Journal of Applied Mathematics and Computational Mechanics 2013, 12(4), 13-21

THE INFLUENCE OF THERMAL ACTIONS AND COMPLEX SUPPORT CONDITIONS ON THE MECHANICAL STATE OF SANDWICH STRUCTURE

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Abstract. The paper concerns the problem of the influence of thermal actions on the structural behavior of sandwich panels with unspecified elastic supports. An ordinary sandwich panel theory is used. The boundary conditions have the arbitrary form of elastic supports. The solution of a statically undetermined system with limitations of horizontal displacement and rotation is derived. The illustrative examples are presented and the problem solution is discussed.

Keywords: *sandwich panels, thermal actions, boundary conditions, elastic supports*

Introduction

Due to their specific structure, sandwich panels have good load capacity at low deadweight and excellent thermal insulation. The separation of rigid faces by the thick and light core results in the significant role of thermal actions. These actions bring on displacements, strains and stresses in the structure. This applies in particular to the statically undetermined systems. The influence of thermal actions in this case is usually greater than other impacts.

In the generally accepted theory [1-3], the behavior of the sandwich beam can be described by ordinary differential equations and relatively simple boundary conditions. In fact, sandwich beams and plates are spatial systems with complex support conditions. These conditions are usually different at the upper and lower faces of sandwich. For some time there has been discussion on the accuracy of the theoretical assumptions concerning the boundary conditions. Many controversies surround the effect of admission or restriction of the horizontal displacement of the structure. Complex boundary conditions can be taken into account by using higher-order global-local theories [4-6], but their application usually requires sophisticated computational tools.

The issue of taking into account appropriate boundary conditions is particularly important in the case of thermal actions that have the nature of distortions [7] and reach the extreme values in case of fire. The restraint of the structure deformations causes internal forces and stresses [8]. It was shown in [9, 10] that the structural response can be very sensitive due to the variation of support conditions. Also geometrical shape variations of the structure can have an influence on the load capacity and displacements of sandwich panels [11].

The paper presents the proposal of a refined classical sandwich beam theory, which allows for the easy definition of various boundary conditions. The solution of the thermally loaded undetermined system with arbitrary elastic supports is derived. The problem of the restriction of the horizontal displacement of the structure is in focus. The parametric studies of the influence of different boundary conditions on the internal forces, stresses and deformation of the sandwich structure are presented and discussed.

1. Formulation of the problem

Consider sandwich beam (1D structure) with flat facings and a thick and soft core. In the case of the beam subjected to distributed transverse load q and initial curvature θ, the classical constitutive equations have the form of two differential equations:

$$M = B_S \cdot (\gamma' - w'' - \theta)\,, \tag{1}$$

$$V = G_C A_C \cdot \gamma\,. \tag{2}$$

The curvature θ is induced by the difference $T_2 - T_1$ between the temperatures in lower and upper sandwich faces. The vertical displacement w, shear strain γ, bending moment M and shear force V are functions of the position coordinate x (x, z - coordinates, u, w - displacements). The transverse load q and the curvature θ can also be functions of x. The G_C and A_C denote the shear modulus and the cross-sectional area of the core. The term B_S represents the bending stiffness of the facings with respect to the global center line of the sandwich panel. In the case of flat and thin faces B_S expresses total bending stiffness.

The only variable in the classical theory is the position coordinate x. The typical boundary conditions refer to vertical displacements, rotations caused by bending or internal forces:

a) simply support - vertical displacement and bending moment at the support (point x_0) equal to zero:

$$w(x_0) = 0, \qquad w''(x_0) - \gamma'(x_0) = 0\,, \tag{3}$$

b) clamping - vertical displacement and rotation caused by bending equal to zero:

$$w(x_0) = 0, \qquad w'(x_0) - \gamma(x_0) = 0\,, \tag{4}$$

c) free end:

$$w'''(x_0) = 0, \qquad w''(x_0) = 0 \,. \tag{5}$$

The boundary conditions (3)-(5) are idealized. In practice, there are elastic supports, which allow the displacements proportional to the forces occurring in the supports. This also applies to horizontal displacements. In general, the horizontal support conditions for the two sandwich faces tend to be independent and different.

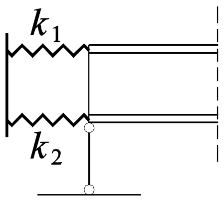


Fig. 1. The scheme of the horizontal elastic supports independently imposed on two rigid sandwich faces; k_1, k_2 - modules of support elasticity

The method of taking into account elastic vertical supports was extensively discussed in [9]. This paper focuses on the problem of the restriction of the horizontal displacement of the structure. The horizontal supporting is assumed as elastic (displacement linearly proportional to the force) and independently imposed on two rigid sandwich faces. The considered type of the support is schematically shown in Figure 1.

2. The solution of the thermally loaded system with elastic supports

Consider the symmetric sandwich panel that is in the initial temperature T_0. The core is not sensitive to temperature change. An increase or decrease in temperature causes a linearly proportional expansion or contraction of sandwich faces, respectively. The size of the faces, expansion or contraction determines the thermal expansion coefficient α of the material of faces. As a result of thermal effects, the temperature of the upper face changed to T_1, whereas the lower face temperature changed to T_2. Any change in temperature can be expressed as the sum of the two effects: uniform change ΔT_N and uneven change ΔT_M:

$$\Delta T_N = \frac{T_1 + T_2}{2} - T_0 \,, \tag{6}$$

$$\Delta T_M = T_2 - T_1 \,. \tag{7}$$

Uniform increase/decrease of temperature in the case of a (thermally) homogeneous or symmetrically non-homogeneous structure is responsible for the expansion/contraction ΔL of the structural element of the original length L:

$$\frac{\Delta L}{L} = \alpha \, \Delta T_N \, . \tag{8}$$

Uneven temperature change causes the curvature of the structural element:

$$\theta = \alpha \frac{\Delta T_M}{e} \, , \tag{9}$$

where e denotes the distance between centroids of sandwich faces (see Fig. 2). Deformation of the free element subjected to thermal excitation is presented in Figure 2. When the deformation of the structure is limited, internal forces appear.

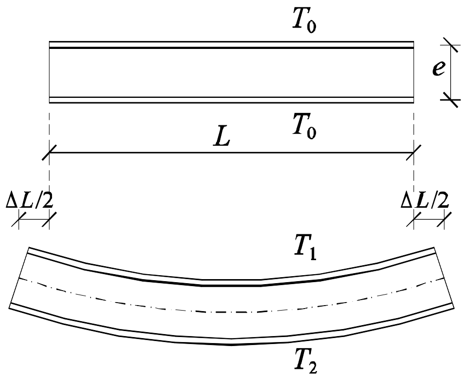


Fig. 2. Deformation of the free element subjected to thermal excitation

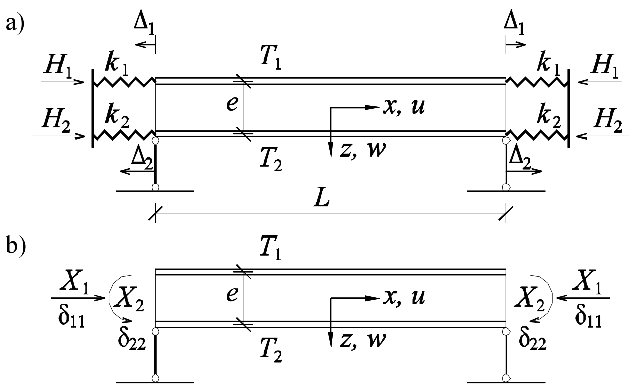


Fig. 3. The one-span basic system with the restriction of the horizontal displacement: a) boundary conditions, b) unknown support forces and displacements

Consider the one-span basic system presented in Figure 3a with the restriction of the horizontal displacement. The structure is symmetric and the sandwich faces have the same thickness. The elasticity of the horizontal supports may take any value. It is assumed that upper and lower horizontal supports have the elasticity coefficients k_1 and k_2, respectively. The structure is subjected to a temperature change - the initial temperature was T_0, the current temperature is T_1 in the upper face and T_2 in the lower face. The thermal action and the restriction of the horizontal displacement cause the horizontal support forces H_1 and H_2. The support forces, as also the internal forces and structure deformations, depend on the geometrical and mechanical parameters of the system, thermal actions and support stiffness. Unknown horizontal displacements at the upper and lower supports are denoted as Δ_1 and Δ_2, respectively. The unknown forces H_1 and H_2 can be represented by the unknown axial force X_1 and bending moment X_2 (see Fig. 3b):

$$H_1 = \frac{X_1}{2} - \frac{X_2}{e}, \tag{10}$$

$$H_2 = \frac{X_1}{2} + \frac{X_2}{e}. \tag{11}$$

From the other point of view, the forces H_1 and H_2 are proportional to the respective support stiffness k and displacement Δ. The final displacements Δ_1, Δ_2 depend on the forces X_1, X_2 and temperature changes ΔT_N, ΔT_M:

$$H_1 = k_1\Delta_1 = k_1\left[-X_1\delta_{11} + \delta_{1N} + \frac{e}{2}\left(X_2\delta_{22} - \delta_{2M}\right)\right], \tag{12}$$

$$H_2 = k_2\Delta_2 = k_2\left[-X_1\delta_{11} + \delta_{1N} + \frac{e}{2}\left(-X_2\delta_{22} + \delta_{2M}\right)\right]. \tag{13}$$

The following terms denote displacements at the end support:
δ_{11} - the horizontal displacement of the structure induced by the virtual force $\overline{X}_1 = 1$,
δ_{22} - the rotation of the structure induced by the virtual moment $\overline{X}_2 = 1$,
δ_{1N} - the horizontal displacement of the free structure induced by ΔT_N,
δ_{2M} - the rotation of the free structure induced by ΔT_M.
In the case of the structure presented in Figure 3a, the horizontal displacements and rotations have the form:

$$\delta_{11} = \frac{1}{E_{F1}A_{F1} + E_{F2}A_{F2}} \cdot \frac{L}{2} = \frac{1}{D_F} \cdot \frac{L}{2}, \tag{14}$$

$$\delta_{22} = \frac{1}{B_S} \cdot \frac{L}{2}, \tag{15}$$

$$\delta_{1N} = \alpha\,\Delta T_N \cdot \frac{L}{2}, \tag{16}$$

$$\delta_{2M} = \frac{\alpha\,\Delta T_M}{e} \cdot \frac{L}{2}, \tag{17}$$

where the denominator $E_{F1}A_{F1} + E_{F2}A_{F2}$ in (14) represents the axial stiffness of the both sandwich faces (E - Young modulus of the face material, A - the area of the cross-section of the face). The subscripts $F1$, $F2$ refer to the upper and lower face. For the simplicity of the presentation, the denominator in (14) is marked with the symbol D_F.

Comparing the right-hand sides of equations (10)-(11) to (12)-(13), taking into account (14)-(17), we finally arrive at the solution:

$$X_1 = 2\alpha L D_F \frac{2k_2 B_S \Delta T_N + k_2 B_S \Delta T_M + k_1 k_2 e^2 L \Delta T_N + 2k_1 B_S \Delta T_N - k_1 B_S \Delta T_M}{8D_F B_S + 4k_1 L B_S + 4k_2 L B_S + 2k_1 k_2 e^2 L^2 + k_1 e^2 L D_F + k_2 e^2 L D_F}, \quad (18)$$

$$X_2 = e\alpha L B_S \frac{2k_2 D_F \Delta T_N + k_2 D_F \Delta T_M + 2k_2 k_1 L \Delta T_M - 2k_1 D_F \Delta T_N + k_1 D_F \Delta T_M}{8D_F B_S + 4k_1 L B_S + 4k_2 L B_S + 2k_1 k_2 e^2 L^2 + k_1 e^2 L D_F + k_2 e^2 L D_F}. \quad (19)$$

The support forces H_1 and H_2 are determined using (10), (11). The stresses (negative stress denotes compression) in the upper and lower face are:

$$\sigma_{F1} = -\frac{H_1}{A_{F1}}, \quad (20)$$

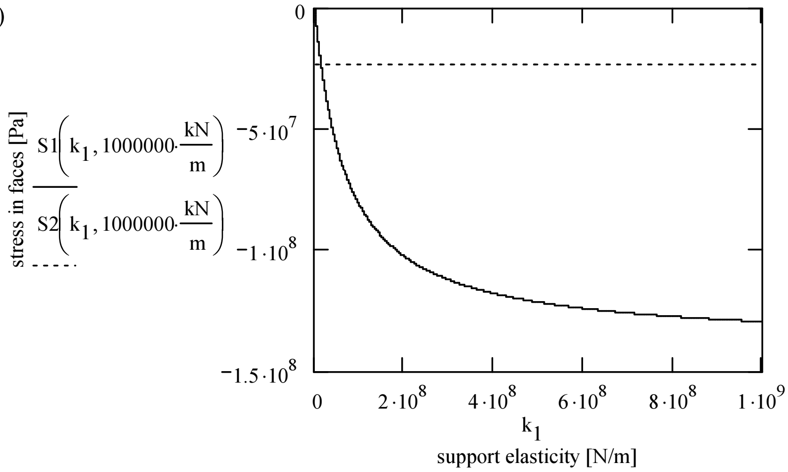
$$\sigma_{F2} = -\frac{H_2}{A_{F2}}. \quad (21)$$

3. The influence of the restriction of the horizontal displacement

To demonstrate the impact of restrictions of the horizontal displacements at supports, the one-span sandwich beam presented in Figure 3a is analyzed. The following geometrical parameters of the beam were assumed: span L = 3.0 m, width b = 1.0 m, depth d = 0.10 m, upper and lower face thickness t = 0.0005 m, distance between centroids of sandwich faces e = 0.0995 m. The Young modulus of the faces is $E_{F1} = E_{F2}$ = 210 GPa and thermal expansion coefficient $\alpha = 12\cdot10^{-6}$ 1/°C. The temperature conditions simulate the typical case of interactions occurring in the summer: T_1 = 65°C, T_2 = 20°C and the temperature of the structure installation T_0 = 10°C.

Various stiffness coefficients of the elastic supports were considered from practically rigid support (k = 1 000 000 kN/m) to the susceptible support (k = 100 kN/m). The elasticity of the supports is common in engineering practice. Figure 4 presents the stress level in the upper face (S1) and in the lower face (S2) as the function of the elasticity coefficient of the upper horizontal support k_1. Figure 4a presents the solution for the elasticity coefficient of the lower horizontal support k_2 = 1 000 000 kN/m, whereas Figure 4b demonstrates the solution for k_2 = 10 000 kN/m.

a)



b)

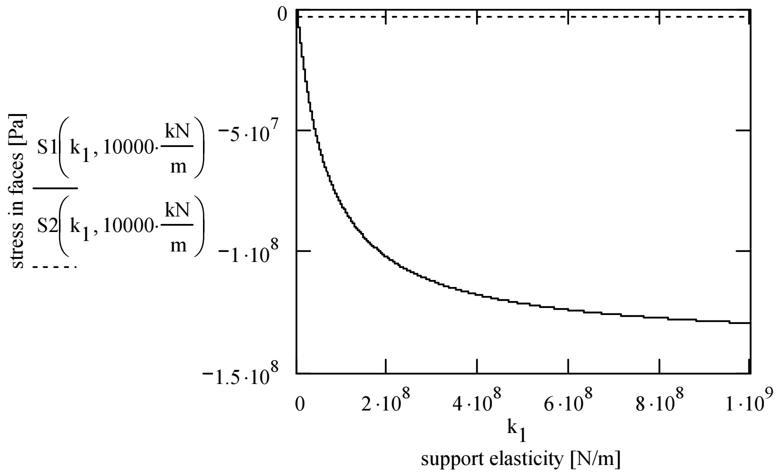


Fig. 4. The stress level in the upper face (S1 - solid line) and in the lower face (S2 - dashed line) as the function of the elasticity coefficient of the upper horizontal support k_1: a) elasticity coefficient of the lower horizontal support k_2 = 1 000 000 kN/m, b) k_2 = 10 000 kN/m

The solutions presented in Figure 4 are very interesting. The stress level in the lower face (S2) depends on the value k_2 but does not depend on the elasticity coefficient of the upper support k_1. The graph S2 is constant with respect to k_1. Analogically, the stress in the upper face will be constant with respect to the variable k_2. It is noteworthy that X_1 and X_2 in (18), (19) are dependent on both k_1 and k_2. In the case of rigid supports (Fig. 4a) the stresses in the upper and lower face reach 129.5 and 23.55 MPa, respectively. These values are huge from the mechanical point of view. The stress values in the faces for the selected parameters k_1, k_2 are shown in Table 1.

Table 1

The stress in the upper face (σ_{F1}) and in the lower face (σ_{F2}) in MPa, for selected values of parameters k_1, k_2

Stress in faces σ [MPa]		Upper face support elasticity k_1 [kN/m]		
		1 000 000	10 000	100
Lower face support elasticity k_2 [kN/m]	1 000 000	$\sigma_{F1} = -129.5$ $\sigma_{F2} = -23.55$	$\sigma_{F1} = -17.32$ $\sigma_{F2} = -23.55$	$\sigma_{F1} = -0.198$ $\sigma_{F2} = -23.55$
	10 000	$\sigma_{F1} = -129.5$ $\sigma_{F2} = -3.150$	$\sigma_{F1} = -17.32$ $\sigma_{F2} = -3.150$	$\sigma_{F1} = -0.198$ $\sigma_{F2} = -3.150$
	100	$\sigma_{F1} = -129.5$ $\sigma_{F2} = -0.004$	$\sigma_{F1} = -17.32$ $\sigma_{F2} = -0.004$	$\sigma_{F1} = -0.198$ $\sigma_{F2} = -0.004$

It should be noted that the rigid horizontal supports generate high stresses in the sandwich faces and large horizontal forces H_1, H_2. Fortunately, in practice, the stiffness of the supports and stresses is more likely such as in the selected cell of the Table 1, namely stresses $\sigma_{F1} = -0.198$ MPa and $\sigma_{F2} = -3.150$ MPa. In this case the horizontal forces are $H_1 = 0.099$ kN and $H_2 = 1.575$ kN. Screws should transfer these forces from the sandwich to the substructure. The horizontal displacements at the level of the upper and lower face reach $\Delta_1 = 0.9886$ mm and $\Delta_2 = 0.1575$ mm. Such vulnerability of the substructure seems to be easily achievable. Despite this, in extreme situations, the impact of the restrictions of the horizontal displacements at the supports of sandwich panels should be carefully evaluated.

Conclusions

The derived formulas and presented results show that limitation of horizontal displacement and rotation at the supports significantly influence the mechanical state of sandwich beams. In the extreme situations (rigid supports and high thermal actions) the compression or tension in faces can be higher than 100 MPa. Installation temperature and uniform change of temperature (neglected so far in the static analysis) substantially change the stress and strain in systems with limited horizontal displacements at supports. Changing temperatures occurring within a year cause that the sandwich faces in a different period of time are tensioned or compressed. Fortunately, structural systems are characterized by certain vulnerability, which decreases the impact of thermal effects.

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