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THE DETERMINANTS OF THE BLOCK BAND MATRICES BASED ON THE *n*-DIMENSIONAL FOURIER EQUATION PART 1

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Abstract. This paper contains the method of calculating the determinant of the block band matrix on the example of *n*-dimensional Fourier equation using the Finite Difference Method.

Keywords: *block matrices, n-band matrices, determinant, Fourier equation*

Introduction

We encounter the block band matrices, inter alia, considering the torsion bars in the theory of vibrations, determining the currents in the eyebolt method in the circuit theory, as well as issues of heat flow using the Finite Difference Method.

The Finite Difference Method (FDM) is based on the introduction of the modeled structure of the grid nodes and replacing a single differential equation (in our case the Fourier equation) by a set of differential algebraic equations. The partial derivatives are approximated by the corresponding difference quotients. The values of the search function (for example temperature values) are calculated only at points of discretization - mesh nodes. The algebraic systems of equations in which the main matrix has the band structure are obtained.

And it is the Fourier equation describing the heat conduction that will serve as an example to illustrate how to calculate the determinants of block band matrix.

1. Solution of the problem

We consider the *n*-dimensional Fourier equation

$$\lambda\left(\frac{\partial^2 T(x_1, x_2, \ldots, x_n, t)}{\partial x_1^2}+\frac{\partial^2 T(x_1, x_2, \ldots, x_n, t)}{\partial x_2^2}+\ldots+\frac{\partial^2 T(x_1, x_2, \ldots, x_n, t)}{\partial x_n^2}\right)= \\ =\rho c\frac{\partial T(x_1, x_2, \ldots, x_n, t)}{\partial t} \tag{1}$$

where λ is a thermal conductivity [W/mK], c is a specific heat [J/kgK], ρ is a mass density [kg/m^3], T is temperature [K], $x_1, x_2, \dots, x_n$ denote the geometrical coordinates and t is time [s].

For the dimension $I_1 \times I_2 \times \dots I_n$ and interval of the time L we get the spatiotemporal grid $\Delta_{i_1 i_2 \dots i_n l}$ and nodes

$$\begin{gathered} x_{1i_1} = i_1 \Delta x_1 \text{ for } 1 \le i_1 \le m_1 - 1, \text{ where } m_1 \Delta x_1 = I_1 \\ x_{2i_2} = i_2 \Delta x_2 \text{ for } 1 \le i_2 \le m_2 - 1, \text{ where } m_2 \Delta x_2 = I_2 \\ \vdots \\ x_{ni_n} = i_n \Delta x_n \text{ for } 1 \le i_n \le m_n - 1, \text{ where } m_n \Delta x_n = I_n \end{gathered}$$

for the spatial coordinates and constant increments $\Delta x_1, \Delta x_2, \dots, \Delta x_n$
and

$$t_l = l \Delta t \text{ for } 1 \le l \le q, \text{ where } q \, \Delta t = L$$

for coordinate of the time.

We assume the following designation $T_{i_1 i_2 \dots i_n l} = T\left(x_{1i_1}, x_{2i_2}, \dots, x_{ni_n}, t_l\right)$.

Then approximations of the second order partial derivatives using MRS are as follows:

$$\begin{aligned} \frac{\partial^2 T}{\partial x_1^2} &= \frac{T_{i_1-1,i_2,\dots,i_n,l} - 2T_{i_1,i_2,\dots,i_n,l} + T_{i_1+1,i_2,\dots,i_n,l}}{\left(\Delta x_1\right)^2}, \quad 1 \le i_1 \le m_1 - 1 \\ \frac{\partial^2 T}{\partial x_2^2} &= \frac{T_{i_1,i_2-1,\dots,i_n,l} - 2T_{i_1,i_2,\dots,i_n,l} + T_{i_1,i_2+1,\dots,i_n,l}}{\left(\Delta x_2\right)^2}, \quad 1 \le i_2 \le m_2 - 1 \\ &\vdots \\ \frac{\partial^2 T}{\partial x_n^2} &= \frac{T_{i_1,i_2,\dots,i_n-1,l} - 2T_{i_1,i_2,\dots,i_n,l} + T_{i_1,i_2,\dots,i_n+1,l}}{\left(\Delta x_n\right)^2}, \quad 1 \le i_n \le m_n - 1 \end{aligned} \tag{2}$$

However, the time derivative approximation takes the following form:

$$\frac{\Delta T}{\Delta t} = \frac{T_{i_1,i_2,\dots,i_n,l} - T_{i_1,i_2,\dots,i_n,l-1}}{\Delta t}, \quad 1 \le l \le q \tag{3}$$

So, the internal iteration corresponding Fourier equation takes the form

$$\lambda\left(\frac{\Delta^2 T}{\Delta x_1^2}+\frac{\Delta^2 T}{\Delta x_2^2}+\ldots+\frac{\Delta^2 T}{\Delta x_n^2}\right)=\rho c\frac{\Delta T}{\Delta t} \tag{4}$$

It leads to the following band system of equations

$$\begin{aligned}
&\frac{\lambda}{\left(\Delta x_1\right)^2}T_{i_1-1,i_2,\ldots,i_n,l}-\frac{2\lambda}{\left(\Delta x_1\right)^2}T_{i_1,i_2,\ldots,i_n,l}+\frac{\lambda}{\left(\Delta x_1\right)^2}T_{i_1+1,i_2,\ldots,i_n,l}+\\
&+\frac{\lambda}{\left(\Delta x_2\right)^2}T_{i_1,i_2-1,\ldots,i_n,l}-\frac{2\lambda}{\left(\Delta x_2\right)^2}T_{i_1,i_2,\ldots,i_n,l}+\frac{\lambda}{\left(\Delta x_2\right)^2}T_{i_1,i_2+1,\ldots,i_n,l}+\\
&\vdots\\
&+\frac{\lambda}{\left(\Delta x_n\right)^2}T_{i_1,i_2,\ldots,i_n-1,l}-\frac{2\lambda}{\left(\Delta x_n\right)^2}T_{i_1,i_2,\ldots,i_n,l}+\frac{\lambda}{\left(\Delta x_n\right)^2}T_{i_1,i_2,\ldots,i_n+1,l}=\\
&=\frac{\rho c}{\Delta t}T_{i_1,i_2,\ldots,i_n,l}-\frac{\rho c}{\Delta t}T_{i_1,i_2,\ldots,i_n,l-1}
\end{aligned} \tag{5}$$

for each time step l.
The main matrix of this system is a block matrix having the following structure

$$A_n=\begin{bmatrix}
A_{n-1} & D_{n-1} & \cdots & \cdots & \cdots & \cdots & \cdots\\
D_{n-1} & A_{n-1} & D_{n-1} & \cdots & \cdots & \cdots & \cdots\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\
\cdots & \cdots & \cdots & \cdots & D_{n-1} & A_{n-1} & D_{n-1}\\
\cdots & \cdots & \cdots & \cdots & \cdots & D_{n-1} & A_{n-1}
\end{bmatrix}_{m_n\times m_n},\quad n\ge 2 \tag{6}$$

while ($n = 1$)

$$A_1=\begin{bmatrix}
a_1 & b_1 & \cdots & \cdots & \cdots & \cdots & \cdots\\
b_1 & a_1 & b_1 & \cdots & \cdots & \cdots & \cdots\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\
\cdots & \cdots & \cdots & \cdots & b_1 & a_1 & b_1\\
\cdots & \cdots & \cdots & \cdots & \cdots & b_1 & a_1
\end{bmatrix}_{m_1\times m_1} \tag{7}$$

where the elements of the matrix A_1 are as follows:

$$a_1=\frac{2\lambda}{\left(\Delta x_1\right)^2}+\frac{2\lambda}{\left(\Delta x_2\right)^2}+\ldots+\frac{2\lambda}{\left(\Delta x_n\right)^2}+\frac{\rho c}{\Delta t},\qquad b_1=-\frac{\lambda}{\left(\Delta x_1\right)^2} \tag{8}$$

Then

$$\det A_1 = b_1^{m_1}\left[\left(\frac{a_1}{b_1}\right)^{m_1} - \binom{m_1-1}{1}\left(\frac{a_1}{b_1}\right)^{m_1-2} + \binom{m_1-2}{2}\left(\frac{a_1}{b_1}\right)^{m_1-4} - \ldots\right] = \\ = a_1^{m_1} - \binom{m_1-1}{1}a_1^{m_1-2}b_1^2 + \binom{m_1-2}{2}a_1^{m_1-4}b_1^4 + \ldots \tag{9}$$

The calculation of the above determinant is given in the article [1].
Then

$$A_2 = \begin{bmatrix} A_1 & D_1 & \cdots & \cdots & \cdots & \cdots & \cdots \\ D_1 & A_1 & D_1 & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & D_1 & A_1 & D_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & D_1 & A_1 \end{bmatrix}_{m_2\times m_2} \tag{10}$$

where

$$D_1 = \begin{bmatrix} d_1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & d_1 & 0 & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & 0 & d_1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0 & d_1 \end{bmatrix}_{m_1\times m_1}, \quad d_1 = -\frac{\lambda}{\left(\Delta x_2\right)^2} \tag{11}$$

Therefore

$$\det A_2 = d_1^{m_1\cdot m_2}\det\left[\left(\frac{A_1}{d_1}\right)^{m_2} - \binom{m_2-1}{1}\left(\frac{A_1}{d_1}\right)^{m_2-2} + \binom{m_2-2}{2}\left(\frac{A_1}{d_1}\right)^{m_2-4} - \ldots\right] = \\ = d_1^{m_1\cdot m_2}\det\left[\left(\frac{A_1}{d_1} - p_{1,1}I_1\right)\left(\frac{A_1}{d_1} - p_{1,2}I_1\right)\ldots\left(\frac{A_1}{d_1} - p_{1,m_1}I_1\right)\right] = \\ = d_1^{m_1\cdot m_2}\det\left(\frac{A_1}{d_1} - p_{1,1}I_1\right)\cdot\det\left(\frac{A_1}{d_1} - p_{1,2}I_1\right)\cdot\ldots\cdot\det\left(\frac{A_1}{d_1} - p_{1,m_1}I_1\right) = \\ = \det\left(A_1 - d_1p_{1,1}I_1\right)\cdot\det\left(A_1 - d_1p_{1,2}I_1\right)\cdot\ldots\cdot\det\left(A_1 - d_1p_{1,m_1}I_1\right) \tag{12}$$

where p_{1,i_1}, $1 \le i_1 \le m_1$ are zeros of polynominal

$$f_1(x) = x^{m_1} - \binom{m_1 - 1}{1} x^{m_1 - 2} + \binom{m_1 - 2}{2} x^{m_1 - 4} + \ldots = \\ = \left(x - p_{1,1}\right) \cdot \left(x - p_{1,2}\right) \cdot \ldots \cdot \left(x - p_{1,m_1}\right) \tag{13}$$

Consecutively

$$A_3 = \begin{bmatrix} A_2 & D_2 & \cdots & \cdots & \cdots & \cdots & \cdots \\ D_2 & A_2 & D_2 & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & D_2 & A_2 & D_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & D_2 & A_2 \end{bmatrix}_{m_3 \times m_3} \tag{14}$$

where

$$D_2 = \begin{bmatrix} C_1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & C_1 & 0 & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & 0 & C_1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0 & C_1 \end{bmatrix}_{m_2 \times m_2} \tag{15}$$

$$C_1 = \begin{bmatrix} d_2 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & d_2 & 0 & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & 0 & d_2 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0 & d_2 \end{bmatrix}_{m_1 \times m_1} , \quad d_2 = -\frac{\lambda}{\left(\Delta x_3\right)^2} \tag{16}$$

Then we obtain

$$\det A_3 = d_2^{m_1 \cdot m_2 \cdot m_3} \det\left[\left(\frac{A_2}{d_2}\right)^{m_3} - \binom{m_3 - 1}{1}\left(\frac{A_2}{d_2}\right)^{m_3 - 2} + \binom{m_3 - 2}{2}\left(\frac{A_2}{d_2}\right)^{m_3 - 4} - \ldots\right] = \\ = d_2^{m_1 \cdot m_2 \cdot m_3} \det\left[\left(\frac{A_2}{d_2} - p_{2,1} I_2\right)\left(\frac{A_2}{d_2} - p_{2,2} I_2\right) \ldots \left(\frac{A_2}{d_2} - p_{2,m_2} I_2\right)\right] = \\ = d_2^{m_1 \cdot m_2 \cdot m_3} \det\left(\frac{A_2}{d_2} - p_{2,1} I_2\right) \cdot \det\left(\frac{A_2}{d_2} - p_{2,2} I_2\right) \cdot \ldots \cdot \det\left(\frac{A_2}{d_2} - p_{2,m_2} I_2\right) = \\ = \det\left(A_2 - d_2 p_{2,1} I_2\right) \cdot \det\left(A_2 - d_2 p_{2,2} I_2\right) \cdot \ldots \cdot \det\left(A_2 - d_2 p_{2,m_2} I_2\right) \tag{17}$$

where p_{2,i_2}, $1 \le i_2 \le m_2$ are zeros of polynomial

$$f_2(x) = x^{m_2} - \binom{m_2 - 1}{1} x^{m_2 - 2} + \binom{m_2 - 2}{2} x^{m_2 - 4} + \dots \tag{18}$$

where

$$\begin{aligned}
&\det\left(A_2 - d_2 p_{2,1} I_2\right) = d_1^{m_1 \cdot m_2} \det\left(\frac{A_2}{d_1} - \frac{d_2}{d_1} p_{2,1} I_2\right) = \\
&= d_1^{m_1 \cdot m_2} \det\left[\left(\frac{A_1}{d_1} - p_{1,1} I_1\right) - \frac{d_2}{d_1} p_{2,1} I_1\right] \cdot \det\left[\left(\frac{A_1}{d_1} - p_{1,2} I_1\right) - \frac{d_2}{d_1} p_{2,1} I_1\right] \cdot \ldots \cdot \\
&\cdot \det\left[\left(\frac{A_1}{d_1} - p_{1,m_1} I_1\right) - \frac{d_2}{d_1} p_{2,1} I_1\right] = \\
&= \det\left[A_1 - \left(d_1 p_{1,1} + d_2 p_{2,1}\right) I_1\right] \det\left[A_1 - \left(d_1 p_{1,2} + d_2 p_{2,1}\right) I_1\right] \cdot \ldots \cdot \\
&\cdot \det\left[A_1 - \left(d_1 p_{1,m_1} + d_2 p_{2,1}\right) I_1\right]
\end{aligned} \tag{19}$$

and analogously

$$\begin{aligned}
&\det\left(A_2 - d_2 p_{2,m_2} I_2\right) = d_1^{m_1 \cdot m_2} \det\left(\frac{A_2}{d_1} - \frac{d_2}{d_1} p_{2,m_2} I_2\right) = \\
&= d_1^{m_1 \cdot m_2} \det\left[\left(\frac{A_1}{d_1} - p_{1,1} I_1\right) - \frac{d_2}{d_1} p_{2,m_2} I_1\right] \cdot \det\left[\left(\frac{A_1}{d_1} - p_{1,2} I_1\right) - \frac{d_2}{d_1} p_{2,m_2} I_1\right] \cdot \ldots \cdot \\
&\cdot \det\left[\left(\frac{A_1}{d_1} - p_{1,m_1} I_1\right) - \frac{d_2}{d_1} p_{2,m_2} I_1\right] = \\
&= \det\left[A_1 - \left(d_1 p_{1,1} + d_2 p_{2,m_2}\right) I_1\right] \cdot \det\left[A_1 - \left(d_1 p_{1,2} + d_2 p_{2,m_2}\right) I_1\right] \cdot \ldots \cdot \\
&\cdot \det\left[A_1 - \left(d_1 p_{1,m_1} + d_2 p_{2,m_2}\right) I_1\right]
\end{aligned} \tag{20}$$

Generally

$$\begin{aligned}
&\det A_n = \\
&= d_{n-1}^{m_1 \cdot m_2 \cdot m_3 \cdot \ldots \cdot m_n} \cdot \\
&\cdot \det\left[\left(\frac{A_{n-1}}{d_{n-1}}\right)^{m_n} - \binom{m_n - 1}{1}\left(\frac{A_{n-1}}{d_{n-1}}\right)^{m_n - 2} + \binom{m_n - 2}{2}\left(\frac{A_{n-1}}{d_{n-1}}\right)^{m_n - 4} - \ldots\right] = \\
&= d_{n-1}^{m_1 \cdot m_2 \cdot m_3 \cdot \ldots \cdot m_n} \cdot \\
&\cdot \det\left[\left(\frac{A_{n-1}}{d_{n-1}} - p_{n-1,1} I_{n-1}\right)\left(\frac{A_{n-1}}{d_{n-1}} - p_{n-1,2} I_{n-1}\right) \cdot \ldots \cdot \left(\frac{A_{n-1}}{d_{n-1}} - p_{n-1,m_{n-1}} I_{n-1}\right)\right] = \\
&= d_{n-1}^{m_1 \cdot m_2 \cdot m_3 \cdot \ldots \cdot m_n} \cdot \\
&\cdot \det\left(\frac{A_{n-1}}{d_{n-1}} - p_{n-1,1} I_{n-1}\right) \cdot \det\left(\frac{A_{n-1}}{d_{n-1}} - p_{n-1,2} I_{n-1}\right) \cdot \ldots \cdot \det\left(\frac{A_{n-1}}{d_{n-1}} - p_{n-1,m_{n-1}} I_{n-1}\right) = \\
&= \det\left(A_{n-1} - d_{n-1} p_{n-1,1} I_{n-1}\right) \cdot \det\left(A_{n-1} - d_{n-1} p_{n-1,2} I_{n-1}\right) \cdot \ldots \cdot \\
&\cdot \det\left(A_{n-1} - d_{n-1} p_{n-1,m_{n-1}} I_{n-1}\right)
\end{aligned} \tag{21}$$

where $p_{n-1,i_{n-1}}$, $1 \le i_{n-1} \le m_{n-1}$ are zeros of polynomial

$$f_{n-1}(x) = x^{m_{n-1}} - \binom{m_{n-1} - 1}{1} x^{m_{n-1} - 2} + \binom{m_{n-1} - 2}{2} x^{m_{n-1} - 4} + \ldots \tag{22}$$

where the first and the second element of the product (21) are as follows:

$$\begin{aligned}
&\det\left(A_{n-1} - d_{n-1} p_{n-1,1} I_{n-1}\right) = d_{n-2}^{m_1 \cdot m_2 \cdot \ldots \cdot m_{n-1}} \det\left(\frac{A_{n-1}}{d_{n-2}} - \frac{d_{n-1}}{d_{n-2}} p_{n-1,1} I_{n-1}\right) = \\
&= d_{n-2}^{m_1 \cdot m_2 \cdot \ldots \cdot m_{n-1}} \det\left[\left(\frac{A_{n-2}}{d_{n-2}} - p_{1,1} I_{n-2}\right) - \frac{d_{n-1}}{d_{n-2}} p_{n-1,1} I_{n-2}\right] \cdot \\
&\cdot \det\left[\left(\frac{A_{n-2}}{d_{n-2}} - p_{1,2} I_{n-2}\right) - \frac{d_{n-1}}{d_{n-2}} p_{n-1,1} I_{n-2}\right] \cdot \ldots \cdot \\
&\cdot \det\left[\left(\frac{A_{n-2}}{d_{n-2}} - p_{1,m_{n-2}} I_{n-2}\right) - \frac{d_{n-1}}{d_{n-2}} p_{n-1,1} I_{n-2}\right] = \\
&= \det\left[A_{n-2} - \left(d_{n-2} p_{1,1} + d_{n-1} p_{n-1,1}\right) I_{n-2}\right] \cdot \\
&\cdot \det\left[A_{n-2} - \left(d_{n-2} p_{1,2} + d_{n-1} p_{n-1,1}\right) I_{n-2}\right] \cdot \ldots \cdot \\
&\cdot \det\left[A_{n-2} - \left(d_{n-2} p_{1,m_{n-2}} + d_{n-1} p_{n-1,1}\right) I_{n-2}\right]
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\det\left(A_{n-1}-d_{n-1}p_{n-1,2}I_{n-1}\right)=d_{n-2}^{m_1\bullet m_2\bullet\ldots\bullet m_{n-1}}\det\left(\frac{A_{n-1}}{d_{n-2}}-\frac{d_{n-1}}{d_{n-2}}p_{n-1,2}I_{n-1}\right)=\\
&=d_{n-2}^{m_1\bullet m_2\bullet\ldots\bullet m_{n-1}}\det\left[\left(\frac{A_{n-2}}{d_{n-2}}-p_{1,1}I_{n-2}\right)-\frac{d_{n-1}}{d_{n-2}}p_{n-1,2}I_{n-2}\right]\cdot\\
&\cdot\det\left[\left(\frac{A_{n-2}}{d_{n-2}}-p_{1,2}I_{n-2}\right)-\frac{d_{n-1}}{d_{n-2}}p_{n-1,2}I_{n-2}\right]\cdot\ldots\cdot\\
&\cdot\det\left[\left(\frac{A_{n-2}}{d_{n-2}}-p_{1,m_{n-2}}I_{n-2}\right)-\frac{d_{n-1}}{d_{n-2}}p_{n-1,2}I_{n-2}\right]=\\
&=\det\left[A_{n-2}-\left(d_{n-2}p_{1,1}+d_{n-1}p_{n-1,2}\right)I_{n-2}\right]\cdot\\
&\cdot\det\left[A_{n-2}-\left(d_{n-2}p_{1,2}+d_{n-1}p_{n-1,2}\right)I_{n-2}\right]\cdot\ldots\cdot\\
&\cdot\det\left[A_{n-2}-\left(d_{n-2}p_{1,m_{n-2}}+d_{n-1}p_{n-1,2}\right)I_{n-2}\right]
\end{aligned}\tag{24}$$

and the last element is in the form

$$\begin{aligned}
&\det\left(A_{n-1}-d_{n-1}p_{n-1,m_{n-1}}I_{n-1}\right)=d_{n-2}^{m_1\bullet m_2\bullet\ldots\bullet m_{n-1}}\det\left(\frac{A_{n-1}}{d_{n-2}}-\frac{d_{n-1}}{d_{n-2}}p_{n-1,m_{n-1}}I_{n-1}\right)=\\
&=d_{n-2}^{m_1\bullet m_2\bullet\ldots\bullet m_{n-1}}\det\left[\left(\frac{A_{n-2}}{d_{n-2}}-p_{1,1}I_{n-2}\right)-\frac{d_{n-1}}{d_{n-2}}p_{n-1,m_{n-1}}I_{n-2}\right]\cdot\\
&\cdot\det\left[\left(\frac{A_{n-2}}{d_{n-2}}-p_{1,2}I_{n-2}\right)-\frac{d_{n-1}}{d_{n-2}}p_{n-1,m_{n-1}}I_{n-2}\right]\cdot\ldots\cdot\\
&\cdot\det\left[\left(\frac{A_{n-2}}{d_{n-2}}-p_{1,m_{n-2}}I_{n-2}\right)-\frac{d_{n-1}}{d_{n-2}}p_{n-1,m_{n-1}}I_{n-2}\right]=\\
&=\det\left[A_{n-2}-\left(d_{n-2}p_{1,1}+d_{n-1}p_{n-1,m_{n-1}}\right)I_{n-2}\right]\cdot\\
&\cdot\det\left[A_{n-2}-\left(d_{n-2}p_{1,2}+d_{n-1}p_{n-1,m_{n-1}}\right)I_{n-2}\right]\cdot\ldots\cdot\\
&\cdot\det\left[A_{n-2}-\left(d_{n-2}p_{1,m_{n-2}}+d_{n-1}p_{n-1,m_{n-1}}\right)I_{n-2}\right]
\end{aligned}\tag{25}$$

Conclusion

This paper is the presentation of the algebraic method for solving a large-size system of equations occurring, for example, in the heat flow. In the article, the method of calculating the determinant of the main matrix of the system is given.

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