SHAPE SENSITIVITY ANALYSIS WITH RESPECT TO THE PARAMETERS OF INTERNAL HOLE

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Abstract. The Laplace equation describing temperature field in 2D domain supplemented by adequate boundary conditions is considered. The aim of investigations is to estimate the changes of temperature due to changes of shape parameter (e.g. radius or position of internal hole). To solve the problem, the implicit differentiation method of shape sensitivity analysis coupled with the boundary element method is applied.

1. Boundary integral equation for the Laplace equation

The steady state temperature field $T(x, y)$ in domain Ω limited by boundary $\Gamma = \Gamma_1 \cup \Gamma_2$ (Fig.) is described by the Laplace equation

$$(x, y) \in \Omega: \quad \nabla^2 T(x, y) = 0 \tag{1}$$

supplemented by adequate boundary conditions.

The boundary integral equation corresponding to the equation (1) has the following form [1, 2]

$$(\xi, \eta) \in \Gamma: \quad B(\xi, \eta)\, T(\xi, \eta) + \int_\Gamma q(x, y)\, T^*(\xi, \eta, x, y)\, d\Gamma = \int_\Gamma T(x, y)\, q^*(\xi, \eta, x, y)\, d\Gamma \tag{2}$$

where (ξ, η) is the observation point, $q(x, y) = -\lambda\, \mathbf{n} \cdot \nabla T(x, y)$ is the boundary heat flux (λ is the thermal conductivity, $\mathbf{n} = [\cos\alpha \ \cos\beta]$ is the unit outward vector normal to Γ - Figure 1), $B(\xi, \eta) \in (0, 1)$ is the coefficient connected with the local shape of boundary.

Function $T^*(\xi, \eta, x, y)$ is the fundamental solution and for the problem considered it has the following form

$$T^*(\xi, \eta, x, y) = \frac{1}{2\pi\lambda} \ln \frac{1}{r} \tag{3}$$

where r is the distance between points (ξ, η) and (x, y).

The heat flux $q^*(\xi, \eta, x, y) = -\lambda\, \mathrm{n} \cdot \nabla T^*(\xi, \eta, x, y)$ resulting from the fundamental solution can be calculated in analytical way and then

$$q^*(\xi,\ \eta,\ x,\ y) = \frac{d}{2\pi r^2} \tag{4}$$

where

$$d = (x - \xi)\cos\alpha + (y - \eta)\cos\beta \tag{5}$$

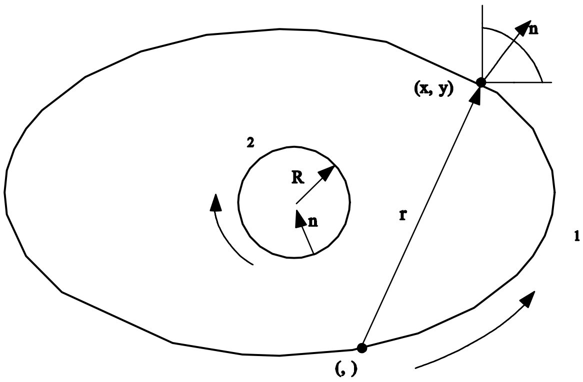

Fig. 1. Domain considered

2. Numerical realization of the boundary element method

To solve the equation (2) the boundary Γ is divided into N boundary elements Γ_j, $j = 1, 2,..., N$ and the integrals appearing in equation (2) are substituted by the sums of integrals over these elements, namely

$$B(\xi_i,\ \eta_i)T(\xi_i,\ \eta_i) + \sum_{j=1}^{N} \int_{\Gamma_j} q(x,\ y)T^*(\xi_i,\ \eta_i,\ x,\ y)\,\mathrm{d}\Gamma_j = \sum_{j=1}^{N} \int_{\Gamma_j} T(x,\ y)q^*(\xi_i,\ \eta_i,\ x,\ y)\,\mathrm{d}\Gamma_j \tag{6}$$

where (ξ_i, η_i) denotes the boundary node.

For linear boundary element Γ_j one has

$$(x,\ y) \in \Gamma_j : \quad \begin{cases} x(\theta) = N_p x_p^j + N_k x_k^j \\ y(\theta) = N_p y_p^j + N_k y_k^j \end{cases} \tag{7}$$

and

$$(x,\ y) \in \Gamma_j : \quad \begin{cases} T(\theta) = N_p\, T(x_p^j,\ y_p^j) + N_k\, T(x_k^j,\ y_k^j) \\ q(\theta) = N_p\, q(x_p^j,\ y_p^j) + N_k\, q(x_k^j,\ y_k^j) \end{cases} \tag{8}$$

where $(x_p^j,\ y_p^j)$, $(x_k^j,\ y_k^j)$ correspond to the beginning and the end of boundary element Γ_j and

$$N_p = \frac{1-\theta}{2}, \qquad N_k = \frac{1+\theta}{2} \tag{9}$$

while $\theta \in [-1, 1]$.

The integrals appearing in Equation (6) can be written as follows

$$\int_{\Gamma_j} q(x,\ y)\, T^*(\xi_i,\ \eta_i,\ x,\ y)\, \mathrm{d}\Gamma_j = G_{ij}^p\, q(x_p^j,\ y_p^j) + G_{ij}^k\, q(x_k^j,\ y_k^j) \tag{10}$$

and

$$\int_{\Gamma_j} T(x,\ y)\, q^*(\xi_i,\ \eta_i,\ x,\ y)\, \mathrm{d}\Gamma_j = \hat{H}_{ij}^p\, T(x_p^j,\ y_p^j) + \hat{H}_{ij}^k\, T(x_k^j,\ y_k^j) \tag{11}$$

where [2]

$$G_{ij}^p = \begin{cases} \dfrac{l_j}{4\pi\lambda} \displaystyle\int_{-1}^{1} N_p \ln \frac{1}{r_{ij}}\, \mathrm{d}\theta, & (\xi_i,\ \eta_i) \neq (x_p^j,\ y_p^j) \text{ and } (\xi_i,\ \eta_i) \neq (x_k^j,\ y_k^j) \\ \dfrac{l_j (3 - 2\ln l_j)}{8\pi\lambda}, & (\xi_i,\ \eta_i) = (x_p^j,\ y_p^j) \\ \dfrac{l_j (1 - 2\ln l_j)}{8\pi\lambda}, & (\xi_i,\ \eta_i) = (x_k^j,\ y_k^j) \end{cases} \tag{12}$$

$$G_{ij}^k = \begin{cases} \dfrac{l_j}{4\pi\lambda} \displaystyle\int_{-1}^{1} N_k \ln \frac{1}{r_{ij}}\, \mathrm{d}\theta, & (\xi_i,\ \eta_i) \neq (x_p^j,\ y_p^j) \text{ and } (\xi_i,\ \eta_i) \neq (x_k^j,\ y_k^j) \\ \dfrac{l_j (1 - 2\ln l_j)}{8\pi\lambda}, & (\xi_i,\ \eta_i) = (x_p^j,\ y_p^j) \\ \dfrac{l_j (3 - 2\ln l_j)}{8\pi\lambda}, & (\xi_i,\ \eta_i) = (x_k^j,\ y_k^j) \end{cases} \tag{13}$$

and

$$\hat{H}_{ij}^{p} = \begin{cases} \dfrac{1}{4\pi}\displaystyle\int_{-1}^{1} N_p \dfrac{r_x^j l_y^j - r_y^j l_x^j}{r_{ij}^2}\,\mathrm{d}\theta, & (\xi_i,\ \eta_i) \neq (x_p^j,\ y_p^j) \text{ and } (\xi_i,\ \eta_i) \neq (x_k^j,\ y_k^j) \\ 0, & (\xi_i,\ \eta_i) = (x_p^j,\ y_p^j) \text{ or } (\xi_i,\ \eta_i) = (x_k^j,\ y_k^j) \end{cases} \tag{14}$$

$$\hat{H}_{ij}^{k} = \begin{cases} \dfrac{1}{4\pi}\displaystyle\int_{-1}^{1} N_k \dfrac{r_x^j l_y^j - r_y^j l_x^j}{r_{ij}^2}\,\mathrm{d}\theta, & (\xi_i,\ \eta_i) \neq (x_p^j,\ y_p^j) \text{ and } (\xi_i,\ \eta_i) \neq (x_k^j,\ y_k^j) \\ 0, & (\xi_i,\ \eta_i) = (x_p^j,\ y_p^j) \text{ or } (\xi_i,\ \eta_i) = (x_k^j,\ y_k^j) \end{cases} \tag{15}$$

In Equations (12), (13), (14), (15) r_{ij} is the distance between the observation point (ξ_i, η_i) and the point (x, y) on the linear boundary element Γ_j

$$r_{ij} = \sqrt{(r_x^j)^2 + (r_y^j)^2} \tag{16}$$

where

$$\begin{aligned} r_x^j &= N_p\, x_p^j + N_k\, x_k^j - \xi_i \\ r_y^j &= N_p\, y_p^j + N_k\, y_k^j - \eta_i \end{aligned} \tag{17}$$

and l_j is the length of the boundary element Γ_j

$$l_{ij} = \sqrt{(l_x^j)^2 + (l_y^j)^2} \tag{18}$$

where

$$\begin{aligned} l_x^j &= x_k^j - x_p^j \\ l_y^j &= y_k^j - y_p^j \end{aligned} \tag{19}$$

For the single node r being the end of the boundary element Γ_j and being the beginning of the boundary element Γ_{j+1} we have

$$\begin{aligned} G_{ir} &= G_{ij}^{k} + G_{ij+1}^{p} \\ \hat{H}_{ir} &= \hat{H}_{ij}^{k} + \hat{H}_{ij+1}^{p} \end{aligned} \tag{20}$$

while for double node r, r+1

$$\begin{aligned} G_{ir} &= G_{ij}^{k}, \qquad & G_{ir+1} &= G_{ij+1}^{p} \\ \hat{H}_{ir} &= \hat{H}_{ij}^{k}, \qquad & \hat{H}_{ir+1} &= \hat{H}_{ij+1}^{p} \end{aligned} \tag{21}$$

Finally, one obtains the following system of equations (i = 1, 2,..., R, R is the number of boundary nodes)

$$\sum_{r=1}^{R} G_{ir}\, q = \sum_{r=1}^{R} H_{ir}\, T_r \tag{22}$$

where

$$H_{ir} = \begin{cases} \hat{H}_{ir}, & i \neq r \\ -\sum\limits_{\substack{r=1 \\ i\neq r}}^{R} \hat{H}_{ir}, & i = r \end{cases} \tag{23}$$

and $T_r = T(x_r, y_r)$, $q_r = q(x_r, y_r)$.

The system of equations (22) can be written in the matrix form

$$\mathbf{G}\,\mathbf{q} = \mathbf{H}\,\mathbf{T} \tag{24}$$

It should be pointed out that in the system of equations (24) part of the boundary values (temperatures or heat fluxes) is known from the boundary conditions, while the remaining R boundary values (heat fluxes or temperatures) should be determined. Next, the temperatures at the optional internal nodes (ξ_i, η_i) are calculated using the formula

$$T_i = \sum_{r=1}^{R} H_{ir}\, T - \sum_{r=1}^{R} G_{ir}\, q_r \tag{25}$$

3. Shape sensitivity analysis - implicit approach

We assume that the shape parameter b corresponds to the radius R of internal hole or corresponds to the position of its centre, this means $b = x_s$ or $b = y_s$, where (x_s, y_s) is the centre of circle - Figure 1. The implicit differentiation method [3, 4] of sensitivity analysis starts with the algebraic system of equations (24). The differentiation of (24) with respect to b leads to the following system of equations

$$\frac{\mathrm{D}\mathbf{G}}{\mathrm{D}b}\mathbf{q} + \mathbf{G}\frac{\mathrm{D}\mathbf{q}}{\mathrm{D}b} = \frac{\mathrm{D}\mathbf{H}}{\mathrm{D}b}\mathbf{T} + \mathbf{H}\frac{\mathrm{D}\mathbf{T}}{\mathrm{D}b} \tag{26}$$

or

$$\mathbf{G}\frac{\mathrm{D}\mathbf{q}}{\mathrm{D}b} = \mathbf{H}\frac{\mathrm{D}\mathbf{T}}{\mathrm{D}b} + \frac{\mathrm{D}\mathbf{H}}{\mathrm{D}b}\mathbf{T} - \frac{\mathrm{D}\mathbf{G}}{\mathrm{D}b}\mathbf{q} \tag{27}$$

This approach of shape sensitivity analysis requires the differentiation of elements of matrices **G** and **H**, this means the differentiation of $\mathbf{G}^p$, $\mathbf{G}^k$, $\mathbf{H}^p$, $\mathbf{H}^k$ (c.f. Equations (12), (13), (14), (15)) with respect to parameter b.

Non-zero elements of these matrices are connected with [5]:
1. the integrals over the boundary elements approximating the external boundary of the domain considered for which the observation point (ξ_i, η_i) belongs to the circle (Figure 2),
2. the integrals over the boundary elements approximating the internal circle (Figure 3).

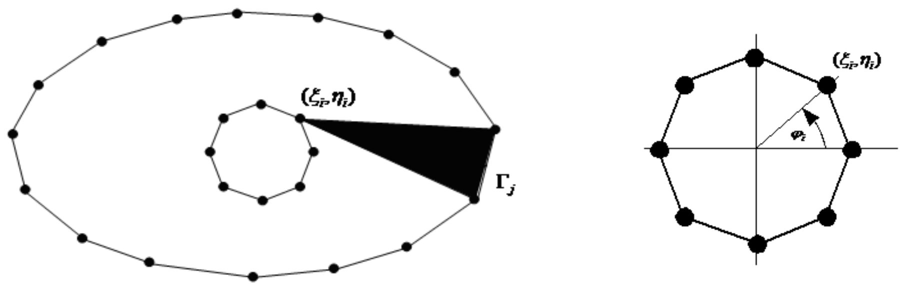

Fig. 2. Integration over the 'external' linear boundary element

In the first case the observation point (ξ_i, η_i) can be expressed as follows (Fig. 2)

$$\begin{aligned} \xi_i &= x_s + R\cos\varphi_i \\ \eta_i &= y_s + R\sin\varphi_i \end{aligned} \tag{28}$$

where (x_s, y_s) is the circle centre. Taking into account the formulas (7), (18), (19) one has

$$\frac{\partial l_x^j}{\partial b} = 0, \qquad \frac{\partial l_y^j}{\partial b} = 0, \qquad \frac{\partial l_j}{\partial b} = 0 \tag{29}$$

while (c.f. Equation (17))

$$\begin{aligned} r_x^j &= N_p x_p^j + N_k x_k^j - (x_s + R\cos\varphi_i) \\ r_y^j &= N_p y_p^j + N_k y_k^j - (y_s + R\sin\varphi_i) \end{aligned} \tag{30}$$

from which results that

$$\frac{\partial r_x^j}{\partial b} = \begin{cases} -\cos\varphi_i, & b = R \\ -1, & b = x_s \\ 0, & b = y_s \end{cases}, \qquad \frac{\partial r_y^j}{\partial b} = \begin{cases} -\sin\varphi_i, & b = R \\ -1, & b = x_s \\ 0, & b = y_s \end{cases} \tag{31}$$

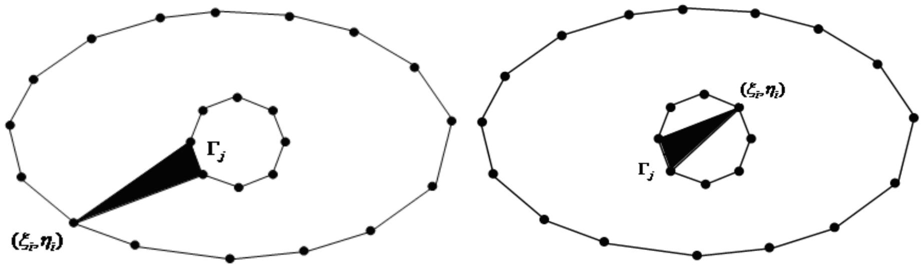

Fig. 3. Integration over the 'internal' linear boundary element

In the second case (Fig. 3) each point on the linear boundary element Γ_j can be expressed as follows (Fig. 4)

$$(x,\ y)\in\Gamma_j:\quad \begin{cases} x = N_p(x_s + R\cos\varphi_p^j) + N_k(x_s + R\cos\varphi_k^j) \\ y = N_p(y_s + R\sin\varphi_p^j) + N_k(y_s + R\sin\varphi_k^j) \end{cases} \tag{32}$$

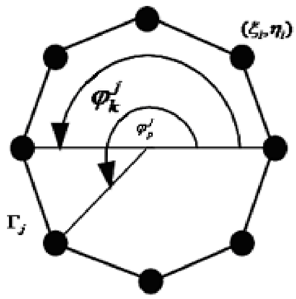

Fig. 4. 'Internal' element

It is easy to check that (c.f. Equation (9))

$$\begin{aligned} l_x^j &= x_k^j - x_p^j = R(\cos\varphi_k^j - \cos\varphi_p^j) \\ l_y^j &= y_k^j - y_p^j = R(\sin\varphi_k^j - \sin\varphi_p^j) \end{aligned} \tag{33}$$

So (c.f. Equation (18))

$$\frac{\partial l_j}{\partial b} = \frac{1}{l_j}\left(l_x^j \frac{\partial l_x^j}{\partial b} + l_y^j \frac{\partial l_y^j}{\partial b} \right) \tag{34}$$

where

$$\frac{\partial l_x^j}{\partial b} = \begin{cases} \cos\varphi_k^j - \cos\varphi_p^j, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases}, \qquad \frac{\partial l_y^j}{\partial b} = \begin{cases} \sin\varphi_k^j - \sin\varphi_p^j, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases} \tag{35}$$

If the observation point (ξ_i, η_i) does not belong to the circle (Fig. 3a) then

$$\begin{aligned} r_x^j &= N_p(x_s + R\cos\varphi_p^j) + N_k(x_s + R\cos\varphi_k^j) - \xi_i \\ r_y^j &= N_p(y_s + R\sin\varphi_p^j) + N_k(y_s + R\sin\varphi_k^j) - \eta_i \end{aligned} \tag{36}$$

and

$$\frac{\partial r_x^j}{\partial b} = \begin{cases} N_p\cos\varphi_p^j + N_k\cos\varphi_k^j, & b = R \\ 1, & b = x_s \\ 0, & b = y_s \end{cases}$$
$$\frac{\partial r_y^j}{\partial b} = \begin{cases} N_p\sin\varphi_p^j + N_k\sin\varphi_k^j, & b = R \\ 1, & b = x_s \\ 0, & b = y_s \end{cases} \tag{37}$$

If the observation point (ξ_i, η_i) belongs to the circle but $(\xi_i, \eta_i) \neq (x_p^j, y_p^j)$ and $(\xi_i, \eta_i) \neq (x_k^j, y_k^j)$ (Fig. 3b) then

$$\begin{aligned} r_x^j &= N_p(x_s + R\cos\varphi_p^j) + N_k(x_s + R\cos\varphi_k^j) - (x_s + R\cos\varphi_i) \\ r_y^j &= N_p(y_s + R\sin\varphi_p^j) + N_k(y_s + R\sin\varphi_k^j) - (y_s + R\sin\varphi_i) \end{aligned} \tag{38}$$

and

$$\frac{\partial r_x^j}{\partial b} = \begin{cases} N_p\cos\varphi_p^j + N_k\cos\varphi_k^j - \cos\varphi_i, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases}$$
$$\frac{\partial r_y^j}{\partial b} = \begin{cases} N_p\sin\varphi_p^j + N_k\sin\varphi_k^j - \sin\varphi_i, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases} \tag{39}$$

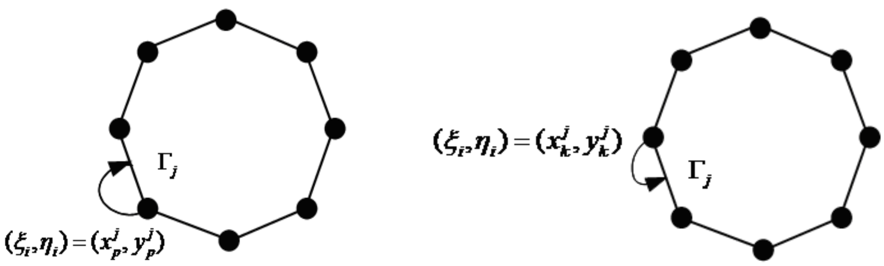

Fig. 5. Singular integrals over the 'internal' boundary elements

For $(\xi_i, \eta_i) = (x_p^{\,j}, y_p^{\,j})$ (Fig. 5) one has

$$\begin{aligned} r_x^j &= N_p(x_s + R\cos\varphi_p^k) + N_k(x_s + R\cos\varphi_k^j) - (x_s + R\cos\varphi_p^j) \\ r_y^j &= N_p(y_s + R\sin\varphi_p^k) + N_k(y_s + R\sin\varphi_k^j) - (y_s + R\sin\varphi_p^j) \end{aligned} \tag{40}$$

and then

$$\begin{aligned} \frac{\partial r_x^j}{\partial b} &= \begin{cases} (N_p - 1)\cos\varphi_p^j + N_k \cos\varphi_k^j, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases} \\ \frac{\partial r_y^j}{\partial b} &= \begin{cases} (N_p - 1)\sin\varphi_p^j + N_k \sin\varphi_k^j, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases} \end{aligned} \tag{41}$$

For $(\xi_i, \eta_i) = (x_k^{\,j}, y_k^{\,j})$ one obtains

$$\begin{aligned} r_x^j &= N_p(x_s + R\cos\varphi_p^j) + N_k(x_s + R\cos\varphi_k^j) - (x_s + R\cos\varphi_k^j) \\ r_y^j &= N_p(y_s + R\sin\varphi_p^j) + N_k(y_s + R\sin\varphi_k^j) - (y_s + R\sin\varphi_k^j) \end{aligned} \tag{42}$$

and then

$$\begin{aligned} \frac{\partial r_x^j}{\partial b} &= \begin{cases} N_p\cos\varphi_p^j + (N_k - 1)\cos\varphi_k^j, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases} \\ \frac{\partial r_y^j}{\partial b} &= \begin{cases} N_p\sin\varphi_p^j + (N_k - 1)\sin\varphi_k^j, & b = R \\ 0, & b = x_s \text{ or } b = y_s \end{cases} \end{aligned} \tag{43}$$

Now, we calculate (c.f. Equations (12), (13))

$$\frac{\partial G_{ij}^p}{\partial b} = \begin{cases} \dfrac{1}{4\pi\lambda}\dfrac{\partial l_j}{\partial b}\displaystyle\int_{-1}^{1} N_p \ln\frac{1}{r_{ij}}\,\mathrm{d}\theta + \frac{l_j}{4\pi\lambda}\int_{-1}^{1} N_p \frac{\partial}{\partial b}\left(\ln\frac{1}{r_{ij}}\right)\mathrm{d}\theta, \\ \qquad (\xi_i, \eta_i) \neq (x_p^j, y_p^j) \text{ and } (\xi_i, \eta_i) \neq (x_k^j, y_k^j) \\ \dfrac{1}{8\pi\lambda}\dfrac{\partial l_j}{\partial b}(1 - 2\ln l_j), & (\xi_i, \eta_i) = (x_p^j, y_p^j) \\ \dfrac{1}{8\pi\lambda}\dfrac{\partial l_j}{\partial b}(-1 - 2\ln l_j), & (\xi_i, \eta_i) = (x_k^j, y_k^j) \end{cases} \tag{44}$$

and

$$\frac{\partial G_{ij}^{k}}{\partial b}=\begin{cases}\dfrac{1}{4\pi\lambda}\dfrac{\partial l_j}{\partial b}\displaystyle\int_{-1}^{1}N_k\ln\frac{1}{r_{ij}}\mathrm{d}\theta+\frac{l_j}{4\pi\lambda}\int_{-1}^{1}N_k\frac{\partial}{\partial b}\left(\ln\frac{1}{r_{ij}}\right)\mathrm{d}\theta, \\ \qquad (\xi_i,\eta_i)\neq(x_p^j,y_p^j)\text{ and }(\xi_i,\eta_i)\neq(x_k^j,y_k^j) \\ \dfrac{1}{8\pi\lambda}\dfrac{\partial l_j}{\partial b}(-1-2\ln l_j), \qquad (\xi_i,\eta_i)=(x_p^j,y_p^j) \\ \dfrac{1}{8\pi\lambda}\dfrac{\partial l_j}{\partial b}(1-2\ln l_j), \qquad (\xi_i,\eta_i)=(x_k^j,y_k^j)\end{cases} \tag{45}$$

where

$$\frac{\partial}{\partial b}\left(\ln\frac{1}{r_{ij}}\right)=-\frac{1}{r_{ij}^2}\left(r_x^j\frac{\partial r_x^j}{\partial b}+r_y^j\frac{\partial r_y^j}{\partial b}\right) \tag{46}$$

In similar way the formulas (14), (15) are differentiated and then for $(\xi_i,\eta_i)\neq(x_p{}^j, y_p{}^j)$ and $(\xi_i,\eta_i)\neq(x_k{}^j, y_k{}^j)$ one obtains

$$\frac{\partial \hat{H}_{ij}^{p}}{\partial b}=\frac{1}{4\pi}\int_{-1}^{1}N_p\frac{\partial}{\partial b}\left(\frac{r_x^j l_y^j-r_y^j l_x^j}{r_{ij}^2}\right)\mathrm{d}\theta \tag{47}$$

and

$$\frac{\partial \hat{H}_{ij}^{k}}{\partial b}=\frac{1}{4\pi}\int_{-1}^{1}N_k\frac{\partial}{\partial b}\left(\frac{r_x^j l_y^j-r_y^j l_x^j}{r_{ij}^2}\right)\mathrm{d}\theta \tag{48}$$

where

$$\begin{gathered}\frac{\partial}{\partial b}\left(\frac{r_x^j l_y^j-r_y^j l_x^j}{r_{ij}^2}\right)=\left(\frac{\partial r_x^j}{\partial b}l_y^j+r_x^j\frac{\partial l_y^j}{\partial b}-\frac{\partial r_y^j}{\partial b}l_x^j-r_y^j\frac{\partial l_x^j}{\partial b}\right)/\,r_{ij}^2- \\ 2\left(r_x^j\frac{\partial r_x^j}{\partial b}+r_y^j\frac{\partial r_y^j}{\partial b}\right)(r_x^j l_y^j-r_y^j l_x^j)/\,r_{ij}^4\end{gathered} \tag{49}$$

It should be pointed out that in the system of equations (27) the values of **T** and **q** are known from the boundary conditions or basic problem solution (c.f. Equation (24)). Differentiation of assumed boundary conditions allows to calculate part of the values D**T** / Db, D**q** / Db, while remaining part should be determined from (27).

Next, the values U_i = DT_i /Db at the optional internal nodes (ξ_i, η_i) are calculated using the formula

$$U_i = \sum_{r=1}^{R} H_{ir} U_r - \sum_{r=1}^{R} G_{ir} Z_r + \sum_{r=1}^{R} \frac{\mathrm{D} H_{ir}}{\mathrm{D} b} T_r - \sum_{r=1}^{R} \frac{\mathrm{D} G_{ir}}{\mathrm{D} b} q_r \tag{50}$$

where Z_r = Dq_r/Db. For internal nodes (ξ_i, η_i) the non-zero elements DG_{ir}/Db, DH_{ir}/Db are connected only with integrals over the 'internal' boundary elements.

4. Example of computations

The square of dimensions 0.05 × 0.05 m is considered - Figure 6. The centre of the circle: x_s = 0.0025, y_s = 0.0025, radius: R = 0.01 m. It is assumed that λ = 1 W/ mK. On the bottom external boundary the Neumann condition $q_b = -10^4$ W/m^2 is accepted, on the remaining part of the external boundary the Dirichlet condition T_{b1} = 500°C is assumed. Along the circle the constant temperature T_{b2} = 700°C is given. The external boundary has been divided into 40 linear elements, while the internal boundary has been divided into 16 linear elements. The temperature distribution is shown in Figure 7. Figures 8, 9, 10 illustrate the distributions of sensitivity functions DT/DR, DT/Dx_s and DT/Dy_s, respectively.

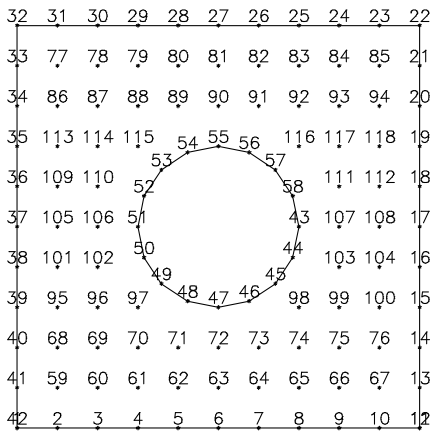

Fig. 6. Discretization and internal nodes

It should be pointed out that using the expansion of function T into the Taylor series

$$T(b+\Delta b) = T(b) + \frac{\mathrm{D}T(b)}{\mathrm{D}b}\Delta b \tag{51}$$

one can estimate the change of temperature due to the change of parameter b.

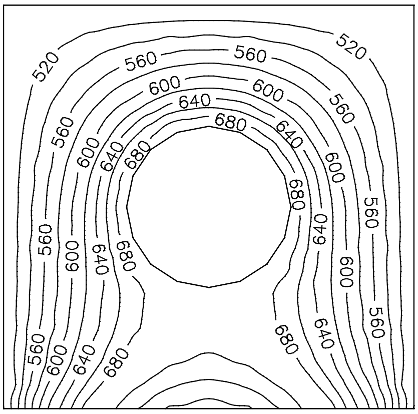

Fig. 7. Temperature distribution

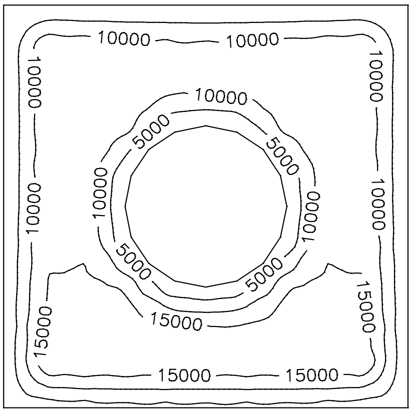

Fig. 8. Distribution of D*T*/D*R*

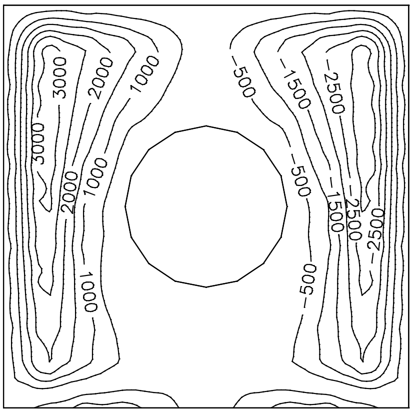

Fig. 9. Distribution of D*T*/Dx_s

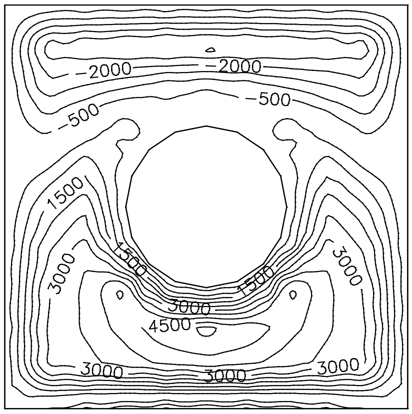

Fig. 10. Distribution of D*T*/Dy_s

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