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STRESS OCCURRING IN THE FRICTION NODE OF ELEMENTS IN THE TOTAL KNEE ENDOPROSTHESIS

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Abstract. The paper presents the numerical analysis of stress occurring in the friction node of bi-condylar knee endoprostheses. The Finite Elements Method allows one to calculate and present the stress distribution in all elements of the computing model.

Keywords: *endoprosthesis, stress, FEM*

1. Introduction

The main objective of the following paper is to present the analysis of stress distribution occurring on the contact surface between sleds and flat polyethylene insert in bi-condylar knee joint endoprosthesis. This analysis was conducted by means of numerical calculations in ADINA System 8.6. Its aim was to find an optimal shape of a sled that would make the stress distribution on the insert surface as favorable as possible. The model cannot interrupt the movement biomechanics of the joint.

Such strict criteria make it very difficult to find a proper sled shape. It is important to apply proper calculating methods, like the finite elements method. All numerical analyses allow one to define and draw the map of stress for each computing variant.

2. Stress analysis in the pair sled - flat insert for bi-condylar endoprosthesis by W.LINK

The calculations were conducted for a total, bi-condylar knee endoprosthesis. It shows reduced stress and contact stress occurring in all types of polyethylene inserts. UHMWPE polyethylene is the weakest point of the endoprosthesis, which is why it is important to present the reduced stress distribution in the inserts.

The formulas presented below have been applied in the analysis; formula (1) presents a short notation of physical equations connecting together the values of stress and strain in the 3D case, in material, isotropic, linear - elastic bodies [1, 2].

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} \tag{1}$$

Applying material constant:
E - Young's modulus (elasticity)
G - Kirchhoff's modulus (non-dilatational strain)
ν - Poisson's coefficient

$$\mu = G = \frac{E}{2(1+\nu)} \tag{2}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \tag{3}$$

After displacing (1) with (2) and (3), we get physical formulas as follows:

$$\sigma_{ij} = 2G\varepsilon_{ij} + \frac{2G\nu}{(1-2\nu)}\varepsilon_{kk}\delta_{ij} = 2G\left[\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right] = \frac{E}{1+\nu}\left[\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right] \tag{4}$$

After displacing *i*, *j*, *k* with 1, 2, 3, we get:

$$\sigma_{11} = \frac{E}{1+\nu}\left[\varepsilon_{11} + \frac{\nu}{1-2\nu}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})\right] \quad \sigma_{12} = \frac{E}{1+\nu}\varepsilon_{12} = 2G\varepsilon_{12} \tag{5}$$

$$\sigma_{22} = \frac{E}{1+\nu}\left[\varepsilon_{22} + \frac{\nu}{1-2\nu}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})\right] \quad \sigma_{13} = \frac{E}{1+\nu}\varepsilon_{13} = 2G\varepsilon_{13} \tag{6}$$

$$\sigma_{33} = \frac{E}{1+\nu}\left[\varepsilon_{33} + \frac{\nu}{1-2\nu}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})\right] \quad \sigma_{23} = \frac{E}{1+\nu}\varepsilon_{23} = 2G\varepsilon_{23} \tag{7}$$

3. The shape of contact surfaces between elements of bi-condylar knee joint endoprosthesis

There are four types of bi-condylar knee joint endoprostheses as far as the shape of polyethylene insert is concerned. Figure 1 presents the shapes of contact surfaces between elements of knee joint endoprosthesis.

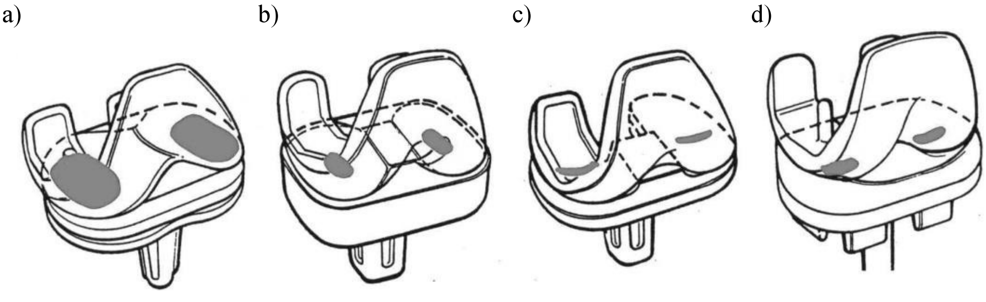

Fig. 1. Shapes of contact surfaces between elements of knee joint endoprosthesis: a) surface contact, b) point contact, c) linear contact, d) quasi-linear contact [3-5]

The durability of the endoprosthesis depends on mechanical and tribological features of its weakest element, which is the polyethylene insert.

4. Bi-condylar total knee endoprosthesis by W.LINK

Figure 2 presents the product by W.LINK, which is an example of bi-condylar knee joint endoprosthesis. The femoral part copies the shape of condyles of femur bones. The femoral part is precisely fixed with two pins into the bone.

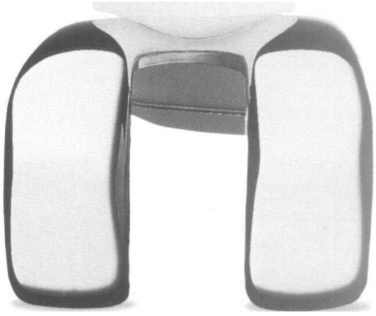

Fig. 2. Bi-condylar knee joint endoprosthesis by W.LINK [6]

The materials used in the above endoprostheses are:

- CoCrMo alloy - femoral and tibial part,
- UHMWPE polyethylene - the insert.

That is the most commonly used pair of materials for knee joint endoprostheses. In some cases, the titanium alloy is used as well due to its high mechanical features and because it is lighter than CoCrMo alloy.

5. Numerical model of knee joint endoprostheses

When designing a simplified numerical model of endoprosthesis, it is important to draw a geometry of the endoprosthesis as true and similar to the real knee joint as possible, not only as far as the shape is concerned, but also considering the anatomical range of movements.

5.1. Criteria taken for numerical calculations

The contact stress occurs between two elements pressed together with force. They take place in certain areas and can reach quite high values even at a respectively low value clamp.

Theoretical criteria for contact stress according to *Hertz's theory* have been applied in the form as follows:

- Contacting elements are made of homogeneous isotropic materials.
- The surfaces are fixed in the contact area of the element with smooth and regularly curved surfaces.
- When subjected to load there are only slight strains in the contact area.
- The contact area is relatively small when compared with the surfaces of the contacting elements.
- On the contact area there only normal strains occur.

5.2. Numerical model of bi-condylar endoprosthesis

The numerical model is a simplified version of the original endoprosthesis, though the sleds' geometry has been maintained. That enables us to keep the general shape of endoprosthesis and to quite closely imagine the strain distribution on the insert's surface. Figure 3 presents the simplified sled model.

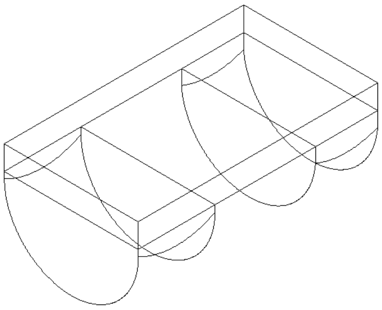

Fig. 3. Simplified sled model used for calculations

The flat polyethylene insert in the model has got the shape adjusted to the real anatomical tibial bone.

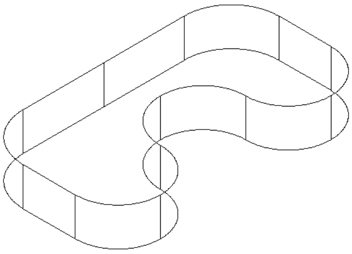

Fig. 4. Simplified model of the polyethylene insert

The main objective of these calculations is to present the strain distribution on, and right underneath the surface of the insert, while using various geometrical options of both elements. Considering the polyethylene insert, it will be its thickness; and considering the sled - it will be its cross-section radius. Furthermore, the sleds might be of a different thickness which might provide us with additional information about the tested contact surfaces. The main dimensions of the sleds are shown in Figure 5.

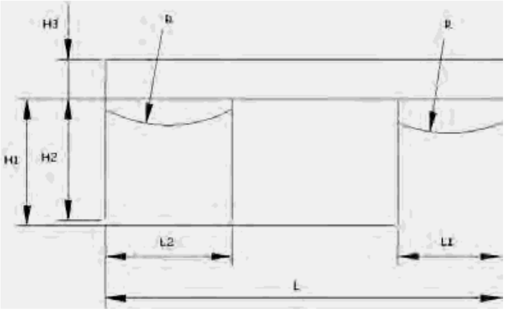

Fig. 5. Geometry of the femoral element of the endoprosthesis. Front view

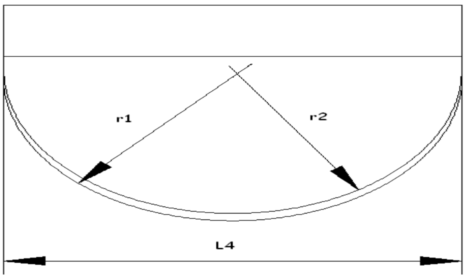

Fig. 6. Geometry of the sled of knee joint endoprosthesis. Side view

Figures 5 and 6 present the geometry of a sled with the bonding element.
Constant geometrical dimensions:
L = 50 mm, L1 = 13 mm, L2 = 16 mm, H3 = 5 mm
Radiuses r1 and r2 are identical and have 14 mm
Calculations were conducted for three different cross-section radiuses of the sleds: R1 = 17 mm, R2 = 26 mm, R3 = 35 mm. Dimensions of the polyethylene insert are presented in Figure 7.

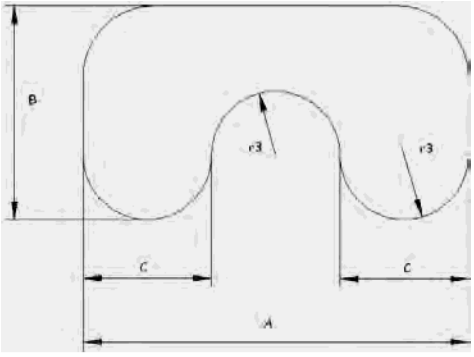

Fig. 7. Geometry and dimensions of polyethylene insert

The polyethylene insert's dimensions are as follows:
A = 54 mm, B = 30 mm, C = 18 mm, r3 = 9 mm
The most significant is the thickness of the insert = G:
G1 = 8 mm, G2 = 13 mm, G3 = 22 mm
Numerical calculations have been conducted for 9 computing variants, using 3 thicknesses of polyethylene inserts cooperating with 3 different sleds. Each pair will be subjected to load F1 = 1500 N.
Table 1 presents computing variants together with models' marks:

Table 1

Computing variants of the analysed models

force F1 = 1500 N			
	sled R1	sled R2	sled R3
insert G1	GR111	GR112	GR113
insert G2	GR121	GR122	GR123
insert G3	GR131	GR132	GR133

Simulations for those loads were conducted with the following physical features of the materials, placed in Table 2.

Table 2

Mechanical features and density of materials used for endoprostheses [7]

	Young's modulus E [MPa]	Poisson's coefficient ν	Density ρ [kg/m^3]
CoCrMo	$2.1 \cdot 10^5$	0.29	8300
UHMWPE	1000	0.4	960

6. The results of numerical calculation conducted with finite elements method and ADINA System 8.6.

All the calculations proved that the stresses in endoprosthesis are concentrated right underneath the contact surfaces of both elements, particularly underneath the insert's surface [6]. Figures 8 and 9 present exemplary results of numerical calculations.

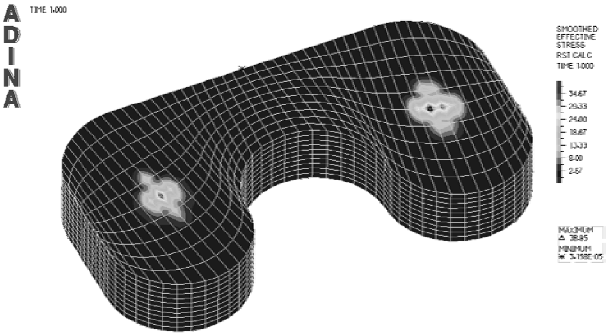

Fig. 8. Model GR111 - reduced stress distribution on the surface of the polyethylene insert

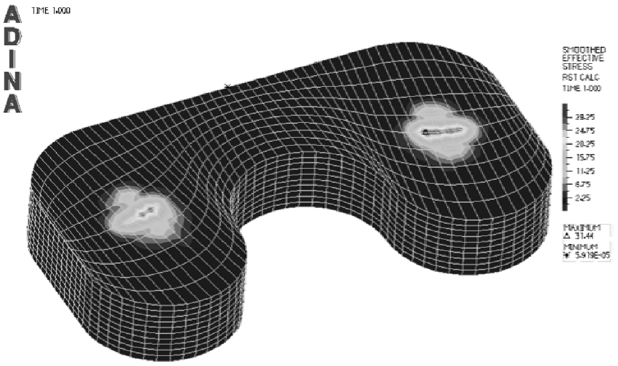

Fig. 9. Model GR113 - reduced stress distribution on the insert's surface

7. Conclusions

The lowest values of reduced strain have been gained with the computing variant: GR123 - 28,91 MPa and GR133 - 26,85 MPa where the cross-section radius of the sled was 35 mm. The highest values of the reduced strain occurred in models GR111 - 38,85 MPa and GR112 - 36,93 MPa where the sled's radius was the smallest and = 17 mm. Figure 10 presents the influence of the sled's cross-section radius and insert's thickness on the value of generated stress in the flat polyethylene insert.

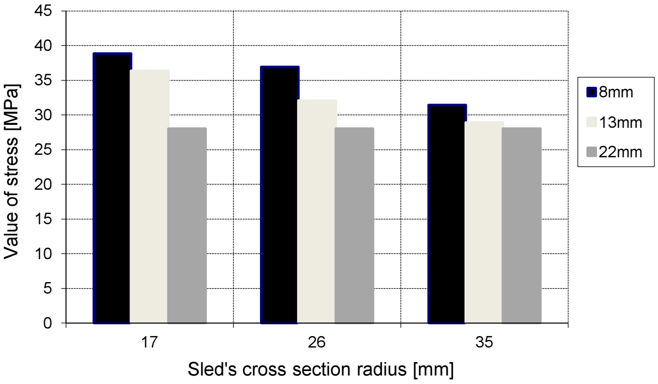

Fig. 10. The influence of the sled's cross-section radius and insert's thickness on the value of generated stress in the flat UHMWPE insert

1. The key issue is the geometry of the contact between both elements of endoprosthesis and the value of the cross-section radius of the sled, which influences the values of generated strains.
2. The knee joint endoprosthesis sled with the cross-section radius of 35 mm assures optimal the contact strain distribution in the polyethylene insert. Unfortunately,

this geometry of the pair: sled - insert, decreases the mobility of the knee after implantation.

3. The thickness of the insert slightly influences the values of strains and displacement in it.
4. In all the considered numerical variants the stress values generated by the sleds have not crossed the yield point of UHMWPE, and the prevalent issue here are the tribological wear processes.
5. Numerical calculations were conducted in order to define the optimal geometry of the friction node of the knee joint endoprosthesis. To design a knee joint endoprosthesis of the best construction, it is important to complete all numerical analysis with empirical research on prototype of endoprosthesis conducted on a knee joint simulator.

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