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## FORCE DISTRIBUTION IN GRANULAR SYSTEMS WITH AND WITHOUT FRICTION

*Jolanta Błaszczuk*

*Institute of Mathematics, Czestochowa University of Technology, Poland  
jolab@imi.pcz.pl*

**Abstract.** Force distribution in granular media is discussed in this paper. The proper estimation of the force network has become a challenging issue because of a tendency to optimize packing processes. The aim of the study is numerical analysis of regular packings of grains in the frictionless and frictional 2D systems, as well as assessment of force distribution in static granular media and analysis of the influence of the load type and system size on force distribution.

### Introduction

Granular materials represent a very important class of materials used in technological processes. In general their properties are similar to solids or liquids [1, 2]. One of the most interesting aspects of granular media is the distribution of forces between individual particles and it is not studied completely to the present day. The geometrical properties of networks have attracted much attention due to progress in the fields of computer science, mathematical biology and statistical physics. Many branches of industry are very interested in this research in particular pharmaceutical, food and the construction industry. Force networks in the granular systems are highly heterogeneous, also in case of static problem. The contact forces in the ensemble of grains are usually unilateral. The set of equations expressing the balance equations for each particle is undetermined. All above mentioned features of granular packings mean that the problem is widely investigated.

Inside a granular material loaded grains form a structure, called the force network, which are studied theoretically [2-10] and experimentally [5, 11]. Many numerical simulations of force distribution in the absence of friction [4, 6-9] and for packings of frictional grains [7] were carried out. The problem of force distribution was examined for various geometry of lattices: triangular „snooker ball” configurations [8, 9, 12], rectangular and hexagonal packings [7].

## 1. Formulation of the problem

The paper examines the regular packings of spheres with the radius of  $r$ . The number of spheres is not limited, however analysis of huge systems is much more time consuming. The ensemble is loaded by unit forces applied to the spheres located on the top. For the first step of studies unit vertical force was acting on each spherical grain and friction wasn't taken into account. In the second step the similar system allowing for Coulomb friction was analysed. The system is bounded by three walls and the surroundings can take over forces perpendicular to the sides and the bottom (in case of friction also tangential forces  $t$ ). The scheme of the ensemble is presented in Figure 1.

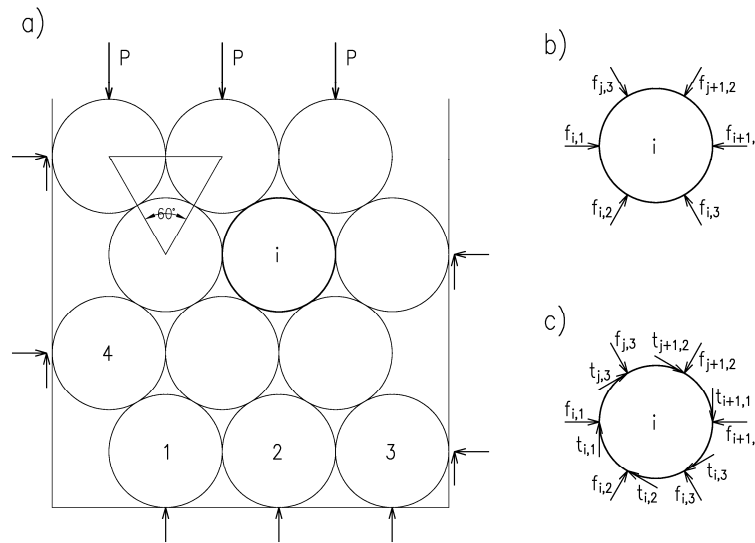


Fig. 1. The system of regular packings: a) the arrangement of grains, b) forces acting on an isolated element in case of the system without friction, c) forces in case of friction

Figure 1 shows that the typical grain has contact with six neighbouring spheres. It means three or six independent contact forces per grain for frictionless and friction systems, respectively. In the frictionless system we can write two independent balance equations for the grain. In case of the friction system additionally the torque balance equation should be satisfied. It causes that the set of balance equation is undetermined. From the mathematical point of view the problem consists in finding the admissible space of solution and then the force distribution. Please note, that the solution may contain only repulsive forces ( $f_i \geq 0$ ) and not all forces can be equal to zero (the zero vector does not belong to the admissible space).

The aim of the study was the analysis of force network in the regular packings of grains. In order to solve the problem many times, a numerical program was created which helps to generate a specific set of balance equations in a matrix form.

At the same time specific boundary conditions are generated. The program enables a search to be carried out for the numerous solutions to the static system. In this way we were able to examine the influence of grain arrangement and boundary conditions on the system response. The structure of the matrix which describes the behaviour of the ensemble was also analysed.

## 2. Numerical examples

To understand the phenomenon of force network distribution, an example of the frictionless system with twelve grains was analysed. The vertical forces acting at the top are equal to the unity. We need 52 equations to find all 52 unknown forces. There are 24 balance equations and 22 boundary conditions. The solutions are achieved by multiple generation of selected internal forces (only six forces are generated) using random numbers following the continuous uniform distribution on the interval  $(0,1)$ . After each generation the condition is tested if all forces are positive. The procedure of searching can be repeated many times providing sufficient information about the space of solutions. The example of generated force network is presented in Figure 2. The thickness of lines connecting respective grains represents the level of force.

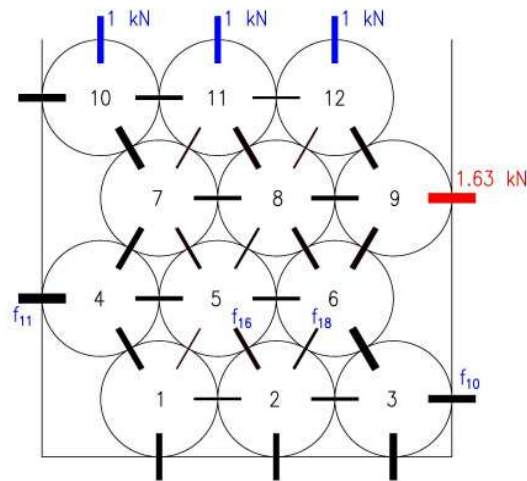


Fig. 2. The force network in the system of 12 particles without friction. The system is loaded by unit forces applied to the spheres located on the top

In the example, the interparticle forces which are presented in Figure 2 vary from 0.0985 to 1.63 kN. It demonstrates that force distribution is quite heterogeneous. Loads are transferred to the ground by random channels. It is interesting that in the case of small systems the forces between grains and walls are relatively high. Force distribution is visible along grains: 11, 8, 6, 3 and 12, 9. Maximum force equals

1.63 kN. Next, different values of four forces are analysed: two horizontal forces  $f_{10}$  and  $f_{11}$  and two at an angle forces  $f_{16}$  and  $f_{18}$ . Distribution of forces is presented in Figure 3. The horizontal forces are bigger than the diagonal that is justified.

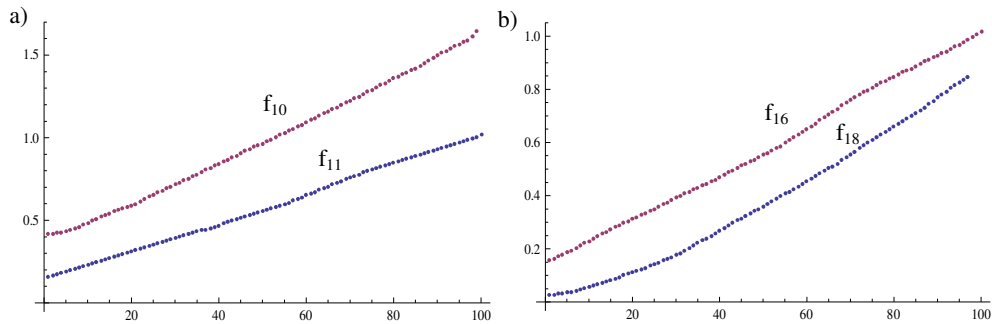


Fig. 3. Distribution of forces: a) two horizontal forces  $f_{10}$  and  $f_{11}$ ,  
b) at an angle forces  $f_{16}$  and  $f_{18}$

Systems which consist of various numbers of particles with different amounts of rows and columns are analysed for the next example (Fig. 4). The aim of the example is checking disproportion between values of extreme forces and various proportion of systems. All the systems behave similarly in the case of similar force excitations.

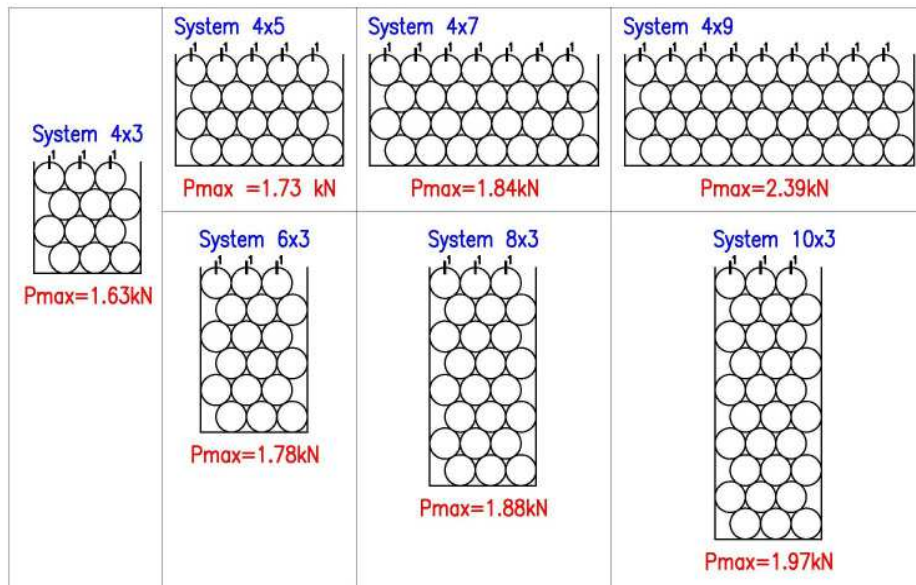


Fig. 4. Systems for various numbers of particles and different numbers of rows and columns

Change of load conditions, e.g. horizontal actions instead of vertical ones or only one grain loaded by unit force applied to the grain located on the top, result in different force distribution. In Figure 5, the influence of the load type on force distribution is presented. It demonstrates that force distribution is quite heterogeneous.

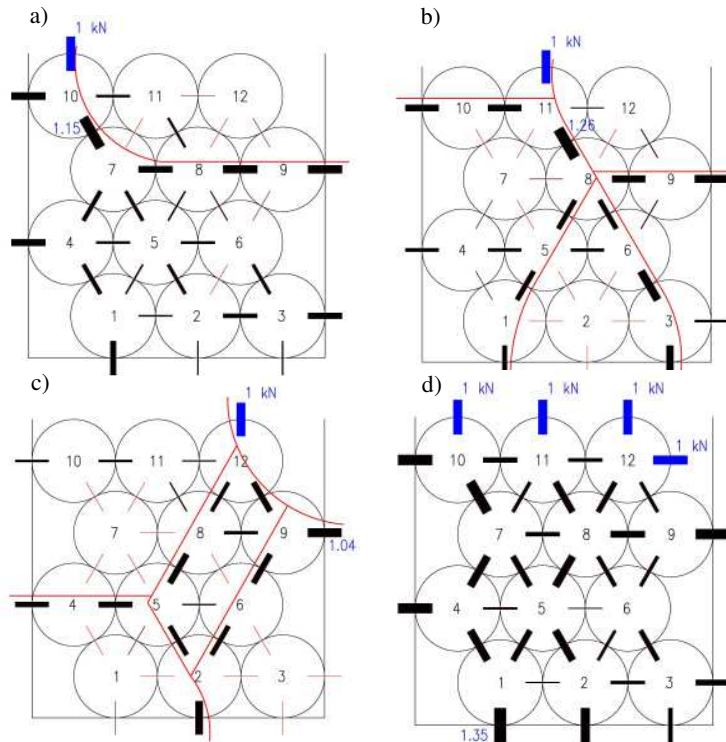


Fig. 5. Systems of grains for the various type of loads acting on the systems

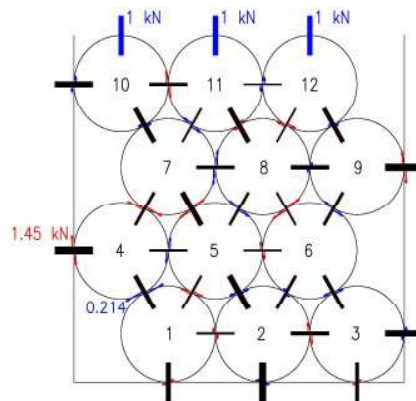


Fig. 6. The system of 12 particles with friction which is loaded by unit forces applied to the spheres located on the top

In Figure 6, an example of the system with friction is presented. The system with twelve grains is described by two static independent balance equations for the grain and additionally the torque balance equation should be satisfied. It is assumed friction coefficient  $\mu = 0.4$ . The number of unknown forces is 82. We must generate 24 forces. Maximum friction force is equal to 0.214 kN.

From the mathematical point of view, a feature of the matrix representing the set of balance equations is its sparseness. It is caused by the fact that only grains localized in the close vicinity participate in the respective balance equations. Therefore, most matrix elements equal zero and improper assumptions concerning force distribution have an effect in the lack of any solution.

### Concluding remarks

The numerical analysis of force networks in granular systems proved that there is a complex problem to find an appropriate and realistic force distribution. The set of balance equations is undetermined and assuming random forces in the system usually leads to unacceptable solution with negative forces. The form of force loads acting on the ensemble play an important role in force distribution.

The presented example verifies the approach to the problem and demonstrates its effectiveness.

The proposed method allows for the numerical prediction of the behaviour of regular packing in static problems with and without friction. The above presented approach can be easily generalized by the introduction of gravity forces and cohesion in normal interactions. The true challenge is appropriate and an efficient analysis of systems with various size particles.

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