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THE RESEARCHES OF FINDING CONSISTENT IN SAATY'S MATRIX JUDGMENTS WITH INTERVAL PARAMETERS

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Abstract. The goal of our proposition is work out the conception coming to satisfy given requires to consistent level in condition of interval parameters in Saaty's matrix. These requires refer to speed of convergence and scale of value or variance of corrective judgments too. At last we made a simple comparison between proposed variants of modification in reference to parameter of level of consistent (inconsistent), number of iteration, variance of corrective increments.

Introduction

Uncertain information may be presented by interval [1-3], fuzzy [4, 5] or approximation [2, 6] character of evaluating, converting and inference [5, 7, 8]. We can consider different but realistic variants of situations and formulated problems. Starching realistic problems we often found situations, when some parameters have deterministic or even "frozen" form or level. The second (last) goal is the proposition of investigation and methodology of improving of judgment matrix [9-11]. It should be done on the possibility earlier stages of task (to eliminate progression of errors). Chosen convention permit us define extra criterion relating on increasing pace of convergence to optimal (possibly in given situation) level of consistent or warranting minimal corrections in experts opinions (judgments). To have permanently control of changes in structure of relative opinions we propose iterative procedure. It warrants, as well, possibility of change extra-criterion character during improving process. Proposed method has some advantages which will be presented in the last two section of this paper. However the method has also disadvantage consist in disturbing of consistent level in the first several stages (iterations) of improving process. We should say that this phenomena don't occurs very often and we may avoid it by using special worked out heuristic (authors regret about not described them in this paper).

1. Adaptation (modification) proposed method of improved consistent to interval conditions

Process of adaptation is simple and consist in narrowed increment correction to ranks of setting interval (Fig. 1).

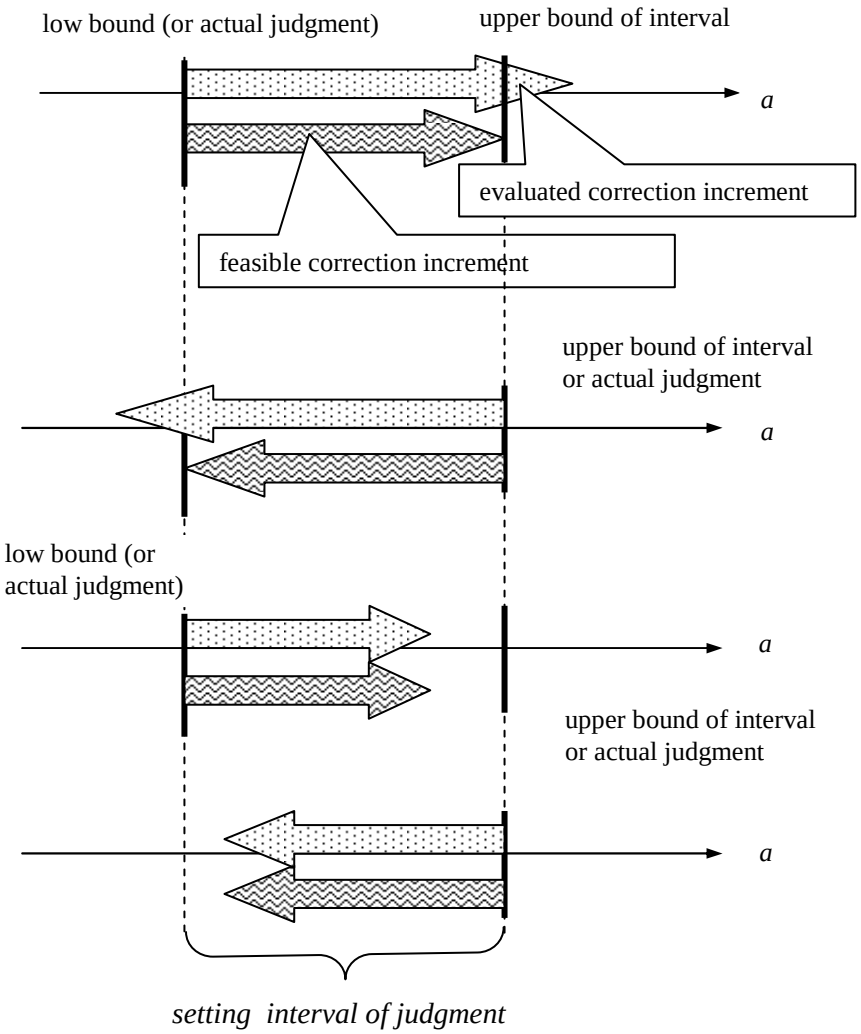

Fig. 1. The rules of adaptation method of improvement level of consistent to interval condition. Actual judgment means judgment after last correction

To realization correction (improvement) method we start from initial point (low or upper bound of interval or actual level of judgment). Then we evaluate the correction increments and cutting its according to width of interval (see Fig. 1).

Choice of correcting increment we can realize like in Figure 2. As example 1 we use date from [12] (Table 1).

Table 1.

Date of relative pairwise judgment from [12]

Goal	Price	Quality stability	Financial serv.	Customer serv.
Price	1	[2,3]	[3,5]	[4,7]
Quality stability	[1/3,1/2]	1	[1,2]	[3,4]
Financial service	[1/5,1/3]	[1/2,1]	1	[2,4]
Customer service	[1/7,1/4]	[1/4,1/3]	[1/4,1/2]	1

2. Experimental approval the conception of finding consistent level

We start from upper bounds of interval as well as use correcting row k = 1 and column k = 4 simultaneously.

Tables 2.

The example of improvement consistent level (variant 1)

1-st iteration

date a(i,j)

1	3	5	7
0,333	1	2	4
0,200	0,500	1	4
0,143	0,250	0,250	1

correcting row k= 1

0	0	0	0
	0	0,333	1,667
		0	2,600
			0

correcting column k= 4

0	1,250	3,250	0,000
	0	1,000	0,000
		0	0,000
			0

sum column

1,533	4,500	8,000	15,000

				sum row	vector w	u=A*w	lambda =u/w
0,652	0,667	0,625	0,467	2,411	0,603	2,543	4,220
0,217	0,222	0,250	0,267	0,956	0,239	1,003	4,196
0,130	0,111	0,125	0,267	0,633	0,158	0,645	4,074
0,093	0,056	0,031	0,067	0,247	0,062	0,247	4,007

sum column

1	1	1	1	sum col>	4,000	sum col>	1,000	sum col>	4,191	average>	4,164

consistent	inconsistent CI	limit
0,06056	0,054502411	

After 7-th iteration we obtain level of inconsistent clause to zero:
7-th iteration

date a(i,j)

1	2,333	3,000	7,000
0,429	1	1,286	3,000
0,333	0,778	1	2,333
0,143	0,333	0,429	1

correcting row k= 1

0	0	0	0
	0	0,000	0,000
		0	0,000
			0

correcting column k= 4

0	0,000	0,000	0,000
	0	0,000	0,000
		0	0,000
			0

sum column

1,762	4,111	5,286	12,333

				sum row	vector w	u=A*w	lambda =u/w
0,568	0,568	0,568	0,568	2,270	0,568	2,270	4,000
0,243	0,243	0,243	0,243	0,973	0,243	0,973	4,000
0,189	0,189	0,189	0,189	0,757	0,189	0,757	4,000
0,081	0,081	0,081	0,081	0,324	0,081	0,324	4,000

sum column

1	1	1	1	sum col>	4,000	sum col>	1,000	sum col>	4,000	average> 4,000

consistent	inconsistent CI	limit
1,5E-09	1,36732E-09	

The presentation all stages of improvement CI is in Figure 2.

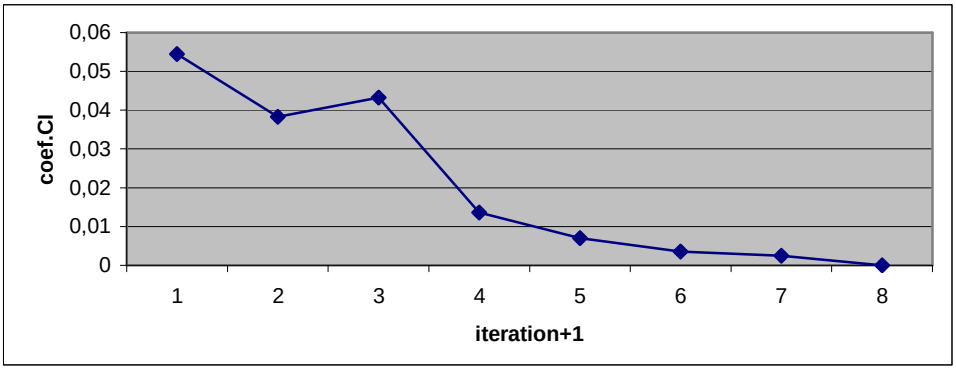

Fig 2. Improvement of CI

In second part of experiment we use averaging method of correction (described above) (Tables 3).

Tables 3.

Results of averaging method of correction without taking into account actual values of judgment (variant 2)

1-st iteration

date a(i,j)

1	3,000	5,000	7,000
0,333	1	2,000	4,000
0,200	0,500	1	4,000
0,143	0,250	0,250	1

sum column

1,533	4,500	8,000	15,000

averaged judgments without actual state

1	2,125	1,75	7
	1	1,333	2,333
		1	1,700
			1

averaged correction

0,000	0,875	3,250	0,000
	0,000	0,667	1,667
		0,000	2,300
			0,000

				sum row	vector w	u=A*w	lambda =u/w
0,652	0,667	0,625	0,467	2,411	0,603	2,543	4,220
0,217	0,222	0,250	0,267	0,956	0,239	1,003	4,196
0,130	0,111	0,125	0,267	0,633	0,158	0,645	4,074
0,093	0,056	0,031	0,067	0,247	0,062	0,247	4,007

sum column

1	1	1	1	sum col>	4,000	sum col>	1,000	sum col>	4,191	average> 4,164

consistent	inconsistent CI	limit
0,06056	0,054502411	

After 7-th iteration we obtain results

date a(i,j)

1	1,313	2,333	7,000
0,762	1	1,777	5,331
0,429	0,563	1	3,000
0,143	0,188	0,333	1

sum column

2,190	2,876	5,110	15,331

averaged judgment without actual state

1	1,313	2,333	7
	1	1,777	5,331
		1	3,000
			1

averaged correction

0,000	0,000	0,000	0,000
	0,000	0,000	0,000
		0,000	0,000
			0,000

				sum row	vector w	u=A*w	lambda =u/w
0,457	0,457	0,457	0,457	1,826	0,457	1,826	4,000
0,348	0,348	0,348	0,348	1,391	0,348	1,391	4,000
0,196	0,196	0,196	0,196	0,783	0,196	0,783	4,000
0,065	0,065	0,065	0,065	0,261	0,065	0,261	4,000

sum column

1	1	1	1	sum col>	4,000	sum col>	1,000	sum col>	4,000	average> 4,000

consistent	inconsistent CI	limit
8,6E-10	7,70166E-10	

The presentation all stages of improvement CI is in Figure 3.

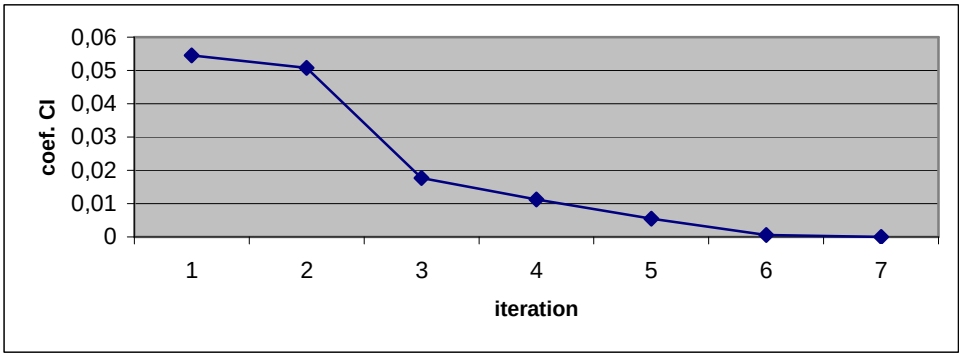

Fig. 3. Improvement of inconsistent CI

We may realize averaging method in two variants: without taking into account actual state (corrected judgment) or with taking into account actual state. In above part of experiment we don't take into account corrected judgment. There is interesting to compare two variants of averaging method (Tables 4).

Tables 4.

Results of averaging method of correction with taking into account actual value of judgment (variant 3)

1-st iteration

date a(i,j)

1	3,000	5,000	7,000
0,333	1	2,000	4,000
0,200	0,500	1	4,000
0,143	0,250	0,250	1

sum column

1,533	4,500	8,000	15,000

0,652	0,667	0,625	0,467
0,217	0,222	0,250	0,267
0,130	0,111	0,125	0,267
0,093	0,056	0,031	0,067

sum column

1	1	1	1

averaged judgments with actual state

1	2,563	3,917	7
	1	1,667	3,444
		1	2,850
			1

averaged correction

0,000	0,438	1,083	0,000
	0,000	0,333	0,556
		0,000	1,150
			0,000

sum row	vector w	u=A*w	lambda =u/w
2,411	0,603	2,543	4,220
0,956	0,239	1,003	4,196
0,633	0,158	0,645	4,074
0,247	0,062	0,247	4,007
sum col> 4,000	sum col> 1,000	sum col> 4,191	average> 4,164

consistent	inconsistent CI	limit
0,06056	0,054502411	

After 14-th iteration we obtain:

14-th iteration

date a(i,j)

1	2,026	3,470	7,000
0,494	1	1,783	3,531
0,288	0,561	1	2,044
0,143	0,283	0,489	1

sum column

1,782	3,587	6,253	12,575

0,561	0,565	0,555	0,557
0,277	0,279	0,285	0,281
0,162	0,156	0,160	0,163
0,080	0,079	0,078	0,080

sum column

1	1	1	1

averaged judgments with actual state

1	1,995	3,455	7
	1	1,752	3,506
		1	2,021
			1

sum row	vector w	u=A*w	lambda =u/w
2,238	0,559	2,238	4,000
1,122	0,280	1,122	4,000
0,641	0,160	0,641	4,000
0,317	0,079	0,317	4,000
sum col> 4,000	sum col> 1,000	sum col> 4,000	average> 4,000

averaged correction

0,000	0,031	0,015	0,000
	0,000	0,031	0,025
		0,000	0,023
			0,000

consistent	inconsistent CI	limit
7,9E-05	7,12468E-05	

The presentation all stages of improvement CI is in Figure 4.

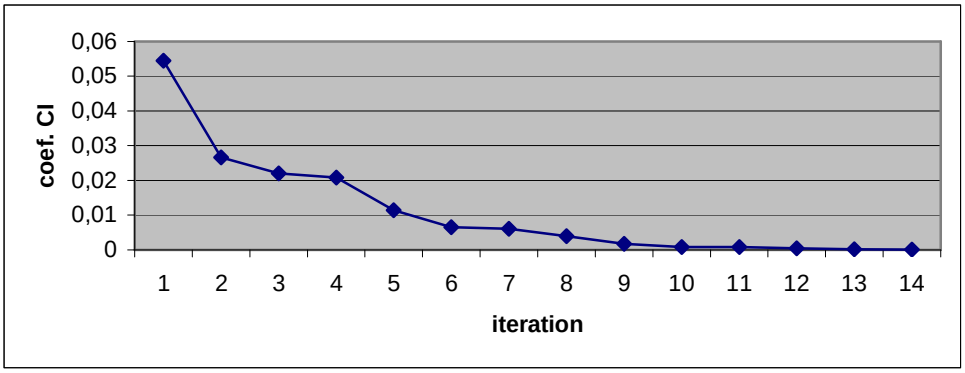

Fig. 4. Improvement of consistent CI

The corrective scale we estimate with help of average and variance of judgments correction. For three last variants (modification) of proposed improving method we obtain results presented in diagrams in Figures 5-7. The process of iteration starts from upper bounds and scale of correction was evaluated according to upper bounds too.

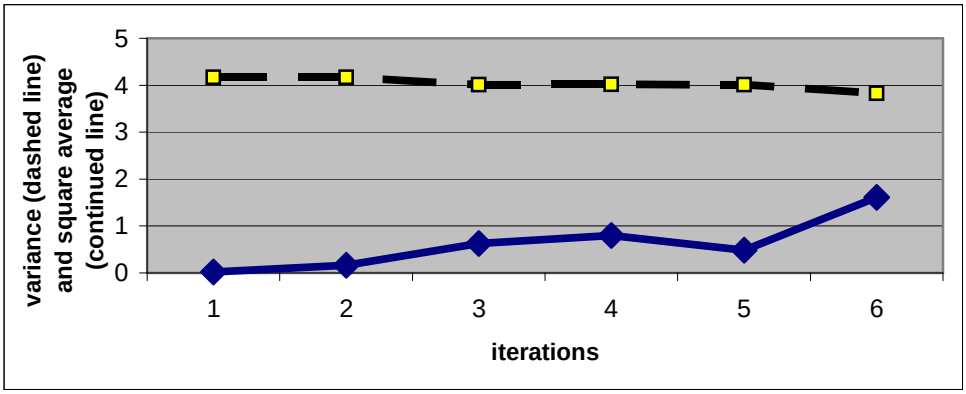

Fig. 5. Scale of judgments correction for variant 1

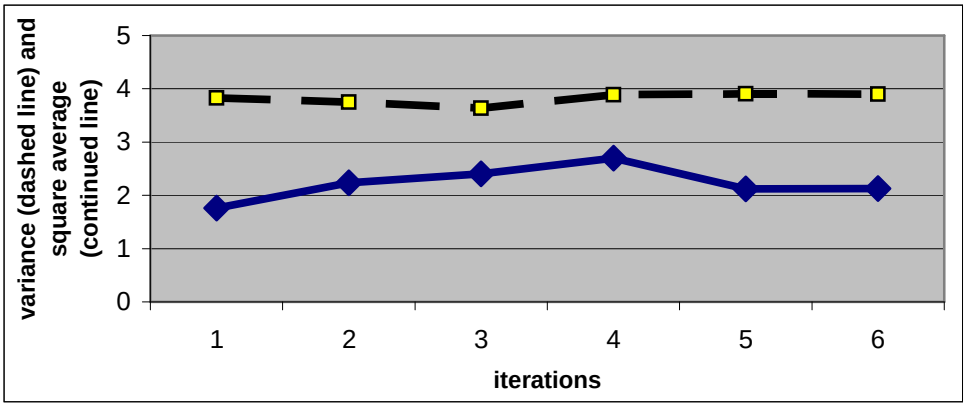

Fig. 6. Scale of judgments correction for variant 2

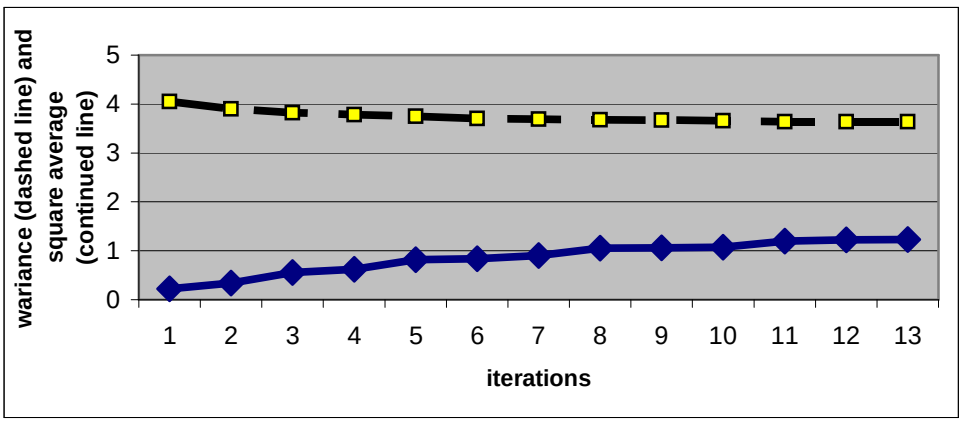

Fig. 7. Scale of judgments correction for variant 3

Conclusions

Proposed three variants of improving method characterized different scale of judgment correction (Figs. 5-7) and different speed of convergence to optimal consistent (Figs. 2-4). Taking into account actual state in averaging procedure we obtain more "smoothly" corrective judgment process but it is realized in much more cycle (about two times longer). Using interval date influents on enlarging time of convergence to consistent level, because same evaluated corrective increments must be abbreviate.

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