Scientific Research of the Institute of Mathematics and Computer Science 1(11) 2012, 15-22

NUMERICAL ANALYSIS OF INTERRELATIONS BETWEEN SKIN SURFACE TEMPERATURE AND BURN WOUND SHAPE

Mariusz Ciesielski[1], Bohdan Mochnacki[2]

[1]Institute of Computer and Information Sciences, [2]Institute of Mathematics
Czestochowa University of Technology, Poland
mariusz.ciesielski@icis.pcz.pl, bohdan.mochnacki@im.pcz.pl

Abstract. Numerous observations show the apparent relations between skin surface temperature and the shape of a burn wound. The tracing of temperature changes allows one to analyze the process of wound healing and a such information can be very essential at the stage of therapy choice. The information concerning the surface temperature distribution can be obtained using, first of all, the thermography methods. In the paper, the numerical method of burn wound shape estimation on the basis of skin temperature measurements is discussed. It is assumed that the thermal processes proceeding in the heterogeneous system healthy tissue-burn wound can be described by a system of two Pennes equations supplemented by the boundary conditions given on the external surface of the tissue domain and the boundary conditions determining heat exchange between sub-domains considered. The healing of wounds proceed very slowly and the problem of temporary wound shape estimation can be treated as the steady one.

Introduction

The non-homogeneous domain being the composition of healthy tissue and burn wound is considered and the temperature distribution at the skin tissue surface is assumed to be known. The first step of computations consists in the analytical solution of the 1D problem which allows one to find the dependence between depth of burn wound and skin surface temperature. From the physical point of view, this solution corresponds to temperature distribution in the vertical pillar dispatched from skin tissue domain under the assumption that its lateral surface is thermally insulated. In this way the knowledge of surface temperature of successive 'pillars' enables preliminary estimation of wound shape. Next, the 3D boundary problem concerning the temperature distribution in the obtained heterogeneous system being the composition of healthy tissue and wound sub-domains is solved (using the numerical methods). The conformability of calculated surface temperature distribution with the measured one confirms the proper estimation of wound shape. If it were not so, the solution obtained would constitute the starting point for the simple numerical procedure realizing the correction of wound geometry.

1. Governing equations

The steady temperature in domain of burn wound ($e = 1$) and healthy tissue ($e = 2$) is described by a system of equations [1, 2]

$$\lambda_e \nabla^2 T_e\left(x, y, z\right) + G_{Be} c_B [T_B - T_e(x, y, z)] + Q_{met} = 0 \tag{1}$$

where λ_e [W/(mK)] is the thermal conductivity, G_{Be} [(m^3 blood/s)/(m^3 tissue)] is the blood perfusion coefficient and Q_{met} [W/m^3] is the metabolic heat source. The parameters c_B [J/(m^3 K)] and T_B correspond to the volumetric specific heat of blood and the arterial temperature, respectively, while $T(x)$ is the temperature, x, y, z denotes the geometrical co-ordinates. For $e = 1$ both $G_{B1} = 0$ and $Q_{m1} = 0$.

At the contact surface between sub-domains the continuity condition is given

$$x \in \Gamma_{1-2}: \quad \begin{cases} -\lambda_1 \dfrac{\partial T_1\left(x, y, z\right)}{\partial n} = -\lambda_2 \dfrac{\partial T_2\left(x, y, z\right)}{\partial n} \\ T_1\left(x, y, z\right) = T_2\left(x, y, z\right) \end{cases} \tag{2}$$

while at the skin surface the Robin condition is taken into account

$$x \in \Gamma_0: \quad -\lambda_1 \frac{\partial T_1\left(x, y, z\right)}{\partial n} = \alpha[T_1(x, y, z) - T_a] \tag{3}$$

where α is a heat transfer coefficient, T_a is an ambient temperature, $\partial/\partial n$ denotes a normal derivative. The internal surface of domain is kept at a constant temperature T_B while at the conventionally assumed lateral surface of the system the no-flux condition is assumed.

2. Solution of 1D problem

Let's the index $e = 1$ corresponds to burn wound, while $e = 2$ to healthy tissue (as previously). The mathematical model of the process create the following equations and conditions (Fig. 1)

$$\begin{cases} \lambda_1 \dfrac{\mathrm{d}^2 T_1(x)}{\mathrm{d}x^2} = 0 \\ \lambda_2 \dfrac{\mathrm{d}^2 T_2(x)}{\mathrm{d}x^2} + G_B c_B [T_B - T_2(x)] + Q_{met} = 0 \end{cases} \tag{4}$$

$$x \in \Gamma_{1-2}: \quad \begin{cases} -\lambda_1 \dfrac{\mathrm{d}T_1\left(x\right)}{\mathrm{d}x} = -\lambda_2 \dfrac{\mathrm{d}T_2\left(x\right)}{\mathrm{d}x} \\ T_1\left(x\right) = T_2\left(x\right) \end{cases} \tag{5}$$

$$x \in \Gamma_0 : \quad \lambda_1 \frac{\mathrm{d}T_1(x)}{\mathrm{d}x} = \alpha[T_1(x) - T_a] \tag{6}$$

$$x \in \Gamma_L : \quad T_2(x) = T_B \tag{7}$$

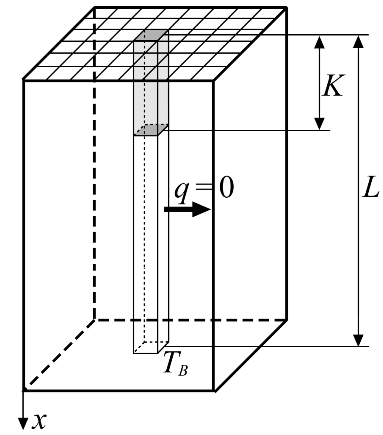

Fig. 1. The 1D tissue domain with burn wound (grey) and healthy tissue (white) sub-domains

The solution of this simple boundary problem is of the form

$$T(x) = \begin{cases} C_1 + C_2 x, & 0 \le x \le K \\ C_3 \exp(ax) + C_4 \exp(-ax) + T_B + Q_{met} / (G_B c_B), & K < x \le L \end{cases} \tag{8}$$

where K is the depth of wound, L is the thickness of skin tissue, while $a = \sqrt{G_B c_B / \lambda_2}$.

Coefficients $C_1 - C_4$ can be found using the boundary conditions, in particular the following system of equations should be solved

$$\begin{bmatrix} 0 & 0 & \exp(aL) & \exp(-aL) \\ -1 & -K & \exp(aK) & \exp(-aK) \\ \alpha & -\lambda_1 & 0 & 0 \\ 0 & -\lambda_1 & \lambda_2 a \exp(aK) & -\lambda_2 a \exp(-aK) \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} -Q_{met} / (G_B c_B) \\ -T_B - Q_{met} / (G_B c_B) \\ \alpha T_a \\ 0 \end{bmatrix} \tag{9}$$

In Figure 2 the temperature profiles for L = 25 mm (this value assures the stabilization of surface temperature) and different values of K are shown, at the same time λ_1 = 0.1 W/(mK), λ_2 = 0.2 W/(mK), G_B = 0.5 m^3 blood/s/m^3 tissue, c_B = 4200 J/(m^{3}K), T_B = 37°C, Q_{met} = 200 W/m^3, T_a = 20°C, α = 3.1 W/(m^{2}K).

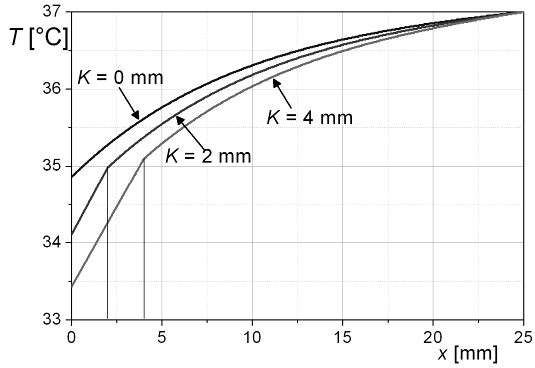

Fig. 2. Temperature profiles for different values of K

Figure 3 illustrates the dependence between skin surface temperature and wound depth.

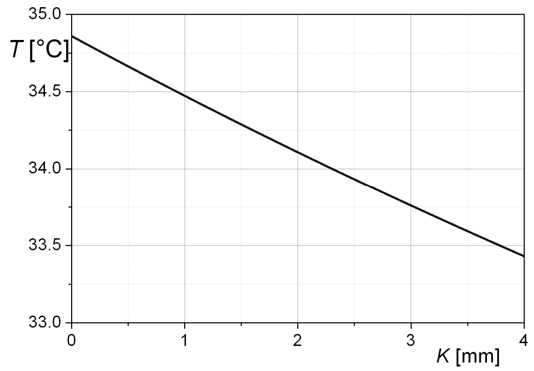

Fig. 3. Surface temperature $T(0)$ for different values of K

It should be pointed out that the solution of the problem discussed is strongly sensitive to the changes of the heat transfer coefficient α. The proper choice of this parameter can be assured on the basis of measurements of healthy tissue temperature $T(0)$. For instance (see: Figure 2) if $T(0)$ = 34.8°C then α is close to 3.1 W/(m^{2}K).

3. FDM algorithm for 3D problem

The version of FDM presented below is described in detail in [3]. The domain considered is divided into control volumes CV (cuboids or cubes). The central points of control volumes correspond to the nodes of geometrical mesh. Node (i, j, k) and adjoining ones $(i+1, j, k)$, $(i-1, j, k)$, $(i, j+1, k)$ etc. create the 7-point stars. To simplify the mathematical formulas the nodes creating a star will be denoted as $w = 0$ (central node) and w = 1, 2, 3, 4, 5, 6 (adjoining ones).

Let us introduce the thermal resistances defined as follows

$$R_w = \frac{0.5\,h_w}{\lambda_0} + \frac{0.5\,h_w}{\lambda_w} \tag{10}$$

where h_w is the distance between node 0 and node w, λ_0, λ_w are the thermal conductivities of volumes 0 and w.

Additionally the shape functions for rectangular mesh will be introduced

$$\Phi_w = \frac{1}{h_w} \tag{11}$$

and then denoting

$$A_w = \frac{\Phi_w}{R_w} \tag{12}$$

one obtains the following form of FDM equation (internal control volumes)

$$\sum_{w=1}^{6} A_w (T_w - T_0) - c_B G_B T_0 = c_B G_B T_B - Q_{met} \tag{13}$$

It can be noticed that for the wound sub-domain the right hand side of equation (13) is equal to zero.

Finally

$$\sum_{w=1}^{6} A_w T_w - T_0 \left(\sum_{w=1}^{6} A_w + c_B G_B\right) = c_B G_B T_B - Q_{met} \tag{14}$$

The similar equations determine the temperatures at the central point of boundary control volumes. Let us assume that direction $w = 1$ corresponds to the boundary surface of CV. To take into account the Robin boundary condition given on the surface considered one should re-define the thermal resistance in direction $w = 1$, in particular [3]

$$R_1 = \frac{0.5\,h_1}{\lambda_0} + \frac{1}{\alpha} \tag{15}$$

at the same time in a place of T_1 the ambient temperature should be introduced. Taking into consideration the no-flux condition requires one to assume a very small (close to zero) value of α, while the Dirichlet condition can be modeled by assumption of very big value of α, while the role of T_1 plays the given boundary temperature. So, in the numerical realization it is convenient to surround the domain using the 'empty nodes' for which the ambient or boundary temperatures are recorded.

The set of FDM approximations (14) creates the linear system of equations and the solution obtained allows one to determine the nodal temperatures of domain considered.

4. Example of computations

The cubical domain of biological tissue (the sizes of surface area: L_x = 100 mm, L_y = 100 mm and thickness: L_z = 25 mm) has been considered. The thermophysical parameters of tissue and other input data are the same as in chapter 3, whereas the temperature distribution on the skin surface ($z = 0$) is shown in Figure 4. The domain considered has been divided into 500 000 cuboids (n_x = 100, n_y = 100, n_z = 50).

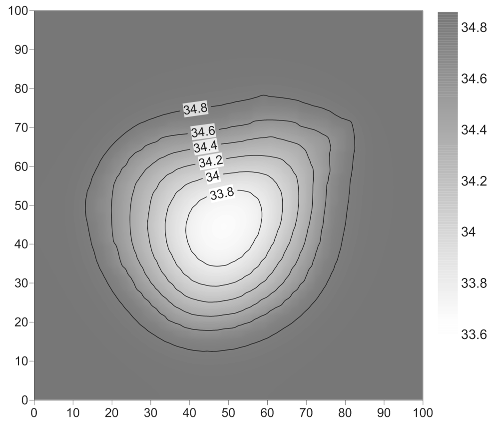

Fig. 4. The input temperature distribution on the skin surface

In Figure 5a the preliminary estimation of the wound shape $K(x, y)$, calculated on the basis of the analytical solution of the 1D problem and the known temperature of skin surface is shown. Next, the nodal temperatures of domain considered have been determined. Figure 5b shows the surface temperature field. One can observe (Figures 4 and 5b) that the calculated surface temperature is higher than the input skin surface temperature. In order to decrease the temperature difference, the numerical procedure realizing the correction of wound geometry (by the changes in depth of wound) has been applied. In Figures 6a and 6b the corrected wound shape and the calculated surface temperature field for this wound shape are presented. One can see that the surface temperature distribution shown in Figures 4 and 6b are very similar. Figure 7 shows isotherms and the profile of the determined depth of the wound in the x-z plane (normal to skin) for $y = L_y/2$.

a) b)

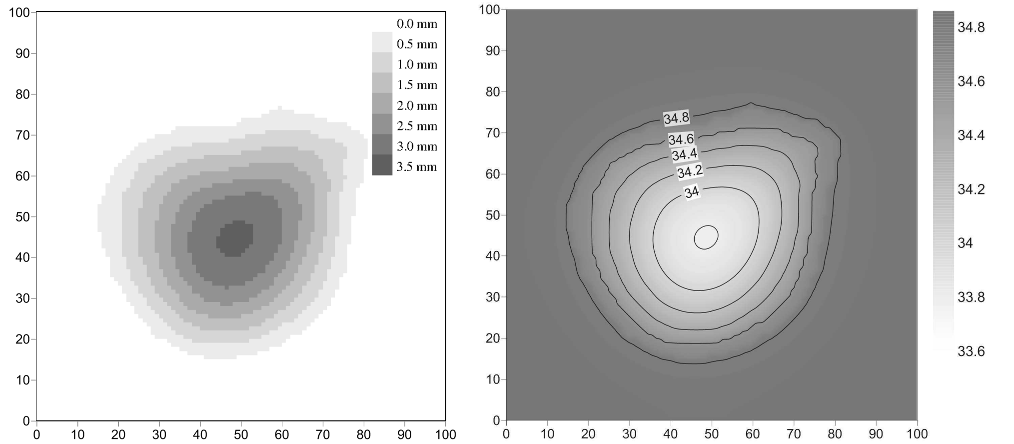

Fig. 5. The preliminary estimation of wound shape (a) and the calculated temperature field of skin surface in the *x-y* plane for $z = 0$ (b)

a) b)

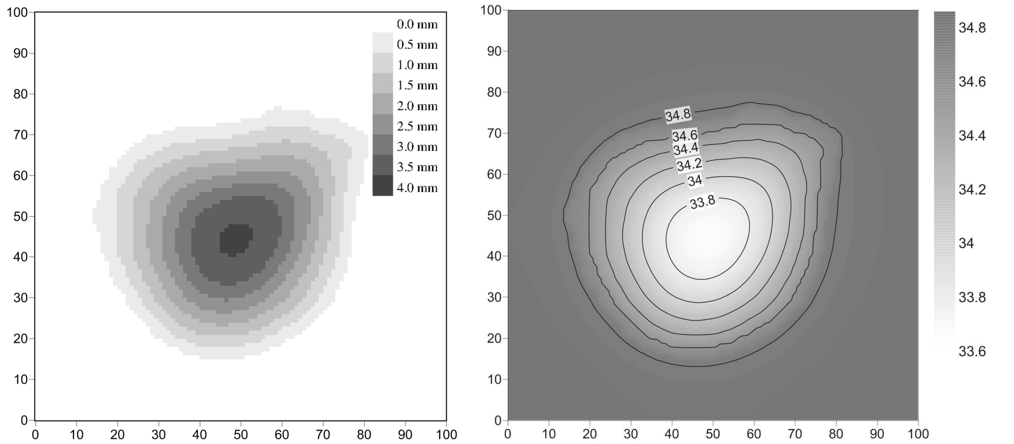

Fig. 6. The corrected of wound shape (a) and the calculated temperature field of skin surface in the *x-y* plane for $z = 0$ (b)

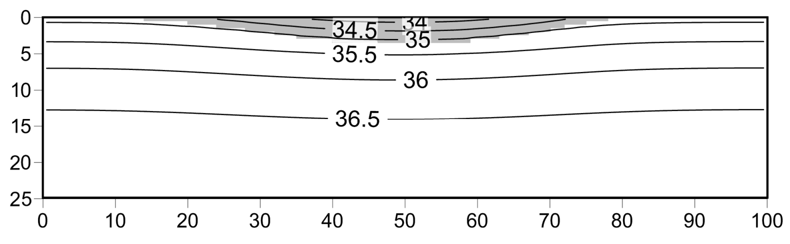

Fig. 7. Isotherms and the profile of wound shape in the *x-z* plane for $y = L_y/2$

Conclusions

The method proposed can be applied as an effective tool of burn wounds shape estimation on a basis of thermal images done in the real conditions (a distribution of skin surface temperature). The process of wound healing can be also analyzed. The problems close to the presented ones are also discussed in [4] and [5].

Acknowledgment

The paper is a part of Project NR-13-0124-10/2010

References

[1] Majchrzak E., Biomechanics, Modelling and analysis of thermal problems, IPPT PAN, 2011, part IV, 223-355.

[2] Majchrzak E., Mochnacki B., Sensitivity analysis and inverse problems in bioheat transfer modelling, CAMES 2006, 13, 85-108.

[3] Majchrzak E., Mochnacki B., Numerical Methods. Theoretical Base, Numerical Aspects and Algorithms, Publ. of Silesian. Univ. of Techn., 2004.

[4] Romero-Méndez R., Jiménez-Lozano J.N., Sen M., González F.J., Analytical solution of the Pennes equation for burn-depth determination from infrared thermographs, Mathematical Medicine and Biology 2010, 27, 21-38.

[5] Majchrzak E., Determination of burn depth on a basis of skin surface temperature - solution of inverse problem using the gradient method, 10th World Congress On Computational Mechanics, Sao-Paulo, Brazil 2012 (in print).