Scientific Research of the Institute of Mathematics and Computer Science 3(11) 2012, 33-44

METHODOLOGY PERTAINING TO COMPONENT EVALUATION OF TURBULENCE ENERGY BALANCE IN A PREMIXED JET FLAME WITH DISSIPATION

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Abstract. This paper consists of an analysis of existing methods and their limitations pertaining to defining the dissipation rate as one of the components of the turbulent kinetic energy balance equation. Careful attention has been paid with reference to the method used during tests conducted in a burning round jet by means of laser Doppler anemometry. This paper covers the developed algorithms for digital signal processing of LDA, examples of output data sets for calculations and chosen profiles of energy balance components of dissipation and production.

Introduction

Tests on structures of turbulent flow with combustion have been limited for many years to the analysis of average temperature fields and visual observations, which did not practically lead to a quantified description. The implementation of laser Doppler anemometry which undoubtedly is a noninvasive measurement method, caused a significant intensification of tests pertaining to phenomena occurring in flames. The interest in the structure of burning jet turbulence was hitherto more concentrated on thick-scale coherent vortices. Papers [1, 2], which pertain to the identification of generation mechanisms and the development of organized vortex structures in free diffusion flame, may be counted among the publications with respect to this group. Observations, made in a series of papers [3, 4], suggest that these structures have impact on the course of chemical reactions which accompany combustion, and that they are responsible for the processes regarding air extraction from the surroundings and its mix with fuel and hot products of combustion. Organized movement in flows with combustion has been, therefore, deemed important with reference to the majority of processes occurring in the flame.

Nevertheless, turbulent velocity field in the flame also contains a random component. Hence, determination of the character of influences on the effectiveness of combustion processes by the random velocity fluctuations in the flame, occurring in the longitudinal and radial direction and their mutual correlation described by Reynolds turbulent stress tensor, is becoming significant.

Conducted literature studies make it possible to state that the existing tests usually pertain to diffusion flames [5, 6]. In a smaller, insufficient degree they focus upon the turbulence generated in strongly kinetic and kinetic-diffusion flames [7, 8], and particularly in the high-temperature sphere in the area of combustion front. From the perspective regarding the effectiveness analysis of combustion process and its modeling, it is a particularly significant range, in which strong gradients of factor, specific volume and displacement effects lead to the intensification of secondary air extraction from the surroundings.

In this paper, a problem concerning the influence of combustion on the flame turbulence structure, and particularly on the phenomena comprising the process of turbulence kinetic energy transportation, has been depicted. The principal aim was to conduct an analysis of turbulence kinetic energy balance in a burning jet on the basis of experimental values of elements representing all forms occurring in the energy transport flow.

1. Turbulence kinetic energy transportation equation in free round jet

The energy equation for axisymmetric jet in a non-dimensional form is presented by the relation

$$\underbrace{\frac{1}{\rho}\frac{d}{\overline{U}_{ox}^{3}}\left[\overline{u_x\frac{\partial}{\partial x}p}+\overline{u_r\frac{\partial}{\partial r}p}\right]}_{1}+\underbrace{\frac{d}{\overline{U}_{ox}^{3}}\left[\overline{u_x\frac{\partial}{\partial x}\frac{q^2}{2}}+\overline{u_r\frac{\partial}{\partial r}\frac{q^2}{2}}\right]}_{2}+\underbrace{\frac{d}{\overline{U}_{ox}^{3}}\left[\overline{U}_x\frac{\partial}{\partial x}\frac{\overline{q^2}}{2}+\overline{U}_r\frac{\partial}{\partial x}\frac{\overline{q^2}}{2}\right]}_{3}+$$

$$+\underbrace{\frac{d}{\overline{U}_{ox}^{3}}\left[\overline{u_x^2}\frac{\partial\overline{U}_x}{\partial x}+\overline{u_xu_r}\left(\frac{\partial\overline{U}_x}{\partial r}+\frac{\partial\overline{U}_r}{\partial x}\right)+\overline{u_r^2}\frac{\partial\overline{U}_r}{\partial r}\right]}_{4}+ \qquad (1)$$

$$+\underbrace{\nu\frac{d}{\overline{U}_{ox}^{3}}\left\{\frac{\partial^2\overline{q^2}}{\partial x^2}+\frac{\partial^2\overline{q^2}}{\partial r^2}+\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(\overline{u_xu_x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\overline{ru_xu_r}\right)\right]+\frac{\partial}{\partial r}\left[\frac{\partial}{\partial x}\left(\overline{u_xu_r}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\overline{ru_ru_r}\right)\right]\right\}}_{5}+\underbrace{\frac{d}{\overline{U}_{ox}^{3}}}_{6}\in=0$$

in which particular elements mean: (1) pressure diffusion, (2) turbulence kinetic energy diffusion, (3) energy convection, (4) production of turbulence kinetic energy, (5) viscous stress work and (6) dissipation of energy into heat.

The first element of the equation (1) depicts a kinetic energy stream generated by work of pressure forces, whereas the second element is responsible for kinetic energy distribution by means of diffusion. Convection term (3) represents the turbulence energy stream transmitted by average movement. Production of turbulence energy (4) consists of energy transportation from average movement (thick-scale) to fluctuating movement (fine-scale). Viscous stress work included in the term (5) of the equation describes an energy stream generated by this work and transmitted within the scope of turbulent movement. Dissipation is the final phase of the inner turbulence kinetic energy transport, occurs in the scope of the smallest vortex structures characterizing a given flow as a result of liquid viscosity and consists of energy conversion into heat. The dissipation value presents an energy stream dispersed into heat.

All components of the given equation may be experimentally obtained except for the term (1) defining the pressure diffusion. It results from the fact that the measurement of velocity-pressure correlation is still insufficiently mastered with respect to metrology. In the dimensional analysis, concerning the evolution of particular terms of the equation of turbulence kinetic energy balance, this element is obtained as the closing value of the energy balance.

2. Review of methods pertaining to obtaining turbulence kinetic energy dissipation

2.1. Establishment of energy dissipation based on the measurement of variances pertaining to dimensional derivatives of fluctuating velocity

Kinetic energy resulting from liquid movement is subjected to conversion into heat as a result of viscosity force influence. In the case of incompressible fluid flow (ρ = const.) dissipation rate of turbulence kinetic energy related to liquid mass unit may be formulated through the relations:

$$\in = \nu \overline{\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)} \tag{2}$$

If the turbulence structure is homogeneous in the dimension, then formula (2) may be presented as follows:

$$\in = \nu \overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}} \tag{3}$$

During the analysis of the above-mentioned relations, the Kolmogorov hypothesis of local isotropy is of the utmost importance, according to which in turbulent flows with sufficiently high Reynolds numbers there is a range of wave numbers, where vortex structures have an isotropic character. Due to the fact that dissipation occurs mainly in the area of the smallest vortices, common isotropic relations are used during its establishment:

$$\overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} = \overline{\left(\frac{\partial u_1}{\partial x_3}\right)^2} = \overline{\left(\frac{\partial u_2}{\partial x_1}\right)^2} = \ldots = 2\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} \tag{4}$$

$$\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} = \overline{\left(\frac{\partial u_2}{\partial x_2}\right)^2} = \overline{\left(\frac{\partial u_3}{\partial x_3}\right)^2}$$

which after the inclusion in the relation (3), give:

$$\in = 15\nu \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} = 7.5\nu \overline{\left(\frac{\partial u_1}{\partial x_2}\right)^2} = 7.5\nu \overline{\left(\frac{\partial u_2}{\partial x_1}\right)^2} \tag{5}$$

In general, dissipation Є may be formulated as follows:

$$\in = \nu \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} G_{ij} \tag{6}$$

where:

$$G_{ij} = \overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2} \Big/ \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2}$$

The usage of the relations (5) requires the measurement of variances pertaining to dimensional derivatives $\partial u_i/\partial x_j$, which in practice is limited to the determination of derivatives $\partial u_i/\partial t$. Nevertheless, it is necessary to use the Taylor hypothesis on the "frozen" character of vortex structures, which creates the possibility to write it as:

$$\overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2} = \overline{U}_1^{-2} \overline{\left(\frac{\partial u_i}{\partial t}\right)^2} \tag{7}$$

where: $\overline{U}_1$- average velocity directed along the axis x_i of local Cartesian coordinate system

$$\overline{U} = \overline{U}_1, \overline{U}_2 = \overline{U}_3 = 0$$

The usage of the Taylor hypothesis allows one to obtain 3 values of derivative variances measured in regard to the coordinate x_1 that is why the dissipation value may be determined by means of the relation:

$$\in = 3\nu \left[\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} + \overline{\left(\frac{\partial u_2}{\partial x_1}\right)^2} + \overline{\left(\frac{\partial u_3}{\partial x_1}\right)^2} \right] = 3\nu \overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} \left(G_{11} + G_{21} + G_{31}\right) \tag{8}$$

for which $G_{11} = 1$

2.2. Determination of energy dissipation on the basis of measuring one-dimensional spectral functions

In the flow with isotropic and homogeneous turbulence structure, dissipation may be obtained by means of one-dimensional spectra functions $F_{ii}(k_1)$ satisfying the conditions:

$$\int_0^\infty F_{ii}(k_1)dk_1 = \overline{u_i^2} \tag{9}$$

because:

$$\overline{\left(\frac{\partial u_i}{\partial x_1}\right)^2} = \int_0^\infty k_1^2 F_{ii}(k_1)dk_1 \tag{10}$$

then in compliance with (3.1.3) one may write:

$$\in = 15\nu \int_0^\infty k_1^2 F_{ii}(k_1)dk_1 \tag{11}$$

$$\in = 7.5\nu \int_0^\infty k_1^2 F_{22}(k_1)dk_1 \tag{12}$$

or

$$\in = 2.5\int_0^\infty k_1^2 [2F_{ii}(k_1) + F_{22}(k_1) + F_{33}(k_1)]dk_1 \tag{13}$$

Relations (11) and (12) are mutually equivalent in the case when isotropic character of turbulence in dissipated range of energy spectrum may be observed. When Reynolds numbers are lower, for which the Kolmogorov condition of local isotropy does not have to be satisfied, it is advised to use the dependences (13), which give an averaged, hence more probable, dissipation value.

2.3. Defining energy dissipation on the basis of turbulence energy spectrum in the inertial subrange of wavenumbers

This method is based on the second Kolmogorov hypothesis, according to which sufficiently high Re numbers in the universal balance of energy spectrum range are accompanied by inertial subrange, in which the following conditions are satisfied:

$$E(k) = \alpha \in^{2/3} k^{-5/3} \tag{14}$$

$$F_{11}(k_1) = \alpha_1 \in^{2/3} k_1^{-5/3} \tag{15}$$

$$F_{22}(k_1) = \alpha_2 \in^{2/3} k_1^{-5/3} \tag{16}$$

where:
α - Kolmogorov universal constant (1.4÷1.7),
α_1 - 18/55α,
α_2 - 24/55α.

If we assume that k^*_1 defines the position of the arbitrary point on the curve $F_{11}(k_1)$ in the inertial range, then the dissipation may be determined by means of relations:

$$\in = \left[\frac{F_{11}(k_1^*)(k_1^*)^{5/3}}{\alpha_1} \right]^{3/2} \tag{17}$$

$$\in = \left[\frac{F_{22}(k_1^*)(k_1^*)^{5/3}}{\frac{4}{3}\alpha_1} \right]^{3/2} \tag{18}$$

$$\in = \left[\frac{F_{11}(k_1^*) + F_{22}(k_1^*) + F_{33}(k_1^*)}{\frac{6}{5}\alpha} (k_1^*)^{5/3} \right]^{3/2} \tag{19}$$

The final dependence allows one to correct potential deviations from isotropy conditions and it is advised in the case when a simultaneous measurement of all three functions $F_{ii}(k_1)$ is possible.

The usage of the given method is possible when an average flow is characterized by a sufficiently high Reynolds number, which forms an inertial subrange in the universal range of energy spectrum. When the value of the Reynolds number is too low to form a sphere satisfying the law (−5/3) in the energy spectrum, then the results obtained by means of this method are characterized by great measurement uncertainty, thus it cannot be recommended.

2.4. Defining energy dissipation on the basis of the velocity correlation function shape (indirect method)

The acknowledgement of the standard autocorrelation time function $R_{11}(\tau)$, which according to the Taylor hypothesis is identified as a function of dimensional and longitudinal correlation $R_{11}(r_1)$, allows one to determine the rate of energy dissipation. This dependence may be depicted in the following form:

$$R_{11}(\tau) = R_{11}(r_1), \quad r_1 = \overline{U}_1 \tau \tag{20}$$

By developing the final dependence into a Taylor series in the zero point range and by transforming it accordingly we receive:

$$\in = \lim_{r_1 \to \infty} \left\{ \frac{30\nu}{r_1^2} \overline{u_1^2} \left[1 - f(r_1) \right] \right\} \tag{21}$$

where: $f(r_1) = R_{11}(r_1)$ is the coordinate of longitudinal correlation with the isotropic turbulence structure.

Dependence (21) is equivalent to the relation:

$$\in = \frac{30\nu\overline{u_1^2}}{\lambda_f^2} = \frac{15\nu\overline{u_1^2}}{\lambda_g^2} \tag{22}$$

where:
λ_f - longitudinal Taylor's microscale of the turbulence,
λ_g - transverse Taylor's microscale of the turbulence.

Nevertheless, the described method of determining dissipation is not recommended in practice, because the implementation of the formulas (21) and (22) requires high precision while defining the function shape $f(r_1)$ in the range of low values of distance correlation r_1, what is an extremely complex task with respect to measurement.

2.5. The evaluation of energy dissipation as an element closing the turbulence energy balance

This method, although described by some authors [9], apart from being significantly demanding, raises certain doubts of a methodological nature. In the turbulence energy balance velocity-pressure correlation is impossible to be experimentally determined, which leads to the conclusion that the closing rate of the balance equation is not only limited to dissipation, but its value is increased by this correlation.

The credibility of the described method regarding free axisymmetric jet is examined in the paper [10], and its results are presented in [11].

3. Implementation examples of the adopted method of obtaining dissipation with regard to burning kinetic-diffusion jet

The method adopted in this paper, pertaining to defining the energy dissipation, is described in a broader scope in the point 2.3. This method has certain limitations resulting from the value of Reynolds number which characterizes a given flow. When Re values are sufficiently high, then fine vortex structures become completely independent of external conditions, defined by the shape of average velocity; however, viscosity influence is increasing, which causes energy dissipation of fine-scale vortices. Within a limited scope of wavenumbers, the structure of the smallest vortices may adopt local isotropic character. The above-mentioned state is revealed in the examined flow when an inertial subrange is extracted from the turbulence energy spectrum.

The necessary condition pertaining to the existence of local isotropy is the implementation of high turbulent Reynolds number $\mathrm{Re}_\lambda = u_1 \lambda / \nu$, which according to [12] should satisfy the ensuring condition that the energy and dissipation vortex ranges are not overlapping.

The extraction of a spectral range, in which the dependence (15) is in effect, constitutes the foundation for the adopted method defining the dissipation of turbulence kinetic energy, based on the dependence (17), where rates $\mathbf{k}^*_1$ and $\mathbf{F}^*_{11}$ are the coordinates of the center point in the inertial subrange. It is also necessary to adopt a constant α_1, which was set in this paper at 0.51 adopted for $\mathrm{Re}_\lambda \geq 100$.

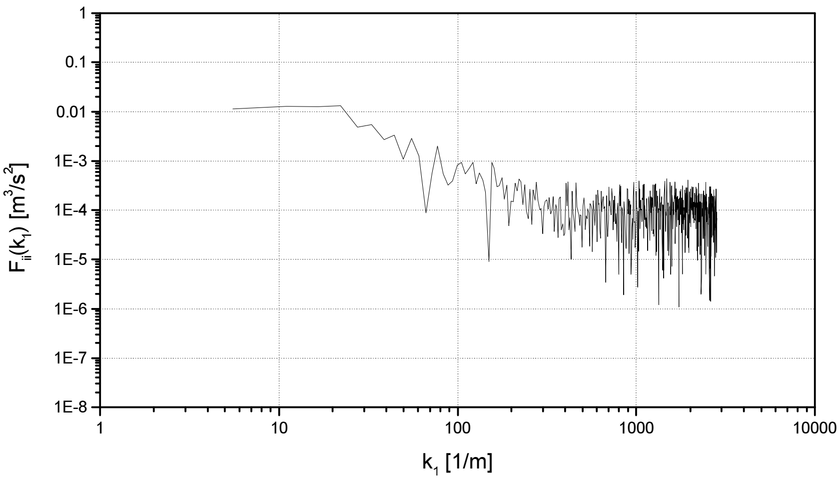

Fig. 1. Spectral function in wavenumber dimension obtained for the kinetic diffusion flame r_m = 18% at a distance from the nozzle exit of x/d = 38

Figure 1 presents an exemplary spectrum of a spectral function obtained for the flame r_m = 18%, for coordinate measurements x/d = 38 and r/d = 0.625. The course of the presented spectral function is accompanied by a significant noise level, which results from the characteristics of LDA system work, which recorded a signal at random intervals. Limitation pertaining to the recording length of datasets was an additional flaw (limitation of software controlling the anemometer). The quantity of measurement sets, from which energy spectra have been extracted, was equal to 30000 samples; however, the fact that they are a randomly sampled time series imposed the necessity of additional averaging thereof. Figure 2 presents a result of the already averaged spectrum and the drawn law (-5/3).

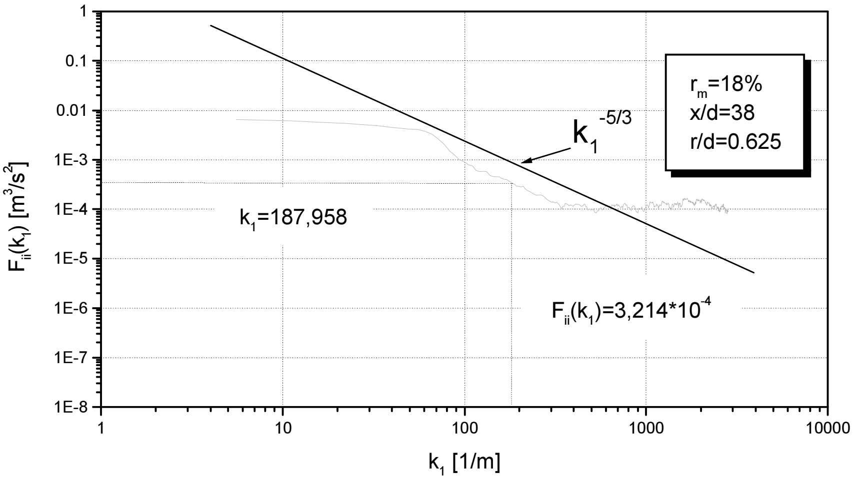

Fig. 2. Averaged spectral function with highlighted curve depicting the law (-5/3)

Figures 3 and 4 depict exemplary radial distribution of determined dissipation in flames with decreasing stoichiometric composition of a combustible mixture. The presented results have been additionally differentiated by the distance from the nozzle exit *x*/*d*.

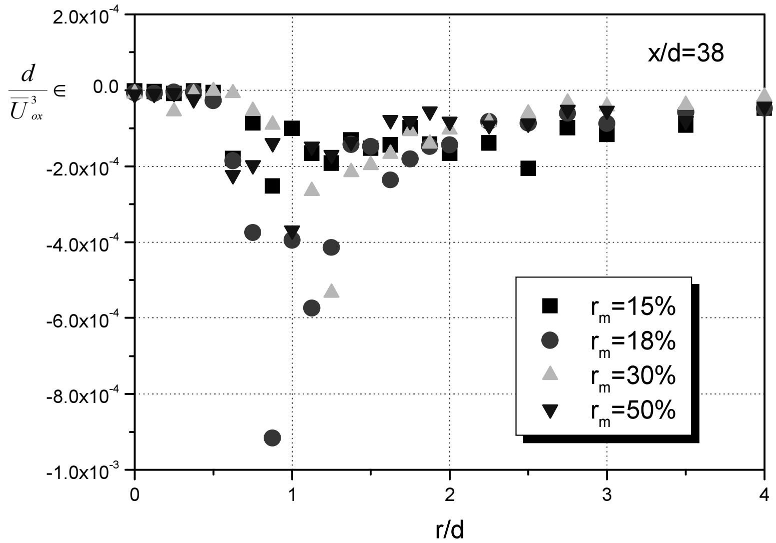

Fig. 3. Radial distributions of dissipation values obtained at the distance $x/d = 38$ for various compositions of a combustible mixture

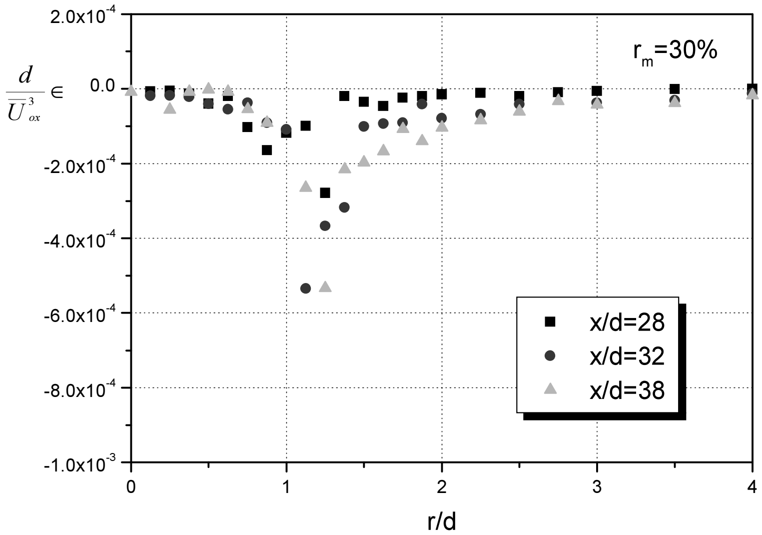

Fig. 4. Radial distributions of dissipation values obtained for the flame $r_m = 30\%$ at various distances from the nozzle exit

The analysis of Figures 3 and 4 indicates that the range of maximal dissipation values for all gatherd flames is located in the region of the coordinate r/d = 1. Moreover, maximal values are noted for flames with a moderate methane content in a combustible mixture i.e. r_m = 18 and 30%. It is also noteworthy that the dissipation value increases as the distance from the nozzle exit grows.

4. Methodology pertaining to production elements of turbulence kinetic energy

In the second point energy equation (1) for axisymmetric jet in a non-dimensional form has been depicted, in which element (4) presents the production of turbulence kinetic energy. During the analysis of particular components thereof and experimental data certain observations have been made, which create a possibility to avoid some of them as insignificant in a general production balance of turbulence kinetic energy. The following element has been chosen for further analysis:

$$\overline{u_x u_r} \frac{\partial \overline{U_x}}{\partial r} \tag{23}$$

Radial distributions of normal stresses in axial and radial direction, shear stresses and average velocity in a longitudinal and transverse direction with respect to the flow direction were the basis of the analysis. These data have been acquired due to the numerical measuring signal-processing recorded by means of the LDA system and they were the foundation for determination of production values of turbulence kinetic energy. An output database, recorded as datasets in the function of the distance from the nozzle exit, has been created for all examined flames.

A standard production value has been determined by means of developed numerical software, in which, inter alia, the following relation has been used:

$$Ep_i = -\frac{d}{U_{ox}^3}\left\{\left[\left(\overline{u_x u_r}\right)_{i-1} + \left(\overline{u_x u_r}\right)_i\right]/2\left(\overline{U}_{x_{i-1}} - \overline{U}_{x_i}\right)/\Delta r\right\} \tag{24}$$

where:

d - nozzle exit diameter $d = 0.008$ m,

$\overline{U}_{ox}$ - average velocity value at the nozzle exit characteristic for the composition of a combustible mixture,

Δr - radial distance between the measurement points (this rate was fluctuant and depending on the distance r/d it had the following values: 0.001, 0.002, 0.004, 0.008).

$\left[\left(\overline{u_x u_r}\right)_{i-1} + \left(\overline{u_x u_r}\right)_i\right]/2$ - arithmetic mean value of shear stresses,

$\left(\overline{U}_{x_{i-1}} - \overline{U}_{x_i}\right)/\Delta r$ - value increase of longitudinal velocity.

Figure 5 depicts radial distributions of energy production and dissipation for exemplary flame and distance from the nozzle exit. The analysis thereof testifies to the fact that radial distributions of this rate indicate to the existence of ranges, in which random fluctuation energy is especially intensively generated.

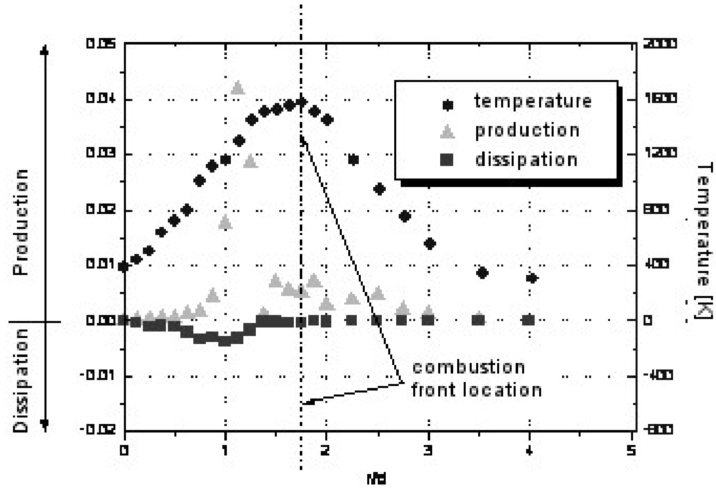

Fig. 5. Radial distributions of production and dissipation associated with the temperature profile of a flame $r_m = 50\%$ obtained at the distance of x/d28

Figure 5. depicts the arrangement of production and dissipation elements against the distribution of temperatures, whereby results pertaining to the flame $r_m = 50\%$ have been obtained at the distance of $x/d = 28$.

Conclusions

This paper presents a method to determine turbulent kinetic energy dissipation. Particular attention was paid to the second method, based on the Kolmogorov hypothesis, according to which, in sufficiently large numbers Re in the energy spectrum of the universal balance is sub inertia. The examples of application of this method in the case of premixed jet flames were seven. The evaluated value of dissipation was compared with the member production of turbulent kinetic energy balance.

The analysis of the presented progresses resulted (Fig. 5) in the establishment of position correctness of spheres regarding maximal turbulence kinetic energy production and maximal dissipation. The fact that this sphere is located in the inner range of the flame is also noteworthy.

References

[1] Asendrych D., Drobniak S., Detection of organized structures in diffusion flames. XI KKMP, Warsaw 1994.

[2] Asendrych D., The characteristics of organized structures in free axisymmetric vortex flows with combustion, PhD Thesis, Czestochowa University of Technology, 1996.

[3] Ballal D.R., Studies of turbulent flow-flame interaction, AIAA Journal 1986, 24, 1148-1154.

[4] Bédat B., Cheng R. K., Effects of buoyancy on premixed flame stabilization, Combustion and Flame 1996, 107, 13-26.

[5] Bilger R.W., Turbulent diffusion flames, Fluid Mech. 1989, 21, 101-135.

[6] Ida T., Ohtake K., Experimental study of turbulent diffusion flame structure and its similarity, JSME International Journal, series B 1997, 40, 2,.

[7] Bray K.N.C., Libby P.A., Masuya G., Moss J.B., Turbulence production in premixed turbulent flames, Combustion Sci. Tech. 1981, 25, 127-140.

[8] Chih Ming Ho, Jakus K., Parker K.H., Temperature fluctuations in a turbulent flame, Combustion and Flame 1976, 27, 113-123.

[9] Chomiak J., On a hypothesis concerning the structure of a turbulent flame front at high Reynolds numbers, Fluid Dynamics Transactions, 5, Part I, 47-59, 1971.

[10] Aldredge R.C., Willimas F.A., Influence of wrinkled premixed-flame dynamics on large-scale, low-intensity turbulent flow. J. Fluid Mech. 1991, 228, 487-511.

[11] Noda S., Matsumoto R., Kawai W., Kimoto K., Nakajima T., A study of the turbulence structure in a rich premixed jet flame, Bulletin of JSME 1985, 28, 239.

[12] Elsner J.W., Flow turbulence, PWN, Warsaw 1987.