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THE COMPROMISE IN MULTICRITERIAL NET OPTIMIZATION

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Abstract. We propose to utility the methodology consist in estimation of concessions for compromise (called here the losses of magnitude). This is practically valuation (estimation) in what degree the got solution for remaining criterions or weight components will correct the suitable value of characteristic for concrete criterion. The additional aspect of multicriteria optimization in nets is the problem of adaptation the methodology, and at the same time the algorithm of estimating losses of magnitude to specific and type of criteria configuration.

Introduction

Finding the optimal solution with point of view all criterions is complex question especially for situations of different trends (directed in different sides) of influence on arguments of criteria function [1-4]. The final stage of multicriteria optimization is finding the compromise. It is neuralgic phase because it seldom happens to find one the predominant solution for group of criterions according to net structure [5, 6]. Similarly as in case of risk we can on many ways define a compromise in dependence on studied situation [3, 7-9]. If all criterions achieve optimum values for concrete set of parameters the value of compromise is equal zero because compromise is unnecessary. Also if all from players don't are satisfied of their payments the value of compromise is also on level zero. In situation when one of players is satisfied, and other not, we can accept that the value of compromise is also even zero [10-12]. Expression "I agree on this condition but I would prefer those" there is the equivalent of partial satisfying [13].

1. The location of compromise in multicriteria net optimization

For location of compromise in multicriteria optimization we use different algorithms [3], which cannot always effectively apply to network models. One of the prime ways (with some defects about which we say somewhat later) is the algorithm using so called the loss of optimum what we can name differently the size of concession for compromise. The idea of such algorithm consist in balance in reference the size of concessions to every aim of optimization is (criterion). The size of these concessions we estimate in following way:

$$Ui = \sum_{\substack{j=1 \\ j \neq i}}^{n\_k} \left| Ch_i(W_i, S_i) - Ch_i(W_i, S_j) \right| \tag{1}$$

where:

Ui - the concession treating to i-th aim (criterion) for benefits of remaining (the loss of value of optimum characteristic solution in relation to i-th criterion for reinforcement solutions characteristic of remaining criteria),

$Ch_i(\,)$ - characteristic treating to i-th criterion,

W_i - weights of elements (nodes or connections) in optimal solution regard i-th criterion,

S_i - estimation of optimum solution structure regard i-th criterion,

n_k - the number of criterions.

To assure the comparability of criteria characteristics we can use with version standardized:

$$Ui = \sum_{\substack{j=1 \\ j \neq i}}^{n\_k} \left| Ch_i(W_i, S_i) - Ch_i(W_i, S_j) \right| / Ch_i(W_i, S_i) \tag{2}$$

Among remembered aims (criterions) of optimization in nets and connected with characteristics we can distinguish next profiles:

- critical path,
- shortest path,
- minimal spanning tree,
- maximal flow,
- minimal cost flow,
- cardinality matching,
- Hamilton road,
- optimal task scheduling.

Usually we present network structures with help of graph or matrix:

$$\begin{pmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,n} \\ w_{2,1} & w_{2,2} & \dots & w_{2,n} \\ \dots & \dots & \dots & \dots \\ w_{n,1} & w_{n,2} & \dots & w_{n,n} \end{pmatrix} \tag{3}$$

Matrices imitate on example the character of connections and treating them weight values. Introduced in Figure 1 net structure can be described as matrix (Table 1).

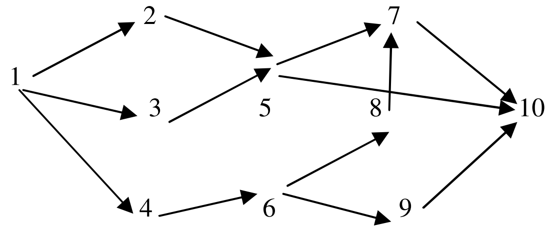

Fig. 1. Example of network structure

Table 1. Net (fig. 1) connections and value of their capacity (the weight)

	1	2	3	4	5	6	7	8	9	10	sum_w
1		6	4	5							15
2					7						7
3					6						6
4						3					3
5							9			4	13
6								6	4		10
7										6	6
8							7				7
9										5	5
10											0
sum_col	0	6	4	5	13	3	16	6	4	15	
min_w_c		6	4	3	13	3	6	6	4		

Preliminary and auxiliary parameters which will used to calculation net maximum capacity are showed in Table 1. The sums in individual rows which reflect the level of maximum size floating media in given node are placed in the last column. Obviously the quantity of medias floating in node (sum_w(i)) is equal quantity of floating out (sum_col(i)). The capacity of node will be equal:

$$min\_w\_k(i) = min\{sum\_w(i), sum\_col(i)\} \tag{4}$$

2. Compromise with taking into account different weight compositions

The defining the losses value of optimization formula in such case will subject to modification:

$$Ui = \sum_{\substack{j=1 \\ j \neq i}}^{n\_k} \left| Ch_i(W_i, S_i) - Ch_i(W_j, S_i) \right| / Ch_i(W_i, S_i) \tag{5}$$

Taking into account introduced conditions we can, with help of Solver, find maximum flow through given net i.e. from start point 1 to target point 10 (Fig. 1). The results of optimization is showed in Table 2.

Table 2. Maximal flow in net from Figure 1

	6	4	3							13
				6						6
				4						4
					3					3
						6			4	10
							0	3		3
									6	6
						0				0
									3	3
										0
0	6	4	3	10	3	6	0	3	13	max_p

Increasing flow capacity of connection between nodes 4 and 6 (W4,6) from 3 to 8 (Fig. 1) we lead to enlargement the maximum flow from 13 to15 (Tables 2 and 3).

Table 3. The results of optimization on the base of maximum flow algorithm in net from Figure 1 after increment the capacity W4,6 from 3 to 8

	6	4	5							15
				6						6
				4						4
					5					5
						5			5	10
							1	4		5
									6	6
						1				1
									4	4
										0
0	6	4	5	10	5	6	1	4	15	max_p

The next correction of weights in our example consist in decrease in connection 4-6 weight to zero. Introduced in board 4. solution is an effect of this analyze. Assuming the invariability of structures (S = const), despite of appearing zero values in connections in optimal solutions) we obtain values of main characteristic:

$Ch(W_{4,6} = 3, S = \text{const}) = 13,$
$Ch(W_{4,6} = 8, S = \text{const}) = 15,$
$Ch(W_{4,6} = 0, S = \text{const}) = 10$

Table 4. The results of net optimization (from Fig. 1) relating to maximum flow after decreasing the capacity *W*4,6 from 3 to 0

	6	4	0							10
				6						6
				4						4
					0					0
						6			4	10
							0	0		0
									6	6
						0				0
									0	0
										0
0	6	4	0	10	0	6	0	0	10	max_p

Applying methodology leaning on (5) we receive the value of loss for individual weight sets:

$U(1) = |13\text{-}15|/13 + |13\text{-}10|/13 = 0{,}385$
$U(2) = |15\text{-}13|/15 + |15\text{-}10|/15 = 0{,}467$
$U(3) = |10\text{-}13|/10 + |10\text{-}15|/10 = 0{,}800$

Minimization of variance leads to conclusions relating the corrections of value of *function loss* (Table 5, Figure 2):

Table 5. The correction of value of loss (concessions on compromise)

Variance	U(i)	correct_u	average	variance
1	0,385	0,165667	0,550667	
2	0,467	0,083667	0,550667	0
3	0,8	–0,24933	0,550667	

If we find place in sense of flows levels where the average of value of *function loss* is equal 0,5506. (Table 5) and the variance of corrections of flow is minimal (on example with help of Solver), it will be situated according with placed in Table 6 description.

How to see, in such approach it does not guarantee unambiguous localization of compromise.

Assuming deterministic valuation of compromise e.g. one level of its value, we obtain the solution responding the minimum value of flow corrections variance (Table 7).

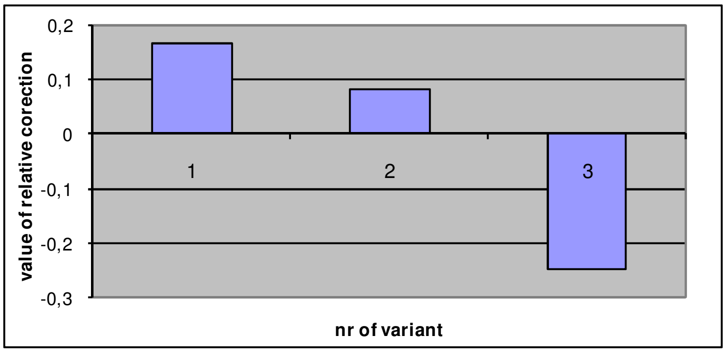

Fig. 2. Scale of corrections of value function loss

Table 6. The correction of flows for found compromise (in interval variant)

after_corr max1	after_corr max2	after_corr max3	after_corr U1	after_corr U2	after_corr U3	average loss
12,9949	14,9949	9,9949	0,3848	0,4668	0,8004	0,5507
befor_corr max1	befor_corr max2	befor_corr max3				
13,000	15,000	10,000				
corr max1	corr max2	corr max3				variance corr
0,0051	0,0051	0,0051				0,0000

Table 7. The correction of flows for compromise (in deterministic variant)

after_corr max1	after_corr max2	after_corr max3	after_corr U1	after_corr U2	after_corr U3	average loss
12,0000	12,0000	12,0000	0,0000	0,0000	0,0000	0,0000
befor_corr max1	befor_corr max2	befor_corr max3				
13,000	15,000	10,000				
corr max1	corr max2	corr max3				variance corr
1,0000	3,0000	2,0000				1,0000

More simply method consist in find the parameters of maximum flow and answering them the weights.
Chosen the parameter of maximum flow can be calculated by methods of the smallest squares, eigenvectors or expected values [6].
If from among proposed sets of weights we found the average the value of maximum flow, that for such chosen parameters we can fit the optimum values of weights. In our example the average of flows is equal 12,67. We match weights with help of

Solver. So we define values of criteria as averages of flow value, such in Table 8 (in aim cell of Solver program):

Table 8. Result of searching weights for chosen set (conciliatory variant) flow:12,67

	5,55	3,55	3,55							12,66
				5,55						5,55
				3,55						3,55
					3,55					3,55
						6			3,11	9,11
							0	3,55		3,55
									6	6
						0				0
									3,55	3,55
										0
0	5,55	3,55	3,55	9,11	3,55	6	0	3,55	12,7	max_p

For the compromise responding flow level 12,00 (Table 7) we receive the following weights (Table 9).

Table 9. Searching of weights for set compromise flow: 12,0

	5,22	3,55	3,22							12
				5,22						5,22
				3,55						3,55
					3,22					3,22
						6			2,78	8,78
							0	3,22		3,22
									6	6
						0				0
									3,22	3,22
										0
0	5,22	3,55	3,22	8,78	3,22	6	0	3,22	12	max_p

We would emphasize, that different weight sets (responded flow capacities) guarantee the same level of flow, e.g. - Table 10.

So we can introduce the additional criteria of weights configuration relating, on example, to criterion of capacity unification (which would answer the equal diameters of pipes transporting gas or liquid media).

Table 10. Invariantly weight set guaranteeing flow level 12,0

	5,33	3,33	3,33							12
				5,33						5,33
				3,33						3,33
					3,33					3,33
						6			2,67	8,67
							0	3,33		3,33
									6	6
						0				0
									3,33	3,33
										0
0	5,33	3,33	3,33	8,67	3,33	6	0	3,33	12	max_p

3. Compromise for different compositions of criteria

The inclusion next criterions is difficult and complex question and require procedural (algorithmic) adjustment of finding optimum methods assuring flow on level 12,0. For example the connection of criterions of the largest flow and the shortest path seems algorithmically independent, and in majority cases one treats it in such way. However if we identify (or only connect) length of path with costs, and flow with capacity of flow, which influences also on costs, then the matter a little complicates and requires additional analysis.

If to deal out costs component connected with capacity and on this basis find the cheapest connection (Fig. 1) we obtain solution introduced with help of Table 11 and Figure 3.

Table 11. The shortest path from point 1 to point 10 (Fig. 1):1-3-5-10

			węzeł	2	3	4	5	6	7	8	9	10
1				6	4	5	*	*	*	*	*	*
2			6	*	*	*	13	*	*	*	*	*
3			4	*	*	*	10	*	*	*	*	*
4			5	*	*	*	*	8	*	*	*	*
5			10	*	*	*	*	*	19	*	*	14
6			8	*	*	*	*	*	*	14	12	*
7			19	*	*	*	*	*	*	*	*	15
8			14	*	*	*	*	*	21	*	*	*
9			12	*	*	*	*	*	*	*	*	17
			min	6	4	5	10	8	19	14	12	14

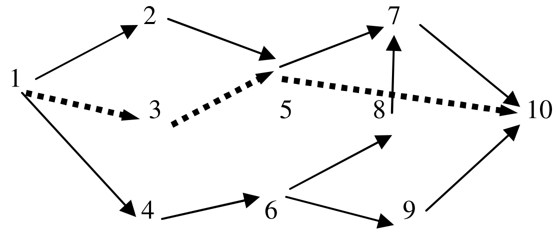

Fig. 3. The shortest road in net from weights from table 1

The length of critical path is named $Ch2(W2,S2)$ and equals 4+ 6 + 4 = 14. Applying methodology (5) we count the scale of loss of values $U1,U2$ for both criterions.

$U_1 = |13 - 13|/13 = 0$
$U_2 = |14 - 12|/14 = 0{,}143$

If for any criterion the function of loss will be equal zero that means the resulting from regard of remaining criterions changes do not influence on optimum solution according with given criterion. In our case it concerns the criterion of maximum flow $U1 = 0$. Defining firstly the length of the shortest path we can examine if the structure of remaining structure may protect (guarantee) the invariability of flow level. In our example aiming to decrease of *loss value* ($U_2 \rightarrow 0$) we admit that without damage for level of flow we can reduce weight connection 3-5 from 6 to 4 and at the some time it corrects shortest path from 14 to 12. In this case the *loss value* will be equal:

$U_2 = |12 - 12|/12 = 0$

The composition of criterions: minimum spanning tree as well as maximum flow is much closer to concrete real situations. The minimum spanning tree shape described with help of analysis provided in Table10 is presented in Figure 4.

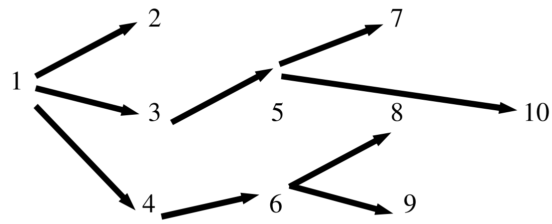

Fig. 4. The minimum spanning tree structure (shape) on base of net from Figure 1

For spanning tree from Figure 4 we present weight table (Table 12) as well as we count on example with help of Solver maximum flow.

Table 12. The estimations of characteristic $Ch1(W2,S2)= 4$

	0	4	0							4,00
										0,00
				4						4,00
					0					0,00
						0			4	4,00
							0	0		0,00
										0,00
										0,00
										0,00
										0,00
0	0	4	0	4	0	0	0	0	4	max_p

Different way of generalization of formula (2) is simultaneous regarding both the changeability of weights and the changeability of structure of solution:

$$Ui = \sum_{\substack{j=1 \\ j\neq i}}^{n\_k} \left| Ch_i(W_i,S_i) - Ch_i(W_j,S_j) \right| / Ch_i(W_i,S_i) \tag{6}$$

Let's present the remaining components values of *loss value* (2) yet:

$Ch_1(W_1,S_1) = 13$
$Ch_1(W_2,S_2) = 4$
$Ch_2(W_2,S_2) = (6+4+5+6+3+9+6+4+4) = 47$
$Ch_2(W_1,S_1) = (6+4+3+4+3+6+0+3+4) = 33$

The suitable levels of *loss value* will equal:

$U_1 = |13-4|/13 = 0{,}692$
$U_2 = |47-33|/47 = 0{,}085$

Joining to spanning tree set of connections 2-5, 7-10, 9-10 gives protection of maximum flow *max_p* = 13. Connection 6-8 wouldn't transport medium. The levels of loss value will be counted as follows:

$U_1 = |13-13|/13 = 0{,}0$
$U_2 = |47-33|/47 = 0{,}085$

The variance of loss values $U1$ and $U2$ strong decreases what indicates on closeness of compromise.

Conclusions

1. Adopting methodology leaning on estimation loss value of (concessions on compromise working out) we solve among other the problem of choice between functions (2), (5) and (6). The analysis will consist in decision if task of multicriteria optimization will be based on change weights or structure or both of them.
2. Utilization methodology on calculation loss values can lead both to deterministic solutions and uncertain (on example interval's; Table 6).
3. Different task connected with searching of compromise in optimization in nets is problem of investigation, here called, network structure sensibility, what would help in finding compromise (in aspect of rate of convergence). This problem has not been developed in presented material.
4. Drawback of method studying the level of loss value is possibility of obtainment of compromise, which it will be "the lost" in reference to all of majority criterions. We can remove it considerably preferring the minimization of lose values over minimization variances of these losses.

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