\sqrt{r}I_{3/2}(\lambda r)\right] G_2(\zeta), \tag{14}$$

where $I_{3/2}(r)$, $K_{3/2}(r)$ are modified Bessel functions of the first and second kind and $G_2(\zeta) = \frac{1}{2}(1-\zeta^2)$ is Gegenbauer polynomial.

The micro-rotation components $v_\varphi^e$ and $v_\varphi^i$ for external and internal flow are

$$v_\varphi^e = \frac{1}{r\sin\theta}\{-\frac{B_1}{r} + \frac{\lambda^2}{N}C_1\sqrt{r}K_{3/2}(\lambda r)\}G_2(\zeta) \tag{15}$$

$$v_\varphi^i = \frac{1}{r\sin\theta}\{5B_2 r^2 + \frac{\lambda^2}{N}C_2\sqrt{r}I_{3/2}(\lambda r)\}G_2(\zeta) \tag{16}$$

## 4. Boundary conditions

The boundary conditions to be satisfied at the surface of the permeable sphere, which are physically realistic and mathematically consistent for this proposed problem, can be taken as:

- Continuity of normal velocity on the boundary i.e.

$$\frac{\partial\psi_e}{\partial\zeta} = \frac{\partial\psi_i}{\partial\zeta} \text{ on } r = a. \tag{17}$$

- No-slip condition across the surface i.e.

$$\frac{\partial\psi_e}{\partial r} = \frac{\partial\psi_i}{\partial r} = 0 \text{ on } r = a. \tag{18}$$

- The micro-rotation vector on the boundary is assumed to be proportional to the rotation rate of velocity which provides

$$v_\varphi = \frac{\tau}{2r\sin\theta}E^2\psi \text{ on } r = a. \tag{19}$$

- The condition at infinity for uniform stream as

$$\lim_{r\to\infty} \psi_e = \frac{1}{2} U r^2 \sin^2\theta. \tag{20}$$

- The pressure difference across the permeable boundary obeys Darcy's law i.e.

$$\Delta P = \frac{\mu}{k} (\text{Normal filtration velocity}), \tag{21}$$

where $k$ is the permeability ceofficient coefficient of the surface.

### 4.1. Determination of arbitrary constant

By applying the above boundary conditions (17)-(20) we obtain the following linear equations:

$$A_1 + B_1 + C_1\, K_{3/2}(\lambda) - \mathrm{A}_2 - \mathrm{B}_2 - \mathrm{C}_2 I_{3/2}(\lambda) = -1,$$

$$A_1 - B_1 + C_1\left[\lambda K_{1/2}(\lambda) + K_{3/2}(\lambda)\right] = 2,$$

$$2A_2 + 4B_2 + C_2\left[\lambda I_{1/2}(\lambda) - I_{3/2}(\lambda)\right] = 0,$$

$$2B_1(1-\tau) - C_1\lambda^2\left(\frac{2}{N} - \tau\right) K_{3/2}(\lambda) = 0,$$

$$10B_2(1-\tau) + C_2\lambda^2\left(\frac{2}{N} - \tau\right) I_{3/2}(\lambda) = 0,$$

The above five linear equations involve six arbitrary constants. Consequently, the unknown constants $A_1, B_1, C_1, \mathrm{B}_2$ and $\mathrm{C}_2$ are expressed in terms of $A_2 = \xi$. On solving above equations, we get

$$A_1 = \frac{1}{2} + \frac{\xi}{2}$$

$$+ \frac{\frac{\xi\epsilon_1}{\lambda}\left(\lambda I_{1/2}(\lambda) - 5I_{3/2}(\lambda)\right)(2N\lambda K_{1/2}(\lambda) - 2N\tau\lambda K_{1/2}(\lambda) + 2NK_{3/2}(\lambda) - 2N\tau K_{3/2}(\lambda) - 2\lambda^2 K_{3/2}(\lambda) + N\tau\lambda^2 K_{3/2}(\lambda))}{\epsilon_2 \nabla_1 \nabla_2}$$

$$+ \frac{2(-3+\xi)(1-\tau)(\frac{1}{2}\lambda K_{1/2}(\lambda) + K_{3/2}(\lambda))}{\epsilon_3} + \frac{2\xi(-\frac{1}{2} I_{3/2}(\lambda) - \frac{2(1-\tau) I_{3/2}(\lambda)(\frac{1}{2}\lambda K_{1/2}(\lambda) + K_{3/2}(\lambda))}{\epsilon_3})}{\epsilon_2},$$

$$B_1 = \frac{\left(-3\lambda I_{1/2}(\lambda) + \xi\lambda I_{1/2}(\lambda) + 3I_{3/2}(\lambda) - 3\xi I_{3/2}(\lambda)\right)(-2\lambda K_{3/2}(\lambda) + N\tau\lambda K_{3/2}(\lambda))}{2\epsilon_2 \nabla_1}$$

$$+ \frac{\xi\epsilon_1\left(\lambda I_{1/2}(\lambda) - 5I_{3/2}(\lambda)\right)(-2\lambda K_{3/2}(\lambda) + N\tau\lambda K_{3/2}(\lambda))}{\epsilon_2 \nabla_1 \nabla_2},$$

$$C_1 = \frac{2\xi N(-1+\tau)\frac{\epsilon_1}{\lambda}\left(\lambda I_{1/2}(\lambda)-5I_{3/2}(\lambda)\right)}{\epsilon_2\nabla_1\nabla_2} + \frac{N(-1+\tau)(-3\lambda I_{1/2}(\lambda)+\xi\lambda I_{1/2}(\lambda)+3I_{3/2}(\lambda)-3\xi I_{3/2}(\lambda))}{\lambda\epsilon_2\nabla_1},$$

$$B_2 = \frac{2\xi\epsilon_1}{\nabla_2}, \quad C_2 = \frac{10\xi N(1-\tau)}{-5N(1-\tau)\epsilon_2+2N\epsilon_1},$$

Where

$$\epsilon_1 = \left(\frac{2}{N}-\tau\right)\lambda^2 I_{3/2}(\lambda), \qquad \epsilon_2 = \lambda I_{1/2}(\lambda) - I_{3/2}(\lambda),$$

$$\epsilon_3 = 2(1-\tau)\lambda K_{1/2}(\lambda) - 2\left(\frac{2}{N}-\tau\right)\lambda^2 K_{3/2}(\lambda),$$

$$\nabla_1 = NK_{1/2}(\lambda) - N\tau K_{1/2}(\lambda) - 2\lambda K_{3/2}(\lambda) + N\tau\lambda K_{3/2}(\lambda),$$

$$\nabla_2 = 10(1-\tau)\left(\lambda I_{1/2}(\lambda) - I_{3/2}(\lambda) - 4\left(\frac{2}{N}-\tau\right)\lambda^2 I_{3/2}(\lambda)\right),$$

We considered $A_2 = \xi$ as the measure of permeability of the boundary surface which reduces the uncertainty in the equation. In the case of $A_2 = \xi = 0$ the stream function for the internal region vanishes identically and this corresponds to the case of an impermeable boundary. Here, we take the problem of permeable sphere for non-zero $\xi$.

## 5. Pressure distribution

Using equation (2) the pressure for the external flow is

$$\frac{\partial p}{\partial r} = \frac{(2-N)}{(1-N)}\frac{B_1}{r^3}\cos\theta, \qquad \frac{\partial p}{\partial \theta} = \frac{(2-N)}{(1-N)}\frac{B_1}{2r^2}\sin\theta \tag{22}$$

and for the internal flow

$$\frac{\partial p}{\partial r} = -5\frac{(2-N)}{(1-N)}B_2\cos\theta, \qquad \frac{\partial p}{\partial \theta} = 5\frac{(2-N)}{(1-N)}B_2 r\sin\theta. \tag{23}$$

Solving Eq. (22) we get the external pressure

$$P_e = -\frac{(2-N)}{(1-N)}\frac{B_1}{2r^2}\cos\theta \tag{24}$$

by Eq. (23) the internal pressure

$$P_i = -\frac{(2-N)}{(1-N)}5B_2 r\cos\theta \tag{25}$$

### 5.1. Bounds for permeability parameter $\xi$

We assume that there is a pressure difference at the surface of the sphere, due to which fluid enters and leaves the surface of sphere. At the boundary, the normal filtration velocity

$$V(\zeta) = -\frac{1}{r^2}\frac{\partial \psi}{\partial \zeta} \tag{26}$$

Bounds of permeability parameter for physically possible flow are determined by Leonov [3], from obvious conditions

$$V < 0,\ \ \Delta P = P_e - P_i > 0 \text{ for } 0 < \theta < \frac{\pi}{2} \text{ and } V > 0, \Delta P < 0 \text{ for } \frac{\pi}{2} < \theta < \pi,$$

using these conditions, we get

$$0 < \xi < \frac{3[5N(-1+\tau)\lambda I_{1/2}(\lambda) + (4\lambda^2 + N(5-5\tau-2\tau\lambda^2)I_{3/2}(\lambda)]K_{3/2}(\lambda)}{5N(-1+\tau)\lambda I_{1/2}(\lambda)K_{3/2}(\lambda) + I_{3/2}(\lambda)[20N(-1+\tau)\lambda K_{1/2}(\lambda) + 3\left(14\lambda^2 - (-5+5\tau+7\tau\lambda^2)N\right)K_{3/2}(\lambda)]}. \tag{27}$$

Putting $\tau = 0$ in equation (27), we obtained

$$0 < \xi < \frac{3[5N\lambda I_{1/2}(\lambda) - (4\lambda^2 + 5N)I_{3/2}(\lambda)K_{3/2}(\lambda)]}{5N\lambda I_{1/2}(\lambda)K_{3/2}(\lambda) + I_{3/2}(\lambda)[20N\lambda K_{1/2}(\lambda) - 3\left(14\lambda^2 + 5N\right)K_{3/2}(\lambda)]} \tag{28}$$

this is calculated by Aparna and Murthy [21].

## 6. Evaluation of drag force

In order to determine the hydrodynamic drag force $D$ acting on the sphere, which is directed along the symmetrical axis, we have used the formula

$$D = \pi \int \bar{A}^3 \frac{\partial}{\partial r}\left(\frac{(\mu + \kappa)E^2\psi + \kappa v_\varphi}{\bar{A}^2}\right) r d\theta, \tag{29}$$

where $\bar{A} = r\sin\theta$. Using the equation (13) to (16), the drag on the permeable sphere due to external flow

$$D = 2\pi U a(\,2\mu + \kappa)B_1 = 2\pi\mu U a\frac{(2-N)}{(1-N)}B_1. \tag{30}$$

The non-dimensional drag is defined as

$$D_N = \frac{D}{-6\pi\mu U a} \tag{31}$$

## 7. Result and discussion

At the outset, it is instructive to consider some limiting situations of the drag force as discussed below:

(a) **Drag for no-spin on the boundary**

It is interesting to know that by putting $\tau = 0$ in the calculated drag force (30), we get

$$D = \frac{2(-2+N)\lambda\mu U\pi a(5(-3+d)N\lambda I_{1/2}(\lambda)+(-15(-1+d)N-2(-6+d)\lambda^2)I_{3/2}(\lambda))K_{3/2}(\lambda)}{(-1+N)\left(5N\lambda I_{1/2}(\lambda)-(5N+4\lambda^2)I_{3/2}(\lambda)\right)(NK_{1/2}(\lambda)-2\lambda K_{3/2}(\lambda))}, \quad (32)$$

this result is same as earlier reported by Aparna and Murthy [21].

(b) **Case of viscous fluid**

By putting $N = 0, \tau = 0$ and $\lambda = \infty$ on the constants $A_1, B_1, C_1, B_2, C_2$ for viscous fluid, the stream functions are obtained for external and internal flow, as given below

$$\psi_e = \left(r^2 + \frac{(2+\tau)}{4r} + \frac{(\tau-6)}{4}\right)G_2(\zeta) \quad (33)$$

$$\psi_i = -\frac{1}{2}(r^4 - 2r^2)\tau G_2(\zeta). \quad (34)$$

As $\tau = 0$, $\lambda = \infty$ the right hand side of equality (27) tends to 2/7 this leads to $0 < \xi < {}^{2}/_{7}$, these results are all in complete agreement with Leonov [3].

(c) **Micropolar polar fluid past a solid sphere**

Substituting $\xi = 0$ in equation (32) and we get

$$D = \frac{6\,\pi\, Ua\,\lambda\,(\kappa+2\mu)(\kappa+\mu)K_{3/2}(\lambda)}{-\kappa K_{1/2}(\lambda) + 2\lambda(\kappa+\mu)K_{3/2}(\lambda)}, \quad (35)$$

this is well known result has been reported by Ramkisoon and Majumdar [12]. The non-dimensional stream function $\psi$ at the surface is given by

$$\psi(1,0) = \frac{\left\{1 + A_1 + B_1 + C_1 K_{3/2}(\lambda)\right\}}{2} \quad (36)$$

The normal velocity on the surface of the sphere decreases as $\theta$ increase for 0 to $\frac{\pi}{2}$ and at $\theta = \frac{\pi}{2}$ the normal velocity is zero. On the axis $\theta = 0$ the normal velocity is given by

$$V = U\left\{\tau + B_2 + C_2 I_{3/2}(\lambda)\right\} = \tau U\{1 + A_1 + B_1 + C_1 K_{3/2}(\lambda). \qquad (37)$$

This velocity is less than the velocity $U$ of the fluid at infinity.

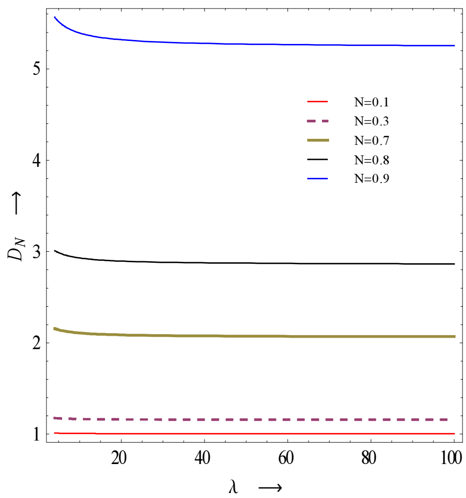


Fig. 2. Variation in $D_N$ w.r.t. $\lambda$

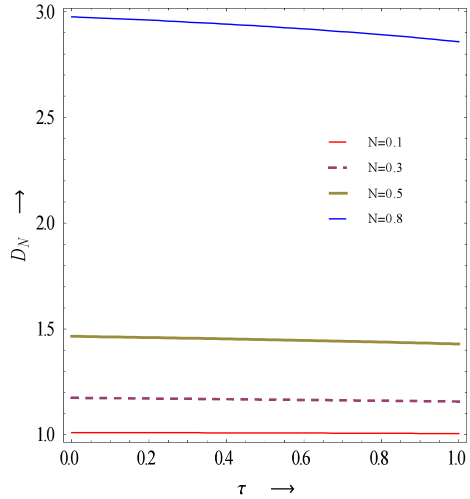


Fig. 3. Dependence of drag w.r.t. $\tau$

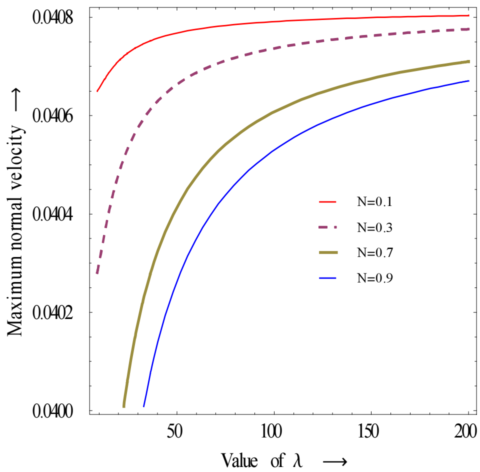


Fig. 4. Variation of $V$ w.r.t. $\lambda$

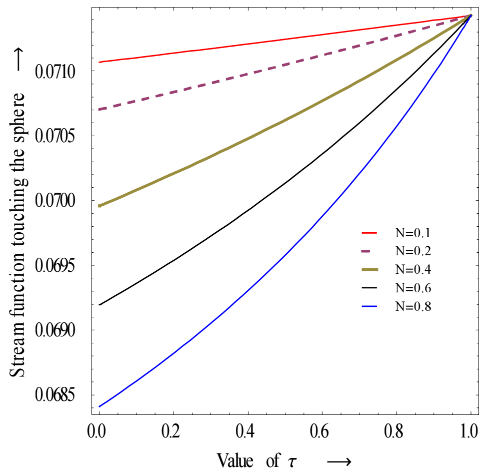


Fig. 5. Dependence of $\psi(1,0)$ w.r.t. $\tau$

The variation in non-dimensional drag $D_N$ with respect to micropolar parameter $\lambda$ is shown in Figure 2. It is clear from the figure that firstly $D_N$ decreases at the small values of $\lambda$ and then drag force reaches almost constant for fix value of $\tau = 0.5$ and $\xi = 2/7$ and $D_N$ approaches unity as $N$ decrease to zero.

Variation in the non-dimensional drag $D_N$ against spin parameter $\tau$ (Fig. 3) shows that the $D_N$ decreases with the increase in $\tau$ for $\lambda = 10$, $\xi = 2/7$ and various value of coupling number $N$. The microrotation vector is zero when spin parameter $\tau = 0$, in this case the drag is maximum as shown in Figure 3. This physically shows that the drag is greater in the case of zero microrotation vector than in the case of non-zero microrotation vector.

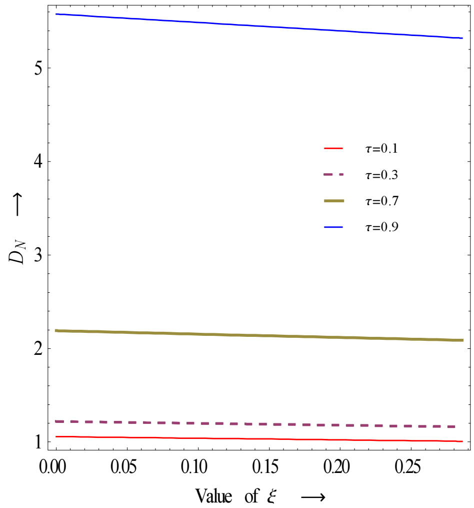


Fig. 6. The variation of $D_N$ versus $\xi$

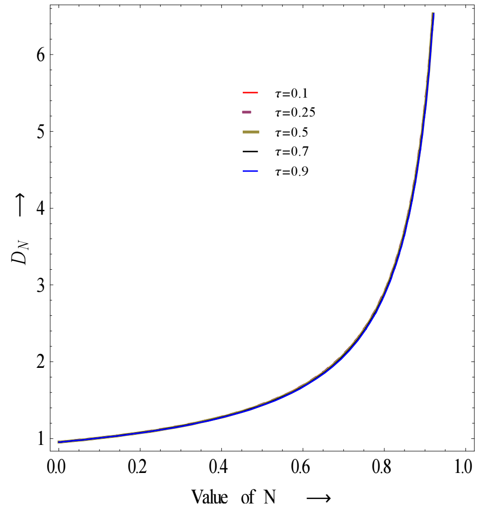


Fig. 7. The variation of $D_N$ versus $N$

Figure 4 represent the variation of non-dimensional velocity $V$ on the sphere on the axis ($\theta = 0$) with respect to micropolar parameter $\lambda$. It is evident as $\lambda$ increases the normal filtration velocity increases rapidly.

The variation between the stream function touching the sphere with regard to value of spin parameter $\tau$ is shown in Figure 5. Represented that stream function increases with increasing value $\tau$ for the fixed value of $\lambda = 20$ and $\xi = 2/7$.

The effect of permeability parameter $\xi$ on the non-dimensional drag $D_N$ is shown in Figure 6. It is evident that for different value of spin parameter $\tau$, drag decreases continiously as increases the value of $\xi$ where $\lambda = 20$, $N = 0.5$ are the fixed value. When permeability parameter $\xi = 0$, the permeable sphere become impermeable. Figure 6 shows that drag is maximum for impermeable sphere.

The variation of $D_N$ with respect to the coupling number $N$ for different values of the spin paeameter $\tau$ shown in Figure 7. It is clear that for $N < 0.7$ correspond to a weak drag and increases rapidly for $N > 0.7$ for fix value of $\lambda = 20$ and $\xi = 2/7$. From Figure 7, it is observed that drag is greater in case of micropolar fluid than that of newtonian fluid.

## 8. Conclusion

The stream function solution to the flow field equation for steady axisymetric creeping flow of micropolar fluid around a permeable sphere are obtained. Different useful results are obtained from the solution, particularly the closed expression for the drag force and the dependence of the drag coefficient on various fluid parameters. It is found that an increase in the spin parameter decreases the drag force experienced by permeable sphere. As the value of permeability parameter increases, the drag force experienced by the sphere decreases continously and, with the increasing of the coupling number, the drag force also increases. The volumetric rate increases with the increasing spin parameter. Maximum normal velocity on the micropolar parameter is also studied. It is observed that the drag is greater in the case of zero microrotation vector than in the case of non-zero microrotation vector.

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