'ONE-SHOT' IDENTIFICATION OF LASER INTENSITY IN A PROCESS OF THIN METAL FILM HEATING

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Abstract. Thermal interactions between thin metal film and external laser pulse are considered. In particular on a basis of the knowledge of surface temperature distribution the laser intensity is estimated. The problem is described by dual-phase-lag model in which two relaxation parameters (relaxation time and thermalization time) are introduced in order to take into account the microscopic thermal interactions. The laser action is taken into account by additional term (source function) supplemented the basic energy equation. In the paper the mathematical model of the thin film heating process is discussed, the inverse problem is formulated and also the example of computations is presented.

1. Formulation of the problem

We will consider a thin film of thickness L and an initial temperature distribution $T(x, 0) = T_0$. The constant thermal properties, this means the thermal conductivity λ [W/(mK)] and the volumetric specific heat c [J/(m^3 K)] are assumed. A front surface $x = 0$ is irradiated by high-intensity and ultra-short duration laser beam as shown in Figure 1. A heat transfer in direction perpendicular to the layer is taken into account (this assumption is entirely acceptable) and then the temperature distribution is described by the following equation [1, 2]

$$c\left[\frac{\partial T(x,t)}{\partial t}+\tau_q\frac{\partial^2 T(x,t)}{\partial t^2}\right]=\lambda\frac{\partial^2 T(x,t)}{\partial x^2}+ \lambda\tau_T\frac{\partial^3 T(x,t)}{\partial t\,\partial x^2}+Q(x,t)+\tau_q\frac{\partial Q(x,t)}{\partial t} \tag{1}$$

where τ_q [s], τ_T [s] are the relaxation time and thermalization time, respectively, $Q(x, t)$ [W/m^3] is the capacity of internal heat sources. Function $Q(x, t)$ results from laser action and it can be expressed as follows [3]

$$Q(x,t)=\sqrt{\frac{\beta}{\pi}}\frac{1-R}{t_p\,\delta}I\exp\left[-\frac{x}{\delta}-\beta\frac{(t-2t_p)^2}{t_p^2}\right] \tag{2}$$

where I [W/m^2] is the laser intensity which is defined as total energy carried by a laser pulse per unit cross-section of laser beam, t_p is the characteristic time of laser pulse, δ is the characteristic transparent length of irradiated photons called the absorption depth, R is the reflectivity of the irradiated surface and $\beta = 4\ln 2$ [3]. The local and temporary value of $Q(x, t)$ results from the distance x between surface subjected to laser action and the point considered.

Taking into account the short period of laser heating, heat losses from front and back surfaces of thin film can be neglected [3], this means

$$q(0, t) = q(L, t) = 0 \tag{3}$$

where $q(x, t)$ is the heat flux.
Additionally, the initial conditions in the form

$$t = 0: \quad T(x, 0) = T_0 \ , \quad \left. \frac{\partial T(x, t)}{\partial t} \right|_{t=0} = 0 \tag{4}$$

are also known.

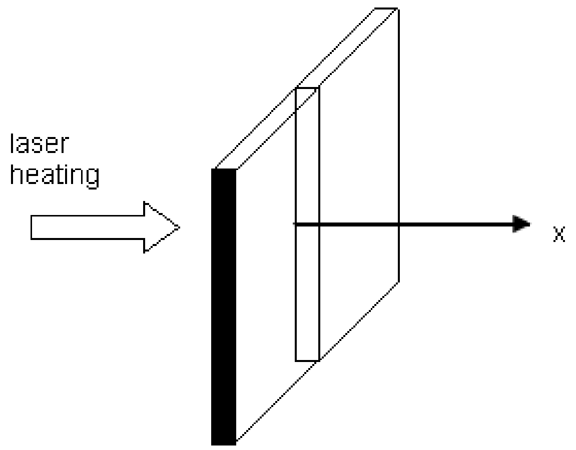

Fig. 1. Thin film

2. Sensitivity analysis

The problem of laser intensity identification has been solved using the least squares criterion in which the sensitivity coefficients appear. So, on the basis of mathematical model here discussed, the sensitivity one concerning the influence of laser intensity I on the perturbations of temperature distribution in domain considered must be formulated. This model can be formulated using the direct approach [4, 5] consisting in the differentiation of energy equation and boundary-initial conditions with respect to intensity I. So, the following additional problem is taken into account

$$
\begin{aligned}
&0<x<L: \quad c\left[\frac{\partial Z(x,t)}{\partial t}+\tau_q\frac{\partial^2 Z(x,t)}{\partial t^2}\right]=\lambda\frac{\partial^2 Z(x,t)}{\partial x^2}+\\
&\lambda\tau_T\frac{\partial^3 Z(x,t)}{\partial t\,\partial x^2}+\frac{\partial Q(x,t)}{\partial I}+\tau_q\frac{\partial}{\partial I}\left[\frac{\partial Q(x,t)}{\partial t}\right]\\
&x=0: \quad \frac{\partial Z(x,t)}{\partial x}=0\\
&x=L: \quad \frac{\partial Z(x,t)}{\partial x}=0\\
&t=0: \quad Z(x,0)=0, \quad \left.\frac{\partial Z(x,t)}{\partial t}\right|_{t=0}=0
\end{aligned}
\tag{5}
$$

The second order sensitivity model can be also formulated, in particular

$$
\begin{aligned}
&0<x<L: \quad c\left[\frac{\partial U(x,t)}{\partial t}+\tau_q\frac{\partial^2 U(x,t)}{\partial t^2}\right]=\lambda\frac{\partial^2 U(x,t)}{\partial x^2}+\lambda\tau_T\frac{\partial^3 U(x,t)}{\partial t\,\partial x^2}\\
&x=0: \quad \frac{\partial U(x,t)}{\partial x}=0\\
&x=L: \quad \frac{\partial U(x,t)}{\partial x}=0\\
&t=0: \quad U(x,0)=0, \quad \left.\frac{\partial U(x,t)}{\partial t}\right|_{t=0}=0
\end{aligned}
\tag{6}
$$

at the same time $Z=\partial T/\partial I$, $U=\partial^2 T/\partial I^2$.

One can see that the solution of boundary-initial problem (6) is trivial, namely $U(x,t)=0$. It results from the form of boundary-initial conditions and the absence of source term in the basic differential equation.

So, in the case considered the Taylor series takes a form

$$
T(x,t,I_0+\Delta I)=T(x,t,I_0)+Z(x,t,I_0)\,\Delta I \tag{7}
$$

and this dependence is exact. It should be pointed out that the linear form of (7) causes that the identification of laser intensity can be realized very quickly (one-shot formula).

3. Inverse problem

The aim of our investigations is to estimate the value of laser intensity I assuring the postulated temperature distribution on the surface of thin film. It should be pointed out that the transient temperature field in the case considered can be 'measured' indirectly by the measurements of reflectivity or transmissivity variation. The variation of reflectivity or transmissivity is proportional to the variation of temperature in this time scale [6], so it can be assumed that the values $T_d^{\,f}$ at the point $x = 0$ for times t^f are known, namely

$$T_d^f = T_d\,(0\,, t^f\,)\,, \qquad f = 1, 2, ..., F \tag{8}$$

To solve the inverse problem, the least squares criterion is applied [7-9]

$$S(I) = \frac{1}{F}\sum_{f=1}^{F}\left(T^f - T_d^{\,f}\right)^2 \tag{9}$$

where $T^f = T(0,\, t^f\,)$ is the calculated temperature at the point $x = 0$ and time t^f for arbitrary assumed value of $I = I_0$.
Criterion (9) is differentiated with respect to unknown laser intensity I and next the necessary condition of optimum is applied

$$\frac{\mathrm{d}\,S}{\mathrm{d}\,I} = \frac{2}{F}\sum_{f=1}^{F}(T^f - T_d^f\,)\left.\frac{\partial\, T^f}{\partial\, I}\right|_{I\,=\,I_0} = 0 \tag{10}$$

where I_0 is an arbitrary assumed value of I. Next, the function T^f is expanded in a Taylor series about known value of I_0, this means

$$T^f = T_0^f + \left.\frac{\partial T^f}{\partial\, I}\right|_{I = I_0}(I - I_0) \tag{11}$$

where T_0^f denotes the surface temperatures for successive time levels calculated under the assumption that $I = I_0$. Putting (11) into (10), after the mathematical manipulations one has

$$I = I_0 + \frac{\sum\limits_{f=1}^{F} Z_0^{\,f}\left(T_d^f - T_0^f\right)}{\sum\limits_{f=1}^{F}\left(Z_0^f\right)^2} \tag{12}$$

where

$$Z_0^f = \left. \frac{\partial T_0^f}{\partial I} \right|_{I=I_0} \tag{13}$$

are the sensitivity coefficients.

4. Example of computations

The thin film of thickness L = 1 μm is considered. Thermophysical parameters of material (gold) are the following: λ = 315 W/(mK), c = 2.4897·10^6 J/(m^3 K), τ_q = = 8.5 ps (1 ps = 10^{-12} s), τ_T = 90 ps.

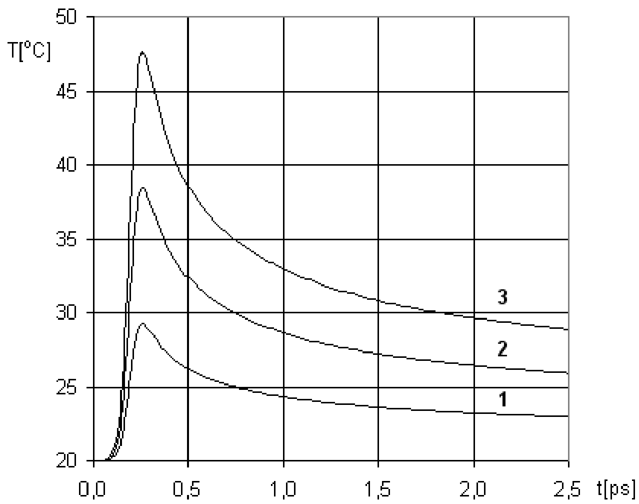

Fig. 2. Course of temperature at x = 0

Initial temperature T_0 = 20°C. The layer is subjected to a short laser pulse irradiation (R = 0.93, t_p = 100 fs, δ = 15.3 nm) and different intensity of laser (I = 10, 20, 30 W/m^2).

This direct problem has been solved by means of the finite difference method [2] under the assumption that mesh step equals h = 2 nm and time step equals Δt = = 0.005 ps. In Figure 2 the course of temperature at the irradiated surface (x = 0) is shown (1 – I = 10, 2 – I = 20, 3 – I = 30 [W/m^2].

Next, the inverse problem has been considered. On the basis of temperature distribution at the point x = 0 the laser intensity I is identified. Using the algorithm presented in chapter 3 and assuming that I_0 = 0 the real value of I has been exactly obtained both in the case of undisturbed (Fig. 2) and also disturbed surface temperatures.

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