ON THE INFLUENCE OF INITIAL TEMPERATURE FLUCTUATIONS ON THE HEAT TRANSFER PROCESS IN TWO-CONSTITUENT MICROPERIODIC LAMINATED RIGID CONDUCTORS

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Abstract. In the note the tolerance averaged technique is applied to investigate a certain aspects of the interrelation between initial temperature fluctuations and the averaged temperature field in two-constituent microperiodic laminated rigid conductors. This influence is measured by proposed in the note influence functions. It was shown that the influence of initial temperature fluctuations onto a heat transfer process is observed only for very small initial time interval named in the note the initial layer interval.

1. Formulation of the problem

In the mathematical investigations of the heat transfer process in microstructural rigid conductors the basic modelling problem is strictly related to answer to the question how to determine a sequence of physical parameters uniquely determining the total temperature field. These parameters should be approximately
described by equations with sufficiently regular coefficients. In the case of rigid conductors with periodic microstructure the total temperature field is usually investigated as a sum of two terms

$$\theta = \vartheta + \theta_{res} \tag{1}$$

determined by the total temperature field and the geometrical properties of the periodical microstucture of a conductor. First term ϑ is usually defined as the averaged temperature field (over periodicity cells). Hence the second residual term is defined by the formula $\theta_{res} \equiv \theta - \vartheta$. However, the mentioned above modelling problem can be related here only to the residual field θ_{res}, i.e. a certain sequence of physical parameters uniquely determined this field is investigated. In the framework of well known homogenization theory as the aforementioned sequence can be taken into account averaged temperature. Indeed, at the related formula for residual field as a function of averaged temperature ϑ arrives the problem on the unit cell well known in the homogenization approach. In the framework of the tolerance averaging technique we deal with the quite different situation in which the residual field is determined by averaged temperature field and uniqueness conditions imposed on the fluctuation amplitudes provided that the sequence of shape functions are previously defined. Since in the framework of the simplified model of a heat conduction process in microperiodic rigid laminated conductors mentioned above uniqueness conditions are reduced to the initial conditions which can be treated as initial temperature fluctuations. Hence, the aforementioned idea arrive at an important possibility to formulate and investigate an influence of initial temperature fluctuations onto a heat transfer process in microperiodic rigid laminated conductors.

In the framework of the tolerance averaged model of heat conduction process in microperiodic rigid laminated conductors the residual temperature (at time instant t and in every point $\mathbf{x}$ from the region Ω occupied by the laminated conductor with l-periodic structure along Ox_3-axis) is assumed as a sum of terms of the form $g^A(x_3)\zeta^A(\mathbf{x},t)$, $A = 1,\ldots,N$, where $g^1(\cdot),\ g^2(\cdot),\ldots,g^N(\cdot)$ are l-periodic shape functions defined in R^3 are postulated *a priori* in every problem formulated in the framework of a certain tolerance averaged model. Moreover, $\zeta^1(\cdot,t),\ \zeta^2(\cdot,t),\ldots,\zeta^N(\cdot,t)$ defined in Ω_0, where

$$\Omega_0 \equiv \{\mathbf{x} = (x_1, x_2, x_3) \in \Omega : V(\mathbf{x}) \subset \Omega\} \tag{2}$$

and $V_\delta(x_1, x_2, x_3) \equiv \{(x_1, x_2, (1-\gamma)(x_3 - l/2) + \gamma(x_3 + l/2)) : 0 < \gamma < 1\}$. The total temperature field in the framework of tolerance averaged models are investigated in the form

$$\theta(\mathbf{x},t) = \vartheta(\mathbf{x},t) + g^1(x_3)\zeta^1(\mathbf{x},t) + g^2(x_3)\zeta^2(\mathbf{x},t) + \ldots + g^N(x_3)\zeta^N(\mathbf{x},t) \tag{3}$$

in which *averaged temperature* ϑ and *fluctuation amplitudes* $\zeta^1(\cdot,t),\ \zeta^2(\cdot,t),\ldots,\zeta^N(\cdot,t)$, which are new basic unknowns in the tolerance model, should be sufficiently regular, i.e. should be slowly varying functions in the Ox_3-axis direction together with all their derivatives applied in the model and shape functions should satisfy a certain additional conditions, [1]. Moreover, averaged temperature $\vartheta(\cdot)$ as well as fluctuations amplitudes $\zeta^1(\cdot,t),\ \zeta^2(\cdot,t),\ldots,\zeta^N(\cdot,t)$ should be defined in Ω_0, where

$$\Omega_0 \equiv \{\mathbf{x} = (x_1, x_2, x_3) \in \Omega : V(\mathbf{x}) \subset \Omega\} \tag{4}$$

and $V_\delta(x_1, x_2, x_3) \equiv \{(x_1, x_2, (1-\gamma)(x_3 - l/2) + \gamma(x_3 + l/2)) : 0 < \gamma < 1\}$. However, in this note we restrict considerations to regions Ω, for which $\Omega_.$ and Ω_0 coincides.

In the framework of tolerance averaged models, the influence of various physical parameters onto a solutions can be investigated. Hence, the aim of this note is to investigate how the initial temperature fluctuations can control heat transfer process in rigid laminated conductors with periodic microstructure. In the framework of this note just mentioned problem is restricted to investigations in the framework of the simplified tolerance model of parabolic heat transfer processes in two-constituent periodic laminated conductor which is illustrated in Figure 1.

It must be emphasized that in the framework of asymptotic homogenization technique the residual temperature field θ_{res} is uniquely determined by the averaged temperature field ϑ and hence the problem of influence of the initial temperature fluctuations onto a heat transfer process in rigid laminated conductors with periodic microstructure cannot be correctly formulated.

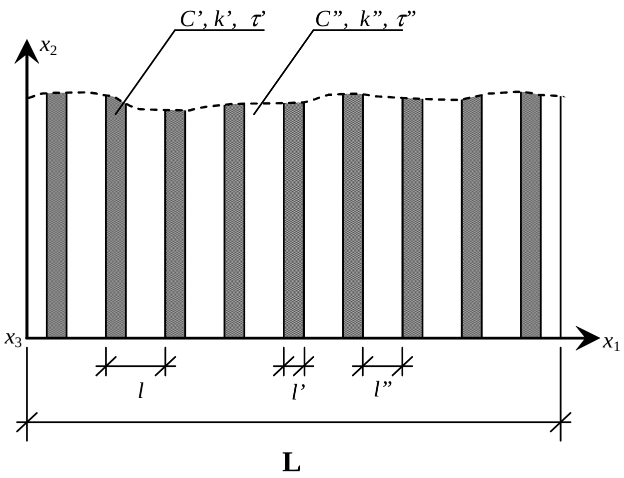

Fig. 1. Laminated two-component conductor made of isotropic constituents

2. Model equations

Considerations are restricted to laminated *l*-periodic rigid conductors having two isotropic constituents satisfying the following *heat proportional assumption*

$$\frac{c_2}{c_1} = \frac{k_2}{k_1} \tag{5}$$

in which c_1, c_2 are heat fluxes constants and k_1, k_2 are conductivity constants. Hence, $\delta \equiv c_2 / c_1$ is a certain parameter. The starting point of the considerations is a system of tolerance averaged model equations of the heat transfer process, [1], which for the aforementioned laminated conductors will be written in the form

$$
\begin{aligned}
&\langle c\rangle\dot{\vartheta} - \langle k\rangle\vartheta_{,ii} = [k]\zeta_{,3} \\
&l^2[\langle c\rangle\dot{\zeta} - \langle k\rangle\zeta_{,\alpha\alpha}] + \{k\}\zeta = -[k]\vartheta_{,3}
\end{aligned}
\tag{6}
$$

where we have introduced the following denotations:

$$
\begin{aligned}
&\langle c\rangle = \eta_1 c_1 + \eta_2 c_2 \in O(l^0) \\
&\{k\} = \langle kg_{,3}\, g_{,3}\rangle = 12\left(\frac{k_1}{\eta_1} + \frac{k_2}{\eta_2}\right) \in O(l^0) \\
&[k] = \langle kg_{,3}\rangle = k_1 - k_2 \in O(l^0)
\end{aligned}
\tag{7}
$$

The averaged temperature and fluctuation amplitudes, which are basic unknowns should satisfy conditions

$$
\vartheta(\cdot,t), \zeta(\cdot,t) \in SV_l(T) \tag{8}
$$

which means that above fields are slowly varying together with all derivative applied in (6) with respect to the aforementioned tolerance system *T* related to the periodic structure of the conductor. It must be emphasized that in the mentioned case of two-constituent laminated conductor we deal with exclusively one shape function *g*, which will be taken in the well known form of saw-like function illustrated in the Figure 2.

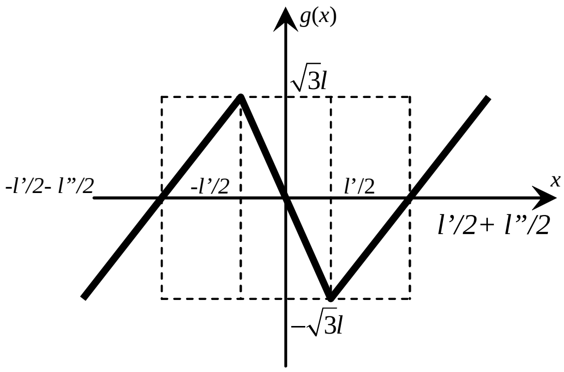

Fig. 2. Diagrams of the periodic shape functions

Hence, the total temperature field will be examined here in the form

$$
\theta(\mathbf{x},t) = \vartheta(\mathbf{x},t) + g(x_3)\zeta(\mathbf{x},t) \tag{9}
$$

where $\mathbf{x} \in \Omega$ and $t \geq 0$. Due to the fact that equations $(6)_2$ are ordinary differential equations with respect to ζ we have to formulate boundary conditions only for the macroscopic temperature field ϑ. The initial conditions for a temperature field θ are given by

$$\theta(\mathbf{x},0)=\overline{\theta}(\mathbf{x}),\ \mathbf{x}\in\Omega \tag{10}$$

where $\overline{\theta}(\mathbf{x})$ and $\tilde{\theta}(\mathbf{x})$ are the known functions. By means of the decomposition (5) we obtain

$$\vartheta(\mathbf{x},0)+g(\mathbf{x})\zeta(\mathbf{x},0)=\overline{\theta}(\mathbf{x}),\ \mathbf{x}\in\Omega \tag{11}$$

Hence we shall restrict considerations to the class of initial data in conditions (11) given by

$$\overline{\theta}(\mathbf{x})=\overline{\vartheta}(\mathbf{x})+g(\mathbf{x})\overline{\zeta}(\mathbf{x}),\ \mathbf{x}\in\Omega \tag{12}$$

Under the aforementioned restriction the initial conditions for equations (6) take the form

$$\vartheta(\mathbf{x},0)=\overline{\vartheta}(\mathbf{x}),\quad \zeta(\mathbf{x},0)=\overline{\zeta}(\mathbf{x}) \tag{13}$$

where $\overline{\vartheta}$, $\overline{\zeta}$ are assumed to be the known slowly-varying functions.

Let us observe that for a homogeneous rigid conductor $c_1 = c_2$, $k_1 = k_2$, and hence by means of $\langle g\rangle=0$ equation $(6)_1$ reduces to the known heat conduction equation

$$c\dot{\vartheta}-k\vartheta_{,ii}=0$$

At the same time from $(6)_2$ we obtain the ordinary differential equation for fluctuation amplitude ζ. If initial temperature fluctuation $\overline{\zeta}$ is equal to zero, then $\zeta=0$ for every $t\geq 0$. It means that initial conditions for ζ describe fluctuations of temperature also for a homogeneous conductor provided that the temperature distribution can be approximated by formula (9).

3. Analysis

In the subsequent considerations we shall assume that the initial values of ϑ and $\dot{\vartheta}$ in conditions (12) are constant: $\overline{\vartheta}=\text{const.}$, $\tilde{\vartheta}=\text{const.}$ In this section we are to show that under the aforementioned restrictions the evolution $\zeta^A(\mathbf{x},t)$, $t\geq 0$ of the initial temperature fluctuations $\overline{\zeta}^A(\mathbf{x})$, $\tilde{\zeta}^A(\mathbf{x})$, $\mathbf{x}\in\Omega_0$, can be described by the system of equations which is independent of the averaged temperature ϑ. To this end we shall decompose fluctuation amplitude ζ into two terms:

$$\zeta = \zeta_{hom} + \chi \tag{14}$$

We shall assume that the first term is uniquely determined by

$$\{k\}\zeta_{hom} + [k]\vartheta_{,3} = 0 \tag{15}$$

and will be termed *a gradient part of the fluctuation amplitude*. Hence, $\chi = \zeta - \zeta_{hom}$ and model equations (6) can be rewritten in the form

$$\begin{aligned} &\langle c\rangle\dot{\vartheta} - \langle k\rangle\vartheta_{,\alpha\alpha} + k^{eff}\vartheta_{,33} = [k]\chi_{,3} \\ &l^2[\langle c\rangle\dot{\chi} - \langle k\rangle\chi_{,\alpha\alpha}] + \{k\}\chi = l^2[\langle c\rangle\dot{\zeta}_{hom} - \langle k\rangle\zeta_{hom,\alpha\alpha}]_{,3} \end{aligned} \tag{16}$$

where

$$k^{eff} = \langle k\rangle - [k]^2/\{k\} = \frac{k_1 k_2}{\eta_1 k_2 + \eta_2 k_1} \tag{17}$$

Since coefficients in $(16)_1$ are of an order $O(l^0)$, RHS of $(16)_2$ are of an order $O(l^2)$. It is mean that in the first approximation right-hand side of $(16)_2$ can be omitted. Hence, model equations (16) takes the form

$$\begin{aligned} &\langle c\rangle\dot{\vartheta} - \langle k\rangle\vartheta_{,\alpha\alpha} + k^{eff}\vartheta_{,33} = [k]\chi_{,3} \\ &l^2[\langle c\rangle\dot{\chi} - \langle k\rangle\chi_{,\alpha\alpha}] + \{k\}\chi = 0 \end{aligned} \tag{18}$$

Equation $(16)_1$ will be referred to as *the mean heat conduction equation*. Initial condition $(13)_2$ under conditions (14) and (15) can be rewritten in the form

$$\chi(\mathbf{x},0) = \bar{\zeta}(\mathbf{x}) + \{k\}^{-1}[k]\bar{\vartheta}(\mathbf{x}),\ \mathbf{x}\in\Omega \tag{19}$$

which, together with $(13)_1$ collect initial conditions for unknowns ϑ and χ. Bearing in mind $(13)_1$, $(16)_1$, (19), (23) for the distribution of temperature in a microperiodic rigid conductor we obtain formula

$$\chi(\mathbf{x},0) = \bar{\zeta}(\mathbf{x}) + \{k\}^{-1}[k]\bar{\vartheta}(\mathbf{x}),\ \mathbf{x}\in\Omega \tag{20}$$

Mean heat conduction equation $(16)_1$ and evolution equation (23) together with initial conditions for averaged temperature ϑ given by $(13)_1$, (19) and related boundary conditions as well as initial conditions (19) for χ constitute the averaged mathematical model for the analysis of initial-boundary value problem. The basic unknowns are averaged temperature ϑ and function χ which will be called *fluctuation variable*.

Using $(13)_1$, $(16)_1$, (19), (23) for the distribution of temperature in a micro-periodic rigid conductor we obtain formula

$$\theta(\mathbf{x},t) = \vartheta(\mathbf{x},t) - g(\mathbf{x})\{k\}^{-1}[k]\vartheta,_3(\mathbf{x},t) + g(\mathbf{x})\chi(\mathbf{x},t) \tag{21}$$

and hence initial condition for total temperature is restricted to the form

$$\chi(\mathbf{x},0) = \bar{\chi}(\mathbf{x}) \equiv \bar{\zeta}(\mathbf{x}) + \{k\}^{-1}[k]\bar{\vartheta}(\mathbf{x}),\ \ \mathbf{x}\in\Omega \tag{22}$$

which holds for every $\mathbf{x}\in\bar{\Omega}$ and $t \geq 0$. From $(18)_2$ it follows that fluctuation variable χ determine only this part of temperature fluctuations which is independent of averaged temperature ϑ. If initial values of χ are constant then fluctuation variable χ depend only on time and the right-hand side of the heat propagation equation $(16)_1$ is equal to zero. In this case the problems of finding averaged temperature ϑ and fluctuation variables χ are uncoupled.

In a special case, in which fluctuation variable is slowly-varying not only in the Ox_3-direction but also in every direction in R^3, fluctuation variable χ can should be governed by the *evolution equation*

$$l^2\langle c\rangle\dot{\chi} + \{k\}\chi = 0 \tag{23}$$

being a certain approximation of ordinary differential equation $(16)_2$. Hence, for given initial value (22) fluctuation variable χ can be determined from the *evolution equation* (23) and hence the as *the mean heat conduction equation* $(16)_1$ takes the form

$$\langle c\rangle\dot{\vartheta}(\mathbf{x},t) - \langle k\rangle\vartheta,_{\alpha\alpha}(\mathbf{x},t) + k^{eff}\vartheta,_{33}(\mathbf{x},t) = s(\mathbf{x},t) \tag{24}$$

where

$$s(\mathbf{x},t) = [k]e^{-l^2\{k\}^{-1}\langle c\rangle t}\bar{\chi},_3(\mathbf{x}) \tag{25}$$

will be termed as *a temperature pseudosource*. The temperature pseudosource has the following properties:

1° the temperature pseudosource depends on the gradient $\bar{\chi},_3(\mathbf{x})$ of initial value of the fluctuation variable $\bar{\chi}(\mathbf{x})$

2° the temperature pseudosource monotonically degrees in the time being

3° the temperature pseudosource is equal to zero provided that the conductor under consideration is a homogeneous one or the gradient $\bar{\chi},_3(\mathbf{x})$ of initial value of the fluctuation variable $\bar{\chi}(\mathbf{x})$ vanishes.

At the same time the total temperature field (21) takes the form

$$\theta(\mathbf{x},t) = \vartheta(\mathbf{x},t) + g(\mathbf{x})\left[\{k\}^{-1}[k]\vartheta,_3(\mathbf{x},t) + e^{-l^2\{k\}^{-1}\langle c\rangle t}\bar{\chi}(\mathbf{x})\right] \tag{26}$$

It must be emphasized that mentioned above properties of the temperature pseudosource general not necessary should be hold in cases in which in the tolerance model of heat conduction in the conductor under consideration more than one shape function is taken into account. This situation takes place for laminated conductors consisting of three and large number of constituents and will be analyzed in the separated paper.

4. Influence functions

To describe the influence of initial values of fluctuation variable $\overline{\chi}(\mathbf{x})$ onto a heat transfer process in the rigid laminated conductors with periodic microstructure the influence function will be defined. To this end we the following family of boundary value problems will be considered:

Find an averaged temperature field $\vartheta(\mathbf{x},t)$ defined for $\mathbf{x}$ from the region Ω occupied by the laminated conductor in the reference configuration and for $t \geq 0$ such that

$$\begin{aligned} &\langle c\rangle \dot{\vartheta}(\mathbf{x},t) - \langle k\rangle \vartheta_{,\alpha\alpha}(\mathbf{x},t) + k^{eff}\vartheta_{,33}(\mathbf{x},t) = s(\mathbf{x},t) \\ &\vartheta(\mathbf{x},0) = \kappa\overline{\vartheta}(\mathbf{x}) \\ &\vartheta|_{\partial\Omega}(\mathbf{x},t) = \kappa\overline{\vartheta}(\mathbf{x}) \end{aligned} \tag{27}$$

for given sufficiently regular temperature field $\overline{\vartheta}(\mathbf{x})$.

Every initial boundary value problem including in the mentioned above family of initial boundary value problems is, for every given *a priori* temperature field $\overline{\vartheta}(\mathbf{x})$, determined by gradient $\overline{\chi}_{,3}(\mathbf{x})$ of initial value of the fluctuation variable $\overline{\chi}(\mathbf{x})$ as well as the positive real parameter κ. Hence, solutions to this problem will be denoted by $\vartheta[\kappa,\overline{\chi}](\mathbf{x},t)$. Now we conclude that after determining averaged temperature field $\vartheta[\kappa,\overline{\chi}](\mathbf{x},t)$ being the solution to the problem (27) by virtue of (26) we can determine the total temperature field

$$\theta(\mathbf{x},t) = \vartheta[\kappa,\overline{\chi}](\mathbf{x},t) + g(\mathbf{x})\left[\frac{[k]}{\{k\}}\vartheta_{,3}[\kappa,\overline{\chi}](\mathbf{x},t) + e^{-l^2\{k\}^{-1}\langle c\rangle t}\,\overline{\chi}(\mathbf{x})\right] \tag{28}$$

It must be emphasized that the above formula is obtained in the framework of simplified tolerance model represented by (24) end hence it cannot describe behaviours like boundary effect phenomena which can be observed in laminated conductors under consideration. Now we are to consider two different initial boundary problems.

In the first initial-boundary value problem we shall assume that initial values of the fluctuation variable are equal to zero and the problem (27) in the considered case reduces to the form

$$
\begin{aligned}
&\langle c\rangle \dot{\vartheta}(\mathbf{x},t) - \langle k\rangle \vartheta,_{\alpha\alpha}(\mathbf{x},t) + k^{eff}\vartheta,_{33}(\mathbf{x},t) = 0 \\
&\vartheta(\mathbf{x},0) = \kappa\bar{\vartheta}(\mathbf{x}) \\
&\vartheta|_{\partial\Omega}(\mathbf{x},t) = \kappa\bar{\vartheta}(\mathbf{x})
\end{aligned}
\tag{29}
$$

Hence, we deal with the solution $\vartheta[\kappa,0](\mathbf{x},t)$ to the special case (29) of the problem (27). In the second case of initial boundary problem we shall assume given *a priori* initial values $\bar{\chi}(\mathbf{x})$ of the fluctuation variable and hence we shall investigate exact solutions $\vartheta[\kappa,\bar{\chi}](\mathbf{x},t)$ to the problem (27).

The influence of initial temperature fluctuations on the heat transfer process in two-constituent microperiodic laminated rigid conductors will be restricted in this note to the analysis the following nondimensional ratio

$$
\delta\theta(\mathbf{x},t) = \frac{\theta^{S}{}_{\kappa,\bar{\chi}^A}(\mathbf{x},t) - \theta^{rel}{}_{\kappa}(\mathbf{x},t)}{\kappa\bar{\vartheta}}
\tag{30}
$$

where temperatures $\theta^{S}_{\kappa,\bar{\upsilon}_0^A}(x_1,x_2,t)$ and $\theta^{rel}_{\kappa}(x_1,x_2,t)$ are obtained from (28) for $\vartheta = \vartheta^{S}_{\kappa,\zeta_0^A}(x_1,x_2,t)$ and $\vartheta = \vartheta^{rel}_{\kappa}(x_1,x_2,t)$, respectively. Hence

$$
\begin{aligned}
&\theta^{\kappa}_{\kappa,\bar{\upsilon}_0^A}(x_1,x_2,t) = \\
&\quad = \vartheta^{s}[\kappa,\bar{\chi}](x_1,x_2,t) + g(x_1,x_2)\left[\frac{[k]}{\{k\}}[\vartheta^{s}[\kappa,\bar{\chi}](x_1,x_2,t)],_3 + e^{-l^2\{k\}^{-1}\langle c\rangle t}\bar{\chi}(x_1,x_2)\right]
\end{aligned}
\tag{31}
$$

and

$$
\begin{aligned}
&\vartheta^{rel}_{\kappa}(x_1,x_2,t) = \\
&\quad = \vartheta[\kappa,\bar{\chi}](x_1,x_2,t) + g(x_1,x_2)\left\{\frac{[k]}{\{k\}}[\vartheta[\kappa,\bar{\chi}](x_1,x_2,t)],_3 + e^{-l^2\{k\}^{-1}\langle c\rangle t}\bar{\chi}(x_1,x_2)\right\}
\end{aligned}
\tag{32}
$$

By virtue of (28) one can obtain

$$
\begin{aligned}
&\delta\theta[\kappa,\bar{\chi}^A](x_1,x_2,t) = \delta\vartheta[\kappa,\bar{\chi}^A](x_1,x_2,t) + \\
&\qquad + g(x_1,x_2)\frac{[k]\vartheta,_3[\kappa,\bar{\chi}^A](x_1,x_2,t)\bar{\chi},_3(x_1,x_2) + e^{-l^2\{k\}^{-1}\langle c\rangle t}\{k\}\bar{\chi}(x_1,x_2)}{\kappa\bar{\vartheta}\{k\}}
\end{aligned}
\tag{33}
$$

where

$$
\delta\vartheta[\kappa,\bar{\chi}](x_1,x_2,t) = \frac{\vartheta^{S}{}_{\kappa,\bar{\chi}^A}(x_1,x_2,t) - \vartheta^{rel}{}_{\kappa}(x_1,x_2,t)}{\kappa\bar{\vartheta}}
\tag{34}
$$

Let us note the interrelation between ratios (30) and (34):

$$\delta\theta[\kappa,\overline{\chi}](x_1,x_2,t)-\delta\vartheta[\kappa,\overline{\chi}](x_1,x_2,t)= \\ =g(x_1,x_2)\frac{[k](\vartheta_{,3}[\kappa,\overline{\chi}])_{,3}(x_1,x_2,t)+e^{-l^2\{k\}^{-1}\langle c\rangle t}\{k\}\overline{\chi}(x_1,x_2)}{\kappa\overline{\vartheta}\{k\}} \tag{35}$$

Let us observe that $\lim\limits_{t\to+\infty}\delta\vartheta[\kappa,\overline{\chi}](x_1,x_2,t)=0$ for every parameter κ and initial values of fluctuation variable $\overline{\chi}(x_1,x_2)$.

No we are interesting in the determination time instants, after which the influence of initial temperature fluctuation onto the total and averaged temperature fields practically vanishes. To this end as a measure of the evolution of influence initial temperature fluctuation on the total temperature field will be taken the ratio

$$v_\theta[\kappa,\overline{\chi}](t)=\frac{\|\delta\theta[\kappa,\overline{\chi}](\cdot,t)\|}{\|\overline{\chi}_{,3}\|} \tag{36}$$

and the ratio

$$v_\vartheta[\kappa,\overline{\chi}](t)=\frac{\|\delta\vartheta[\kappa,\overline{\chi}](\cdot,t)\|}{\|\overline{\chi}_{,3}\|} \tag{37}$$

as a measure of the evolution of influence initial temperature fluctuation on the averaged temperature field. By virtue of (35) we conclude that

$$\lim_{t\to+\infty}v_\vartheta[\kappa,\overline{\chi}](t)=\lim_{t\to+\infty}v_\theta[\kappa,\overline{\chi}](t)=0 \tag{38}$$

and hence sets

$$\begin{aligned} T_\vartheta[\delta,\kappa,\overline{\chi}]&\equiv\{t:v_\vartheta[\kappa,\overline{\chi}](t)<\delta\}\\ T_\theta[\delta,\kappa,\overline{\chi}]&\equiv\{t:v_\theta[\kappa,\overline{\chi}](t)<\delta\}\end{aligned} \tag{39}$$

are nonempty for every $v>0$. Now we are ready to introduce for every $\delta>0$ the time instants

$$\begin{aligned} t_\vartheta(\delta)&\equiv\inf_{t>0}T_\vartheta(\delta)[\kappa,\overline{\chi}]\\ t_\theta(\delta)&\equiv\inf_{t>0}T_\theta(\delta)[\kappa,\overline{\chi}]\end{aligned} \tag{40}$$

It is easy to verify that t_δ is a continuous function of δ and for every $\delta>0$ we have $t_\delta\geq 0$. Moreover, $\lim\limits_{\delta\to+0}t_\vartheta(\delta)=\lim\limits_{\delta\to+0}t_\theta(\delta)=0$. In the subsequent considerations time intervals $[0,t_\vartheta(\delta)]$ and $[0,t_\theta(\delta)]$ will be referred to as *initial effect intervals* (*related to parameter v*) *related to averaged and total temperature field,*

respectively. Hence $t_{\vartheta}(\delta)$ and $t_{\theta}(\delta)$ are lengths of related initial effect intervals. In
the case in which initial an boundary conditions are sufficiently regular for sufficiently large parameter $\delta > 0$ both initial effect intervals are identical with nonnegative reals and for trivial case $\delta = 0$ the initial effect intervals are reduced to $\{0\}$.

5. Nondimensional form of model equations

Let us introduce nondimensional time instant T_0 and nondimensional length instant L. Now, we define nondimensional time coordinate $\tau = t / T_0$ and non-dimensional space coordinates $x_i = \xi_i L$, i = 1,2,3, nondimensional temperature instant. Moreover, let *nondimensional temperature* u and *nondimensional fluctuation variable* υ be defined by

$$\begin{aligned} u(\xi_1,\xi_2,\xi_3,\tau) &= \vartheta(\xi_1 L,\xi_2 L,\xi_3 L,\tau T_0)/\vartheta_0 \\ \upsilon(\xi_1,\xi_2,\xi_3,\tau) &= \chi(\xi_1 L,\xi_2 L,\xi_3 L,\tau T_0)/\zeta_0 \end{aligned} \tag{41}$$

where ϑ_0, ζ_0 are given *a priori* temperature and fluctuation constants. Hence, model equations (18) can be rewritten in the form

$$\begin{aligned} &\frac{\partial u}{\partial \tau} - \kappa \frac{\partial^2 u}{\partial \xi_\alpha \partial \xi_\alpha} - \kappa^{eff} \frac{\partial^2 u}{\partial \xi_3 \partial \xi_3} = \nu \frac{\partial \upsilon}{\partial \xi_3} \\ &\varepsilon\left(\frac{\partial \upsilon}{\partial \tau} + \kappa \frac{\partial^2 \upsilon}{\partial \xi_\alpha \partial \xi_\alpha}\right) + \mu \upsilon = 0 \end{aligned} \tag{42}$$

where $\varepsilon \equiv l^2 / L^2$ and

$$\kappa \equiv \frac{T_0}{L^2}\frac{\langle k \rangle}{\langle c \rangle}, \quad \kappa^{eff} \equiv \frac{T_0}{L^2}\frac{k^{eff}}{\langle c \rangle}, \quad \nu \equiv \frac{T_0 \chi_0}{\vartheta_0 L}\frac{[k]}{\langle c \rangle}, \quad \mu \equiv \frac{T_0}{L^2}\frac{\{k\}}{\langle c \rangle} \tag{43}$$

At the same time (24) gives

$$\frac{\partial u}{\partial \tau}(\xi,\tau) - \kappa \frac{\partial^2 u}{\partial \xi_\alpha \partial \xi_\alpha}(\xi,\tau) - \kappa^{eff} \frac{\partial^2 u}{\partial \xi_3 \partial \xi_3}(\xi,\tau) = \sigma(\xi,\tau) \tag{44}$$

where $\xi = (\xi_1,\xi_2,\xi_3) \in L^{-3}\Omega$, $\tau \geq 0$, and *nondimensional temperature pseudo-source* is defined by

$$\sigma(\xi,\tau) = \nu e^{-\varepsilon^{-1}\mu\tau} \frac{\partial \overline{\upsilon}}{\partial \xi_3}(\xi) \tag{45}$$

for the given initial values of nondimensional temperature field $\overline{\upsilon} = \overline{\upsilon}(\xi)$. The nondimensional temperature pseudosource (45) is interrelated with the temperature pseudosource (25) by the formula

$$\sigma(\xi_1,\xi_2,\xi_3,\tau) = \frac{\tau_0}{L\langle c\rangle \vartheta_0} s(L\xi_1, L\xi_2, L\xi_3, \tau T_0) \tag{46}$$

Now we are to formulate two basic initial-boundary value problems which will be starting point to the subsequent considerations.

Now we are to formulate the nondimensional form of the family of initial-boundary problems (27):

Find a nondimensional averaged temperature field $u(\xi,\tau)$ *defined for* ξ *from the region* Ω/L *and for* $\tau \geq 0$ *such that*

$$\begin{aligned}
&\frac{\partial u}{\partial \tau}(\xi,\tau) - \kappa \frac{\partial^2 u}{\partial \xi_\alpha \partial \xi_\alpha}(\xi,\tau) - \kappa^{eff} \frac{\partial^2 u}{\partial \xi_3 \partial \xi_3}(\xi,\tau) = \sigma(\xi,\tau) \\
&u(\xi,0) = \kappa \overline{u}(\xi) \\
&u|_{\partial\Omega}(\xi,\tau) = \kappa \overline{u}(\xi)
\end{aligned} \tag{47}$$

for given sufficiently regular nondimensional temperature field $\overline{\vartheta}(\mathbf{x})$.

The averaged temperature being the solution to initial-boundary value problem (47) related to the initial values of nondimensional temperature field $\overline{\upsilon} = \overline{\upsilon}(\xi)$ will be denoted by $u[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau)$. Now introducing denotations

$$i[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau) \equiv \frac{u[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau) - u[\kappa,0](\xi_1,\xi_2,\tau)}{\kappa} \tag{48}$$

for *the reduced influence function* and

$$\begin{aligned}
I[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau) &\equiv i[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau) + \\
&+ g(\xi_1 L,\xi_2 L)\frac{[k](u[\kappa,\overline{\upsilon}]),_3(\xi_1 L,\xi_2 L,\tau T_0) + e^{-l^2\{k\}^{-1}\langle c\rangle t}\{k\}\overline{\upsilon}(\xi_1 L,\xi_2 L)}{\kappa\{k\}\overline{\upsilon}(\xi_1 L,\xi_2 L)}
\end{aligned} \tag{49}$$

for *the reduced influence function* the influence function (30) can be written in the form

$$\delta\theta[\kappa,\overline{\chi}](\xi_1 L,\xi_2 L,\tau T_0) = I[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau) \tag{50}$$

Interrelation between (48) and (49) can be written as:

$$I[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau)-i[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau)= \\ =g(\xi_1,\xi_2)\frac{[k](u[\kappa,\overline{\upsilon}]),_3(\xi_1,\xi_2,\tau)+e^{-l^2\{k\}^{-1}\langle c\rangle t}\{k\}\overline{\upsilon}(\xi_1,\xi_2)}{\kappa\overline{\upsilon}\{k\}} \tag{51}$$

Note that

$$\lim_{\tau\to+0} i[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau)=\lim_{\tau\to+0} I[\kappa,\overline{\upsilon}](\xi_1,\xi_2,\tau)=0 \tag{52}$$

As a *measure of the evolution of influence nondimensional initial temperature fluctuation onto a averaged temperature field* will be taken the ratio

$$\mu[\kappa,\overline{\upsilon}](\tau)=\frac{\| i[\kappa,\overline{\upsilon}](\cdot,\tau)\|}{\|\overline{\upsilon},_3\|} \tag{53}$$

and the ratio

$$M[\kappa,\overline{\upsilon}](\tau)=\frac{\| I[\kappa,\overline{\upsilon}](\cdot,\tau)\|}{\|\overline{\upsilon},_3\|} \tag{54}$$

as *a measure of the evolution of influence nondimensional initial temperature fluctuation on the averaged temperature field*. Under (53) and (54) interrelations

$$\begin{aligned}\mu[\kappa,\overline{\upsilon}](\tau)&\equiv v_\vartheta[\kappa,\overline{\upsilon}\zeta_0](\tau T_0)\\ M[\kappa,\overline{\upsilon}](\tau)&\equiv v_\theta[\kappa,\overline{\upsilon}\zeta_0](\tau T_0)\end{aligned} \tag{55}$$

can be concluded. Moreover, formulas (36) and (37) take forms

$$\begin{aligned}\mu[\kappa,\overline{\upsilon}](\tau)&=\frac{\| i[\kappa,\overline{\upsilon}](\cdot,\tau)\|}{\|\overline{\upsilon},_3\|}\\ M[\kappa,\overline{\upsilon}](\tau)&=\frac{\| I[\kappa,\overline{\upsilon}](\cdot,\tau)\|}{\|\overline{\upsilon},_3\|}\end{aligned} \tag{56}$$

Now, if we define nondimensional time instants

$$\begin{aligned}\tau_\mu[\kappa,\overline{\chi}](\delta)&\equiv\inf_{t>0}\square_\mu[\kappa,\overline{\chi}](\delta)\\ \tau_M[\kappa,\overline{\chi}](\delta)&\equiv\inf_{t>0}\square_M[\kappa,\overline{\chi}](\delta)\end{aligned} \tag{57}$$

where

$$\begin{aligned} \Box_{\mu}[\kappa,\overline{\chi}](\delta) \equiv \{t : \mu[\kappa,\overline{\chi}](t) < \delta\} \\ \Box_{M}[\kappa,\overline{\chi}](\delta) \equiv \{t : M[\kappa,\overline{\chi}](t) < \delta\} \end{aligned} \tag{58}$$

Interval $[0,\tau_{\mu}]$ and $[0,\tau_{M}]$ will be referred to as *a nondimensional initial effect intervals* (*related to parameter* v) *for averaged and total temperature fields*, respectively, and hence τ_{v} are *lengths of a related nondimensional initial effect intervals*. In the case in which initial an boundary conditions in (47) are sufficiently regular for sufficiently large parameter $\delta > 0$ both nondimensional initial effect intervals are identical with the set of nonnegative reals and for trivial case $\delta = 0$ these intervals are reduced to $\{0\}$. Moreover as well $\tau_{\mu} = \tau_{\mu}[\kappa,\overline{\chi}](\delta)$ and $\tau_{M} = \tau_{M}[\kappa,\overline{\chi}](\delta)$ are continuous functions of δ. Moreover, for every $\delta > 0$ we have $\tau_{\mu}, \tau_{M} \geq 0$ and

$$\lim_{\delta \to +0} \tau_{\mu}(\delta) = \lim_{\delta \to +0} \tau_{M}(\delta) = 0 \tag{59}$$

Detailed analysis of the concepts mentioned above as well as an illustration by a certain simple example will be given in the framework of an extension version of this paper.

Conclusions

Now we are to observe that: the nondimensional initial effect interval length τ_{μ} (in the case of two-constituent microperiodic laminated rigid conductor) has the following properties:

1^{o} It tends to zero when microstructural parameter ε tends to zero.
2^{o} It tends to zero when nonhomogeneity parameter $|v|$ tends to infinity.
3^{o} It is tends to zero monotonically increasing in time.

It must be emphasized that answer to the question are the above properties valid for the microperiodic laminated rigid conductor consisting of the large number of constituents is still open.

References

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