SIMULATIONS OF THERMAL PROCESSES
IN A RESTORED TOOTH

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Abstract. In this paper the analysis of the temperature distribution in restored tooth is considered. The aim of the research is to evaluate the impact of temperature changes in the pulp layer of tooth under the influence of rapid temperature changes in the ambient fluid. The axially-symmetrical model of the molar tooth (in vitro) without filling and with fillings (amalgam and composite resin) is analyzed. The equations of mathematical model are solved numerically using the control volume method. Finally, some examples of simulations are presented.

Keywords: restored tooth, thermal process, numerical solution

Introduction

Restorative materials [1] are often used to obturate dental cavities. The filled tooth may be sensitive to heat and cold. Temperature fluctuations occur in the oral cavity due to the ingestion of cold or hot food and drink. The tooth has many layers [2]: enamel (the outer layer is made of hard crystal), dentin (the layer is the bone-like substance) and pulp (in the middle of the tooth). The dental pulp contains nerves (also blood vessels) and it is very sensitive to temperature change. The pulp in the healthy tooth is surrounded by enamel and dentine, which are good thermal insulators.

Dental fillings can be made from many types of material [1, 3]. The amalgam fillings (a mixture of mercury, silver, tin, or other metals) is the easiest and least costly material for a dentist to use. The composite resin fillings (a mixture of powdered glass and plastic resin) can look similar to the appearance of the natural tooth (white fillings). These fillings are usually more expensive than the amalgam fillings. Materials used to restore teeth should offer a similar degree of insulation. Additionally, these materials should not produce a large temperature rise when setting in cavities - a similar problem regarding to exothermic photopolymerization reaction was considered in [4].

In this paper the research related to determine the temperature distribution in the tooth are considered. The axially-symmetrical model of the molar tooth (in vitro)

is placed in the external environment (in vitro), i.e. the tooth crown is in contact with the surrounding liquid of variable temperature, and the tooth root is isolated. On the base of these assumptions the mathematical model for the thermal process in the tooth without fillings and with fillings is developed. The aim of the research is to determine the temperature changes in the pulp layer of the tooth with different filling materials under the influence of the same boundary conditions.

1. Mathematical model

The tooth cylindrical domain considered in Figure 1 is presented. The domain consists of many sub-domains: the enamel (Ω_1), the dentin (Ω_2) and the pulp (Ω_3). In the case of the tooth model with restoration, the additional sub-domain (Ω_4) is taken into account. On the outer surface limiting the domain (Γ_{b1} and Γ_{b4}), a fluid having a certain temperature and playing a role of an external chill or warm is assumed. The boundary Γ_0 represents the contact with the gum.
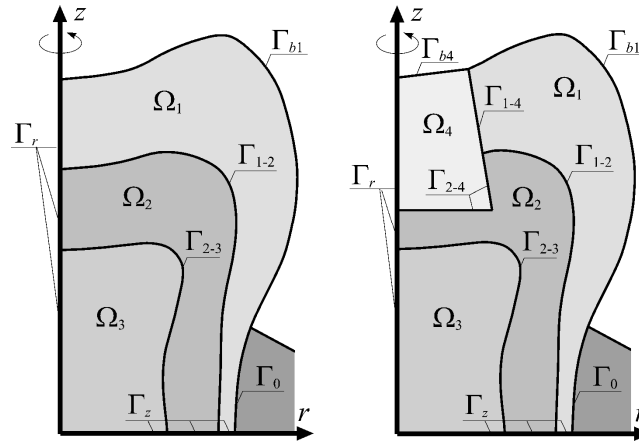

Fig. 1. The tooth domain considered (the cylindrical cross-sections):
without restoration (left-side) and with restoration (right-side)

The temperature field in the system of each sub-domain is described by the system of equations

$$c_m \rho_m \frac{\partial T_m(r,z,t)}{\partial t} = \lambda_m \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_m(r,z,t)}{\partial r} \right) + \frac{\partial^2 T_m(r,z,t)}{\partial z^2} \right), \quad m = 1,2,3,4 \quad (1)$$

where index m identifies the particular sub-domains (1 - the enamel, 2 - the dentin, 3 - the pulp, 4 - the dental restoration material), T [°C] is the temperature, r, z [m], t [s] denote spatial coordinates and time, c [J/(kg °C)], ρ [kg/m^3], λ [W/(m °C)] are the specific heat, the density and the thermal conductivity of materials considered, respectively.

The Eq. (1) is supplemented by the following initial-boundary conditions. For $t = 0$ the initial condition is known

$$T_m\left(r,z,t\right)\big|_{t=0} = T_{init}, \quad m = 1,2,3,4 \tag{2}$$

On the contact surfaces between sub-domains, the IV type of boundary conditions are assumed

$$\Gamma_{k-l}: \begin{cases} -\lambda_k \dfrac{\partial T_k\left(r,z,t\right)}{\partial n} = -\lambda_l \dfrac{\partial T_l\left(r,z,t\right)}{\partial n}, & (k,l) \in \left\{(1,2),(2,3),(1,4),(2,4)\right\} \\ T_k\left(r,z,t\right) = T_l\left(r,z,t\right) \end{cases} \tag{3}$$

where $\partial/\partial n$ is a normal derivative. On the external surface of the sub-domains Ω_1 and Ω_4, the II type (on the surface of the tooth root - Γ_0) and III type (on the surface of the tooth crown - Γ_{1m} and Γ_{4m}) of boundary conditions are given

$$\Gamma_{bm}: -\lambda_m \dfrac{\partial T_m\left(r,z,t\right)}{\partial n} = -\alpha\left(T_{amb}\left(t\right) - T_m\left(r,z,t\right)\right), \quad m = 1,4 \tag{4}$$

$$\Gamma_0: \dfrac{\partial T_1\left(r,z,t\right)}{\partial n} = 0 \tag{5}$$

where α [W/(m^2 °C)] is the convective heat transfer coefficient and T_{amb} [°C] is the temperature of the ambient fluid. In the mathematical model the following simplifications are assumed: the temperature of fluid declines or grows according to a given function, the heat transfer coefficient is treatment as a constant (depends on many parameters: especially the fluid velocity, the surface geometry, the nature of motion, etc.). While on the boundaries Γ_r and Γ_z, the non-flux boundary conditions are given in the form

$$\Gamma_r: \dfrac{\partial T_m\left(r,z,t\right)}{\partial r}\bigg|_{r=0} = 0, \quad m = 1,2,3,4 \tag{6}$$

$$\Gamma_z: \dfrac{\partial T_m\left(r,z,t\right)}{\partial z}\bigg|_{z=0} = 0, \quad m = 1,2,3 \tag{7}$$

2. Examples of computations

Three numerical simulations of a thermal process for the tooth without restoration and with two different dental materials restoration have been executed.
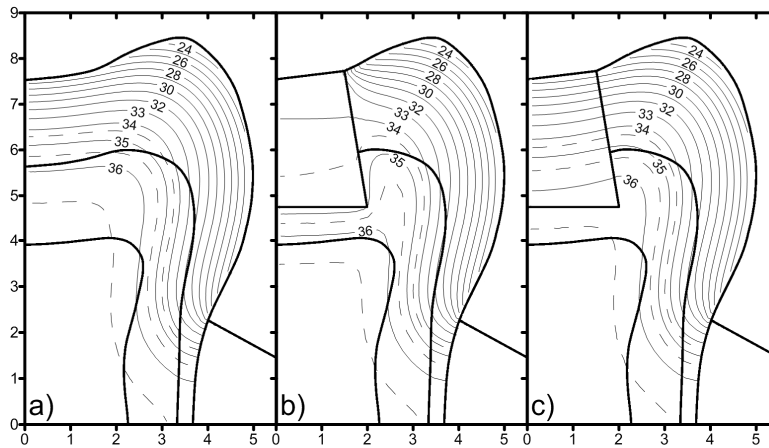
Fig. 3. Temperature distribution in sub-domains at time 2 s: a) tooth without restoration,
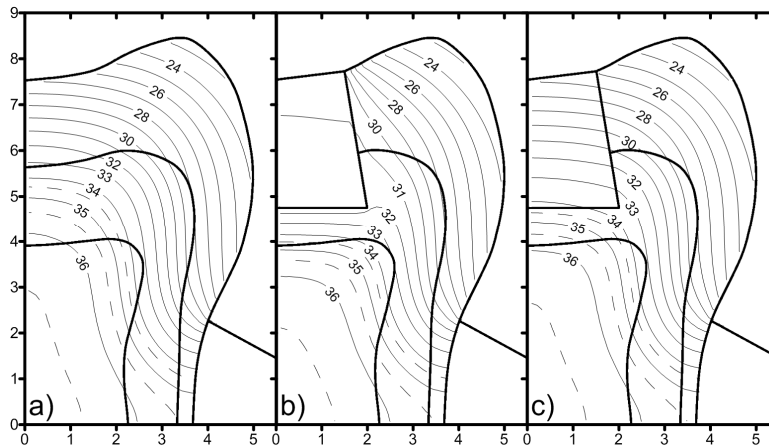b) amalgam restoration, c) composite material restoration

Fig. 4. Temperature distribution in sub-domains at time 5 s: a) tooth without restoration,
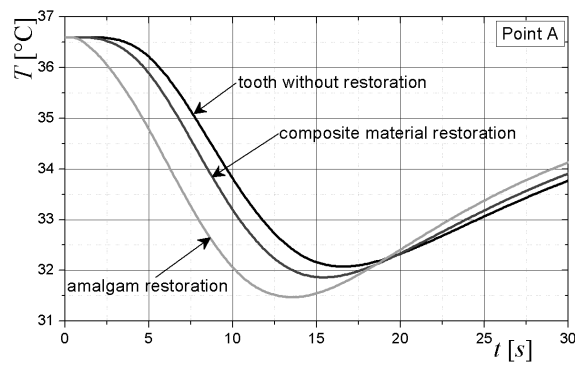b) amalgam restoration, c) composite material restoration

Fig. 5. The kinetics of temperature variation at selected point A (see Fig. 2)

Conclusions

The control volume meshes used in numerical simulations quite accurately determine the geometry of the cylindrical section of the tooth model. Analyzing the kinetics of temperature variation for each type of filling one can note that the amalgam restoration conducts heat very quickly and causes the fastest decrease of temperature in the pulp. In the case of the composite material restoration in the tooth the temperature distribution is similar to the temperature distribution in the tooth without restoration.

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