THE INFLUENCE OF RISK FUZZINESS ON STRATEGIES OF DRAWING UP CONTRACTS

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Abstract. A contract usually consists in weighing risk and motivation [21]. Most contracts are constructed in such a way that both players share of risk. Only one party taking the risk reduces the motivation of the players. When making decisions the players have a range of knowledge on the acts of "Nature", i.e. they are aware that a random factor can influence their payments. One uses, as a rule, the usefulness function in contracts. It is also possible to use functions determining the connection between sums of money and usefulness (e.g. $v(x)$ is the usefulness of receiving the x currency units). One includes the risk concept in considerations in the form of so-called reluctance to risk. If somebody is reluctant to risk this means that he prefers to receive a definite sum of money than to hope for the best which can give either a bigger or a smaller payment. In the present considerations [24], random parameters as well as estimated and non-measurable parameters are subjected to fuzziness. The influence of fuzzy information and any forms of uncertain knowledge is extremely essential in the strategic games and to a high degree decides on the final results of the game.

1. Mathematical and geometrical interpretation of reluctance to risk

Various usefulness functions are found in literature from which a few can be quoted here:

$$v(x) = x \text{ (identity function)}$$
$$v(x) = x/1000$$
$$v(x) = \sqrt{x} \tag{1}$$

Each of them is certainly a growing function, however, representing preferences of the described situation in a different way. The influence of random factors is taken into consideration by estimating the expected value of usefulness. Figures 1 and 2 illustrate the difference between usefulness of choice and expected usefulness of choice.

One concludes from the diagram that for the determined parameters pL, pR and xL, xR the condition $v(x) > pL*v(xL) + pR*v(xR)$ is fulfilled, i.e. one deals with the situation when a player chooses a smaller but surer payment:

$$(v(3000)>pL*v(5000) + pR*v(1000)) \tag{2}$$

It is possible to create a table of reluctance to risk (Table 1 and Figure 3) taking the argument of the usefulness function $v(x)$ as an independent parameter.

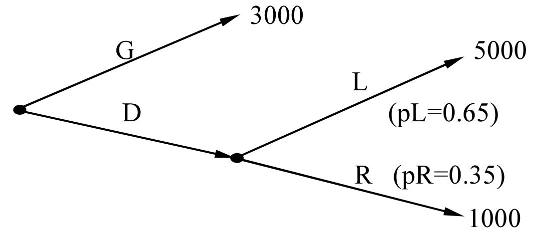

Fig. 1. Result without risk (usefulness of choice G: $v(3000)$) and influence of random factor (expected usefulness of choice D: $pL*v(5000) + pR*v(1000)$)

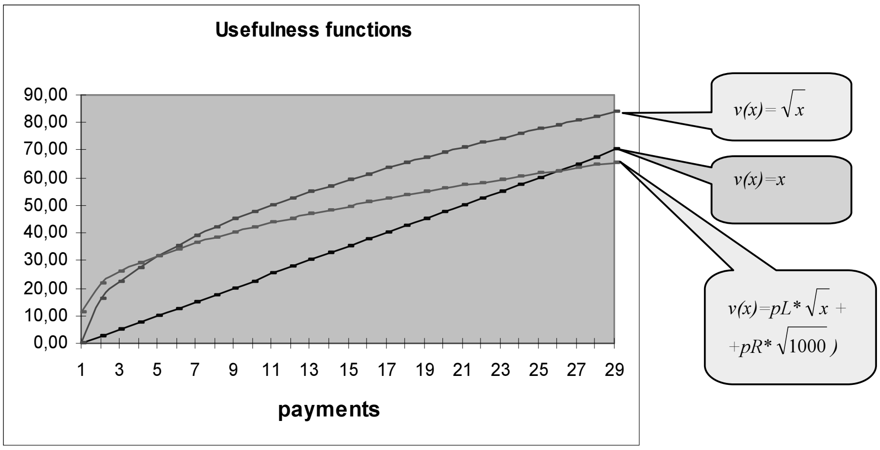

Fig. 2. Usefulness functions: linear and concave

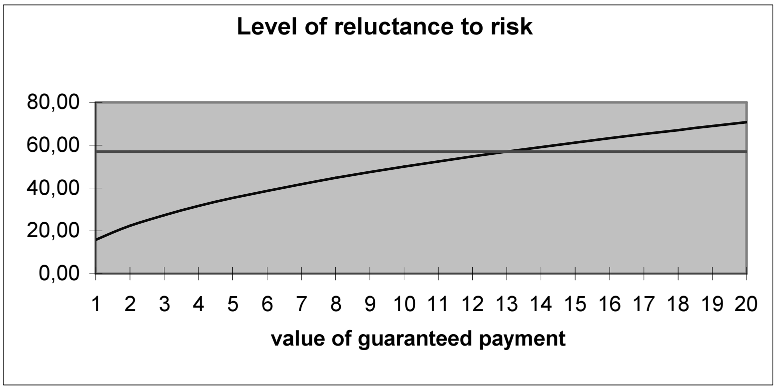

Fig. 3. Appropriate amount of guaranteed payment causes rejection of higher but riskier payment

Table 1

Moment of overcoming reluctance to risk (arguments are guaranteed payments; overcoming occurs in highlighted positions No. 13 and 14)

No.	xL	xR	pL	PR	x	v(x)	vlos(xL..xR)
1	5000	1000	0.65	0.35	250	15.81	57.02991
2	5000	1000	0.65	0.35	500	22.36	57.02991
3	5000	1000	0.65	0.35	750	27.39	57.02991
4	5000	1000	0.65	0.35	1000	31.62	57.02991
5	5000	1000	0.65	0.35	1250	35.36	57.02991
6	5000	1000	0.65	0.35	1500	38.73	57.02991
7	5000	1000	0.65	0.35	1750	41.83	57.02991
8	5000	1000	0.65	0.35	2000	44.72	57.02991
9	5000	1000	0.65	0.35	2250	47.43	57.02991
10	5000	1000	0.65	0.35	2500	50.00	57.02991
11	5000	1000	0.65	0.35	2750	52.44	57.02991
12	5000	1000	0.65	0.35	3000	54.77	57.02991
13	5000	1000	0.65	0.35	3250	57.01	57.02991
14	5000	1000	0.65	0.35	3500	59.16	57.02991
15	5000	1000	0.65	0.35	3750	61.24	57.02991
16	5000	1000	0.65	0.35	4000	63.25	57.02991
17	5000	1000	0.65	0.35	4250	65.19	57.02991
18	5000	1000	0.65	0.35	4500	67.08	57.02991
19	5000	1000	0.65	0.35	4750	68.92	57.02991
20	5000	1000	0.65	0.35	5000	70.71	57.02991

Table 2

Overcoming reluctance to risk depending on random factors *pL* and *pR*

No.	xL	xR	pL	PR	x	v(x)	
1	5000	1000	0.95	0.05	3000	54.77	68.75628
2	5000	1000	0.90	0.10	3000	54.77	66.80189
3	5000	1000	0.85	0.15	3000	54.77	64.84749
4	5000	1000	0.80	0.20	3000	54.77	62.8931
5	5000	1000	0.75	0.25	3000	54.77	60.9387
6	5000	1000	0.70	0.30	3000	54.77	58.98431
7	5000	1000	0.65	0.35	3000	54.77	57.02991
8	5000	1000	0.60	0.40	3000	54.77	55.07552
9	5000	1000	0.55	0.45	3000	54.77	53.12112
10	5000	1000	0.50	0.50	3000	54.77	51.16673
11	5000	1000	0.45	0.55	3000	54.77	49.21233
12	5000	1000	0.40	0.60	3000	54.77	47.25794
13	5000	1000	0.35	0.65	3000	54.77	45.30354
14	5000	1000	0.30	0.70	3000	54.77	43.34915
15	5000	1000	0.25	0.75	3000	54.77	41.39475
16	5000	1000	0.20	0.80	3000	54.77	39.44036
17	5000	1000	0.15	0.85	3000	54.77	37.48596
18	5000	1000	0.10	0.90	3000	54.77	35.53157
19	5000	1000	0.05	0.95	3000	54.77	33.57717
20	5000	1000	0.00	1.00	3000	54.77	31.62278

The influence of the random factor on overcoming reluctance to risk can also be subjected to analysis (Table 2 and Figure 4).

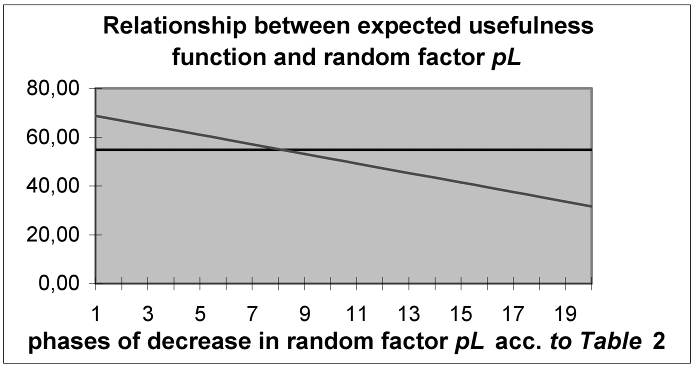

Fig. 4. Illustration of situation when decrease in random factor *pL* in the range of [0.6-0.55] causes change of optimal decision towards choice of guaranteed payment

A decrease in the random factor level *pL* (i.e. obtainment probability of high payment *xL*) leads to the situation when increasing the reluctance to risk makes the guaranteed payment *x* the best choice although this payment is considerably lower than the higher payment (condition (2) will be fulfilled). The graphic illustration of this situation is shown in Figure 4.

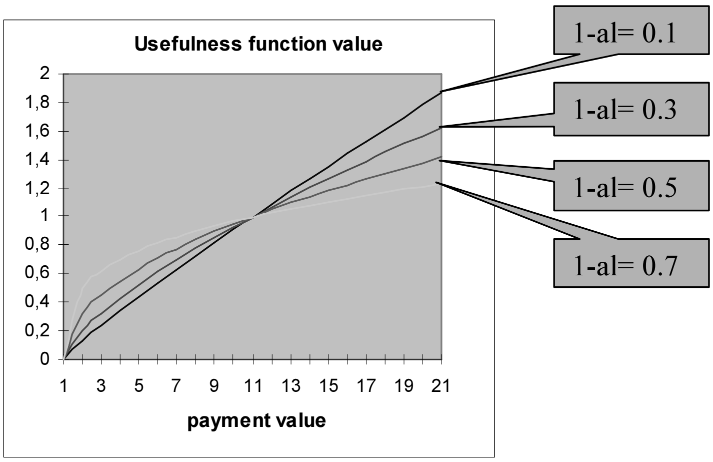

Fig. 5. Various levels of reluctance to risk (measure of reluctance is: 1 – al)

The measure of reluctance to risk can be an Arrow-Pratt coefficient, which is called the relative risk aversion coefficient [29]:

$$coef\_A\text{-}P = -x * v''(x) / v'(x) \qquad (3)$$

where: $v(x) = x^{al}$
$v'(x) = al * x^{al-1}$ (first derivative)
$v''(x) = al * (al - 1) * x^{al-2}$ (second derivative) (4)
and after substitution (4) to (3):

$$coef\_A\text{-}P = -x * al * (al - 1) * x^{al-2} / al * x^{al-1} = 1 - al \quad (5)$$

2. Influence of parameter fuzziness on choice of best response

Reluctance to risk, random factors (probabilities of events) and arguments of payments (constants, bonuses and contributions) can be chosen for the game parameters. Each of these parameters can be estimated with uncertain knowledge [31]. The parameters can also be characterized in given intervals. Knowledge of experts or statistical research can enrich the possessed information with the form of the belonging function on a given interval [14]. Therefore, one will deal with fuzzy sets or the interval analysis [5]. The choice of the optimal decision sequence is significant from the players' and strategy game participants' point of view. Chosen and optimal parameters should be located in the given intervals. At the beginning of the game one does not know the shape of the parameter belonging function as in examples which will be presented below. Thus, one will use the interval analysis for the optimization of the players' strategy. The parameter set, which can have a controllability virtue and will ensure the most favourable activities of individual players, will be the result of such an analysis.

To the interval parameters from the previous section of the paper one can rate: payments x, xL and xR, random factors pL and pR and reluctance to risk $1 - al$. Each of the mentioned parameters influences favourably or unfavourably the players' payments. The analyzed example refers to the game with Nature (player 1) which creates specific situations with determined probability. It is easy to state that the increase in guaranteed payment x brings a profit for the game leader (player 2) and increases the reluctance to risk:

($x\uparrow$ = > $v(x)\uparrow$; (*the game leader – player 2*)
$x\uparrow$ = >*reluctance_to_risk* $\uparrow$).

An increase in the highest payment (success payment) in the random part is profitable for the game leader and for Nature, in addition causes a decrease in the reluctance to risk:

($xL\uparrow$ = > $(pL*v(xL) + pR*v(xR))\uparrow$; (*the game leader – player 2*)
$xL\uparrow$ = > $((1 - pL)*v(xL) + (1 - pR)*v(xR))\uparrow$; (*Nature – player 1*)
$xL\uparrow$ = >*reluctance_to_risk* $\downarrow$).

An increase in the lowest payment (failure payment) in the random part is profitable both the game leader and Nature, in addition causes a decrease in the reluctance to risk:

$(xR\uparrow => (pL*v(xL) + pR*v(xR))\uparrow;$ (*the game leader – player 2*)

$xR\uparrow => ((1 - pL)*v(xL) + (1 - pR)*v(xR))\uparrow;$ (*Nature – player 1*)
$xR\uparrow => reluctance\_to\_risk\downarrow).$

An increase in the random factor leading to the highest payment, i.e. probability of success pL, leads certainly to a decrease in the probability of failure $pR = 1 - pL$ and is simultaneously profitable for the game leader, unprofitable for Nature and also decreases in reluctance to risk:

$(pL\uparrow => (pL*v(xL) + pR*v(xR))\uparrow;$ (*the game leader – player 2*)
$pL\uparrow => ((1 - pL)*v(xL) +(1 - pR)*v(xR))\downarrow;$ (*Nature – player1*)
$pL\uparrow => reluctance\_to\_risk\downarrow).$

An increase in the random factor leading to the lowest payment, i.e. the probability of failure pR, leads certainly to a decrease in the probability of success $pL = 1 - pR$ and is simultaneously unprofitable for the game leader, profitable for Nature and also increases in reluctance to risk:

$(pR\uparrow => (pL*v(xL) + pR*v(xR))\downarrow;$ (*the game leader – player 2*)
$pR\uparrow => ((1 - pL)*v(xL) + (1 - pR)*v(xR))\uparrow;$ (*Nature – player 1*)
$pR\uparrow => reluctance\_to\_risk\uparrow).$

An increase in the relative risk aversion coefficient $(1 - al)$ leads to a decrease of the payment both in the situation when risk is not taken - $v^{al}(x) \geq (pL*v^{al}(xL) + pR*v^{al}(xR))$ - as well as in the reverse situation, i.e. when it is rational to take the risk: $v^{al}(x) < (pL*v^{al}(xL) + pR*v^{al}(xR))$.

$((1 - al)\uparrow => v^{al}(x)\downarrow;$
$(1 - al)\uparrow => (pL*v^{al}(xL) + pR*v^{al}(xR)\downarrow;$ (*the game leader – player 2*)
$(1 - al)\uparrow => ((1 - pL)*v^{al}(xL) + (1 - pR)*v^{al}(xR)\downarrow)$ (*Nature – player 1*)

The above heuristics lead to the choice of lower and upper limits depending on, what is more profitable for a given player by the determination of the specific parameter. Thus, arise conclusions concerning the set of optimal parameters for Nature (player 1) and for the game leader (player 2).
The most profitable parameter set for Nature is:

$$\left\{\underline{x}, \overline{xL}, \overline{xR}, \underline{pL}, \overline{pR}, \underline{(1-al)}\right\}$$

The most profitable parameter set for the game leader is:

$$\left\{\overline{x}, \overline{xL}, \overline{xR}, \overline{pL}, \underline{pR}, \underline{(1-al)}\right\}$$

The "principal-agent" game will be played in the second example whose extensive version is presented in Figure 6.

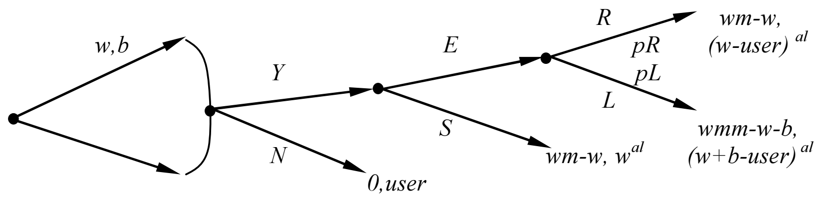

Fig. 6. "Principal-agent" game in extensive version

Description of the game:

1. Offer of player 1 (*w - payment for agent, b - bonus for agent in the case of success*).
2. Player 2 accepts (*Y*) or rejects the offer (*N*). In the case of rejection the payment for player 1 is 0 and for player *2* is equal to useful value *user*.
3. Player 2 works effectively (E) or ineffectively (*S*). In the second case the payment for player 1 is *wm-w*, where *wm* is the effect of lack of success, the payment for player 2 is w^{al}, where $1 - al$ is the measure of reluctance to risk.
4. Success occurs with probability *pL* and failure with probability $pR = (1 - pL)$. Appropriate payment for player 1 *in* the case of success is: *wmm-w-b*, and for player 2: (*w+b-user*). Payment for player 1 in the case of failure: *wm-w*, and for player 2: $(w+b\text{-}user)^{al}$.

Allowance for the expected values creates a chance to simplify the scheme (Fig. 7).

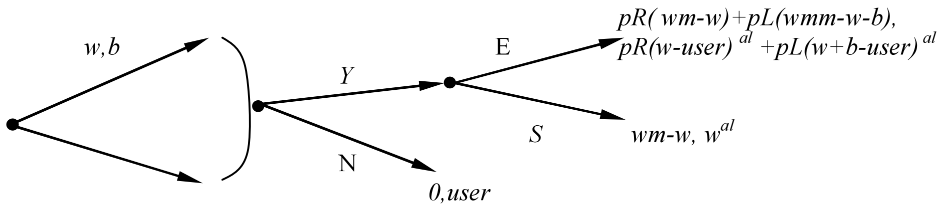

Fig. 7. Scheme of "principal-agent" game taking expected values of payments into consideration

Assuming in the specific example that only reluctance to risk is subjected to fuzziness $1 - al$, one can note down the following heuristics which allows the choice of the most favourable solution concerning payments for both players:

((1 – *al*) ↑ = > (*pR*(*wm-w*) + *pL*(*wmm-w-b*)) *without changes; (payment for "principal", success)*
(1 – *al*) ↑ = > (*pR*(*w-user*) ^{al} + *pL*(*w+b-user*) ^{al}↓; *(payment for "agent", success)*
(1 – *al*) ↑ = > (wm-w) *without changes*; (*payment for "principal", failure*)
(1 – *al*) ↑ = > (w^{al})↓); *(payment for "agent", failure)*

The most favourable parameter value for the "agent": *reluctance to risk* is level $\lceil \overline{al} \rceil$. The parameter value for the "principal": *reluctance to risk* is insignificant in this example.

Conclusions

1. Reluctance to risk is essential for the sake of both the effects of stimulation to action as well as the comfort of action and playing the game. The first aspect is profitable for the player offering work, the second one - for the employee.
2. Fuzziness of parameters directly influencing reluctance to risk allows flexible control of the playing process and choice of the best strategies.
3. The choice optimization of controlled game parameters can be described by means of the heuristics concerning simple rules of changes of the payment values for individual players by increasing (or decreasing) parameter values with the given increment (ΔX).

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