INVESTIGATION OF G-NETWORK WITH RANDOM DELAY OF SIGNALS AT NONSTATIONARY BEHAVIOUR

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Abstract. The object of this research is an open queueing G-network with signals with random delay. The purpose of the research is investigation of such a network at the transient behavior. It is considered the case when the intensity of the incoming flow of positive and negative messages and service intensity of messages do not depend on time. All the systems in the network are one-line. It is described the model of computer system DDoS-attacks, the effect of penetration of the virus in a computer network in the form of G-network with random delay of signals. Approximate expressions are obtained for the time-dependent probabilities of states and the average characteristics of the network. Examples are calculated.

Keywords: *G-network, negative messages, the transient behavior, signals with random delay, DDoS-attack*

Introduction

Previously [1] G-networks with signals were considered (which it could be either negative messages or triggers), the effect of which was manifested immediately, i.e. activation time of any signal equal to zero and therefore not taken into account in the analysis of such networks. Now we assume that entering the queueing system (QS), signal is activated not immediately, but only after a random time. It should be noted that such a network with a one-line QS, at some generalized assumptions about the service time of positive customers, was discussed in [2], the stationary network state probabilities of the states were found in the multiplicative form.

This assumption can be used for modeling real information and telecommunications networks [3]. Negative messages and signals in the model can describe viruses in the systems that start operating after a random time, while the signal “corrected” - becomes a positive customer or “brings destruction” (i.e. becoming a negative message), destroys the positive customer in the system (or “recognized”) or “destroyed” - not rendering any impact on the system.

1. Network description, formulation of the problem

Consider an open G-network with n one-line QS [4, 5]. In QS S_i come from outside (system S_0) independent Poisson flow of positive messages with intensity λ_{0i}^{+} and Poisson flow of signals with intensity λ_{0i}^{-}, $i=\overline{1,n}$. The probability that the positive application gets serviced in S_i during time $[t,t+\Delta t)$, if at the current time moment t in the system there is k messages, is equal to $\mu_i^{+}(k)\Delta t+o(\Delta t)$. Positive message serviced in S_i with probability p_{ij}^{+} comes in QS S_j as a positive message, and with probability p_{ij}^{-} - as a signal, and with probability $p_{i0}=1-\sum_{j=1}^{n}\left(p_{ij}^{+}+p_{ij}^{-}\right)$ comes out from the network to external environment, $i,j=\overline{1,n}$.

Each incoming signal is activated for some random time interval. The probability that the incoming in the QS S_i signal is activated during time $[t,t+\Delta t)$, on condition that, in it QS at moment time t there are l nonactivated signals, is equal to $\mu_i^{-}(l)\Delta t+o(\Delta t)$. After that time (activation time):

– with probability q_{ij}^{+} the signal is activated as a trigger, moving one positive message of QS S_i to QS S_j, while its message remains positive;
– with probability q_{ij}^{-} the signal is activated again as a trigger, moving one positive message of QS S_i to QS S_j, but thus its message in QS S_j becomes a signal;
– with probability $q_{i0}=1-\sum_{j=1}^{n}\left(q_{ij}^{+}+q_{ij}^{-}\right)$ signal is triggered by a negative message, that destroying one positive message in QS S_i and leaves the network.

The message, which moves from QS S_i to QS S_j (as a positive message or signal), terminates its service in the QS S_i. If after the activation signal the system is missing positive messages, then the signal comes out from the network without affecting the functioning of the network as a whole.

The network state at the moment time t will be vector $k(t)=(k,l,t)=((k_1,l_1,t),(k_2,l_2,t),\ldots,(k_n,l_n,t))$, which forms a homogeneous Markov process with a countable number of states, where the state (k_i,l_i,t) means, that at moment time t in QS S_i there are k_i positive messages and l_i nonactivated signals, $i=\overline{1,n}$. We introduce the vectors $(k,t)=(k_1,k_2,\ldots,k_n,t)$ and $(l,t)=(l_1,l_2,\ldots,l_n,t)$, I_i - vector, which is *i*-th component is equal to 1, all the others are 0, $i=\overline{1,n}$.

It is required to find time-dependent state probabilities and average characteristics of the considered network.

Lemma. [4] Let $P(k,l,t)$ - state probability (k,l) at moment time t. State probabilities of the considered network satisfy system of difference-differential equations (DDE):

$$\frac{dP(k,l,t)}{dt} = -\sum_{i=1}^{n}\left[\lambda_{0i}^{+} + \lambda_{0i}^{+} + \mu_i^{+}(k_i)\left(1 - p_{ii}^{+}\right) + \mu_i^{-}(l_i)\right]P(k,l,t) +$$
$$+\sum_{i=1}^{n}\lambda_{0i}^{+}u(k_i)P(k - I_i,l,t) + \sum_{i=1}^{n}\lambda_{0i}^{-}u(l_i)P(k,l - I_i,t) +$$
$$+\sum_{i=1}^{n}\mu_i^{+}(k_i + 1)p_{i0}P(k + I_i,l,t) + \sum_{i=1}^{n}\mu_i^{-}(l_i + 1)q_{i0}P(k + I_i,l + I_i,t) +$$
$$+\sum_{i=1}^{n}\mu_i^{-}(l_i + 1)(1 - u(k_i))P(k,l + I_i,t) + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n}\mu_i^{+}(k_i + 1)u(k_j)p_{ij}^{+}P(k + I_i - I_j,l,t) +$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i^{+}(k_i + 1)u(l_j)p_{ij}^{-}P(k + I_i,l - I_j,t) +$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i^{-}(l_i + 1)u(k_j)q_{ij}^{+}P(k + I_i - I_j,l + I_i,t) +$$
$$+\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n}\mu_i^{-}(l_i + 1)u(l_j)q_{ij}^{-}P(k + I_i,l + I_i - I_j,t) + \sum_{i=1}^{n}\mu_i^{-}(l_i)q_{ii}^{-}P(k + I_i,l,t), \qquad (1)$$

where $\mu_i^{+}(0) = 0$, $\mu_i^{-}(0) = 0$, $u(x) = \begin{cases}1, x > 0\\ 0, x \le 0\end{cases}$ - Heaviside function.

2. Attack model on the computer system, virus penetration effect into the computer network

Damage from the virus penetration into the computer network may be quite different, from a slight increase in the size of outgoing traffic to complete failure of the network or loss of vital information. If the target of the attack is to destroy or steal information, the damage from a successful attack is equal to the value of this information. In case of theft of information - especially in cases of deliberate attacks on obviously some sacrifice - the results can be disastrous for the owner of the data, especially if we are talking about the leak of information critical to a company, organization or even a state. Customer databases, financial and technical documents, bank account numbers, details of offers - the list is endless [6].

The failure of computers and networks, or a sharp slowdown in their work is deliberate or accidental. In the case of a deliberate attack by a virus or a Trojan or the destruction of critical elements of the system, which causes an inoperable or overloaded network DDoS (Distributed Denial of Service) attack, or even in any way affects the efficiency of the system [7].

Among computer intruders DDoS attacks are growing in popularity [8]. Attack purpose is to block the legitimate users access to a website, or to make it extremely difficult. As a result of the attack there are broken or blocked completely legitimate service users, networks, systems and other resources. It uses two types of effects: the attack on the communication channel, which “hammered” a huge mass of specially crafted requests and results in an overflow system or network using a huge amount of information that can not be processed (in this case the network is operating under high load), or use of holes in the software and network protocols to block customers' access to resources information system, in this case, the whole system stops working or the network does as a result of the system sending the data packets that it does not expect, and this leads to a system stop or restarts it [8].

Despite the fact that DDoS-attack can be directed to a separate PC, today this method of network attack is used mainly against government and corporate resources (a well-protected large company or government agency), for which there are several reasons. Network DDoS-attack types by using a botnet (zombie networks) - a large number of special malware infected computers that commanded from the control center (from the attacker) start sending many special requests to the target computer, blocking access to legitimate users [9, 10].

3. DDoS-attack model on a computer network in the form of G-networks with signals with random delay

The model considered above, an attack on a computer network, can be described as an open G-network with signals that are activated after a random time.

In this case, under the positive messages or simply messages, it will be involved requests from different computers. Under negative messages we will understand the data or packets that the system does not expect (the virus program), which leads it to stop or restarts it. In other words, such requests destroy other messages in the system.

Selected nodes are attacked and the attacker gets their admin rights. For each of the captured nodes there are installed Trojans that run in the background. Under the signals in the network we will mean such programs, which are then activated by a random time in the attacker team. They are called zombie computers and their users are not even aware that they are potential participants in DDoS-attack. This means that at any point in time in these hosts there is at least one activated signal.

Next, the attacker sends certain commands to captured computers and those in turn carry out a powerful DDoS-attack on the target computer.

4. Finding state probabilities of the QS network

We will assume that all queuing network systems are one-line, and messages service duration in the QS has an exponential distribution with parameter μ_i^+. Consequently, in this case $\mu_i^+(k_i) = u(k_i)\mu_i^+$, $i = \overline{1,n}$.

Denote by $\Psi_{2n}(z,t)$, where $z = (z_1, z_2, ..., z_n, z_{n+1}, ..., z_{2n})$, generating function of the dimension of $2n$:

$$\Psi_{2n}(z,t) = \sum_{k_1=0}^{\infty} ... \sum_{k_n=0}^{\infty} \sum_{l_1=0}^{\infty} ... \sum_{l_n=0}^{\infty} P(z_1, z_2, ..., z_n, z_{n+1}, ..., z_{2n}) z_1^{k_1} ... z_n^{k_n} z_{n+1}^{l_1} ... z_{2n}^{l_n} =$$

$$= \sum_{k_1=0}^{\infty} ... \sum_{k_n=0}^{\infty} \sum_{l_1=0}^{\infty} ... \sum_{l_n=0}^{\infty} P(k,l,t) \prod_{i=1}^{n} z_i^{k_i} z_{n+i}^{l_i}, \ |z| < 1 \tag{2}$$

the summation is taking for each k_i, l_i from 0 to ∞, $i = \overline{1,n}$.

According to the described DDoS-attack model of our network, find that all the systems in the network operate in high load. We will also assume that $k_i(t) > 0$ $\forall t > 0$, $i = \overline{1,n}$. Furthermore, according to the same model, we will assume that $l_i(t) > 0$ $\forall t > 0$. Multiplying each of the equations (1) to $\prod_{m=1}^{n} z_m^{k_m} z_m^{l_m}$ and summing over all possible values k_m and l_m from 1 to $+\infty$, $m = \overline{1,n}$.

Converting the sums included in the right part of relations (2), we will obtain a homogeneous linear differential equation [4]:

$$\frac{d\Psi_{2n}(z,t)}{dt} = -\sum_{i=1}^{n} \Bigg[\lambda_{0i}^+ + \lambda_{0i}^+ + \mu_i^+ (1 - p_{ii}^+) + \mu_i^- - \lambda_{0i}^+ z_i - \lambda_{0i}^- z_{n+i} - \mu_i^- \frac{q_{i0}}{z_i z_{n+i}} - \mu_i^- \frac{q_{ii}^-}{z_i} -$$
$$- \sum_{j=1}^{n} \left(\mu_i^+ p_{ij}^+ \frac{z_j}{z_i} + \mu_i^+ p_{ij}^- \frac{z_{n+j}}{z_i} + \mu_i^- q_{ij}^+ \frac{z_j}{z_i z_{n+i}} + \mu_i^- q_{ij}^- \frac{z_{n+j}}{z_i z_{n+i}} \right) \Bigg] \Psi_{2n}(z,t).$$

Its solution has the form

$$\Psi_n(z,t) = C_n \exp\Bigg\{ -\sum_{i=1}^{n} \Bigg[\lambda_{0i}^+ + \lambda_{0i}^+ + \mu_i^+ (1 - p_{ii}^+) + \mu_i^- - \lambda_{0i}^+ z_i - \lambda_{0i}^- z_{n+i} - \mu_i^- \frac{q_{i0}}{z_i z_{n+i}} -$$
$$- \mu_i^- \frac{q_{ii}^-}{z_i} - \sum_{j=1}^{n} \left(\mu_i^+ p_{ij}^+ \frac{z_j}{z_i} + \mu_i^+ p_{ij}^- \frac{z_{n+j}}{z_i} + \mu_i^- q_{ij}^+ \frac{z_j}{z_i z_{n+i}} + \mu_i^- q_{ij}^- \frac{z_{n+j}}{z_i z_{n+i}} \right) \Bigg] t \Bigg\}.$$

We assume that at the initial time the network is able to state $(\alpha_1, \alpha_2, ..., \alpha_{2n}, 0)$, $\alpha_i > 0$, $\alpha_{n+i} > 0$, $P(\alpha_1, \alpha_2, ..., \alpha_{2n}, 0) = 1$, $P(k_1, k_2, ..., k_n, l_1, l_2, ..., l_n\ 0) = 0$, $\forall \alpha_i \neq k_i, l_i$,

$i=\overline{1,n}$. Then the initial condition for the last equation will be $\Psi_{2n}(z,0)=P(\alpha_1,\alpha_2,...,\alpha_{2n},0)\prod_{m=1}^{n} z_m^{\alpha_m} z_{n+m}^{\alpha_{n+m}}=\prod_{m=1}^{n} z_m^{\alpha_m} z_{n+m}^{\alpha_{n+m}}$. Using it, we obtain $C_n=1$.

Theorem. If at the initial moment of time the queueing network is in a state $(\alpha_1,\alpha_2,...,\alpha_{2n},0)$, $\alpha_i>0$, $\alpha_{n+i}>0$, $i=\overline{1,n}$, then the expression for the generating function $\Psi_{2n}(z,t)$, then taking into account the decomposition of exponent into Maclaurin series, has the form

$$\Psi_{2n}(z,t)=a_0(t)\sum_{\substack{b_j=0\\ j=\overline{1,n},j\neq i}}^{\infty}\sum_{\substack{c_j=0\\ j=\overline{1,n},j\neq i}}^{\infty}\sum_{\substack{d_j=0\\ j=\overline{1,n},j\neq i}}^{\infty}\sum_{\substack{g_j=0\\ j=\overline{1,n},j\neq i}}^{\infty}\times$$

$$\times\sum_{\substack{h_j=0\\ j=\overline{1,n},j\neq i}}^{\infty}\sum_{\substack{r_j=0\\ j=\overline{1,n},j\neq i}}^{\infty}\sum_{\substack{u_j=0\\ j=\overline{1,n},j\neq i}}^{\infty}\sum_{\substack{w_j=0\\ j=\overline{1,n},j\neq i}}^{\infty} t^{\sum_{i=1}^{n}(b_i+c_i+d_i+g_i+h_i+r_i+u_i+w_i)}\times$$

$$\times\prod_{i=1}^{n}\left[\frac{\left(\lambda_{0i}^{+}\right)^{b_i}\left(\lambda_{0i}^{-}\right)^{c_i}\left(\mu_i^{+}\right)^{h_i+r_i}\left(\mu_i^{-}\right)^{d_i+g_i+u_i+w_i} q_{i0}^{d_i+g_i}\left(q_{ii}^{-}\right)^{g_i}}{b_i!c_i!d_i!g_i!h_i!r_i!u_i!w_i!}\times\right. \quad (3)$$

$$\left.\times\left(\prod_{j=1}^{n}p_{ij}^{+}\right)^{h_i}\left(\prod_{j=1}^{n}p_{ij}^{-}\right)^{r_i}\left(\prod_{j=1}^{n}q_{ij}^{+}\right)^{u_i}\left(\prod_{j=1}^{n}q_{ij}^{-}\right)^{w_i} z_i^{\alpha_i+b_i-d_i-g_i+H-h_i-r_i+U-u_i-w_i} z_{n+i}^{\alpha_{n+i}+c_i-d_i+R-u_i+W-w_i}\right],$$

where $H=\sum_{i=1}^{n}h_i$, $R=\sum_{i=1}^{n}r_i$, $U=\sum_{i=1}^{n}u_i$, $W=\sum_{i=1}^{n}w_i$,

$$a_0(t)=\exp\left\{-\sum_{i=1}^{n}\left[\lambda_{0i}^{+}+\lambda_{0i}^{+}+\mu_i^{+}\left(1-p_{ii}^{+}\right)+\mu_i^{-}\right]t\right\}.$$

Proof. This can be proved in a similar way as for networks with positive and negative messages at transition behaviour [5]. We have:

$$\Psi_n(z,t)=a_0(t)\prod_{i=1}^{n}a_i(z,t)\prod_{m=1}^{n}z_m^{\alpha_m}z_{n+m}^{\alpha_{n+m}},$$

where

$$a_1(z,t)=\exp\left\{t\sum_{i=1}^{n}\lambda_{0i}^{+}z_i\right\}=\prod_{i=1}^{n}\exp\left\{\lambda_{0i}^{+}tz_i\right\}=\prod_{i=1}^{n}\sum_{b_i=0}^{\infty}\frac{\left[\lambda_{0i}^{+}tz_i\right]^{b_i}}{b_i!}=\sum_{b_1=0}^{\infty}\dots\sum_{b_n=0}^{\infty}\prod_{i=1}^{n}\frac{\left[\lambda_{0i}^{+}tz_i\right]^{b_i}}{b_i!}=$$

$$=\sum_{b_1=0}^{\infty}\dots\sum_{b_n=0}^{\infty}\frac{t^{b_1+b_2+\ldots+b_n}}{b_1!b_2!\cdot\ldots\cdot b_n!}\left(\lambda_{01}^{+}\right)^{b_1}\cdot\ldots\cdot\left(\lambda_{0n}^{+}\right)^{b_n}z_1^{b_1}\cdot\ldots\cdot z_n^{b_n},$$

$$a_2(z,t)=\exp\left\{t\sum_{i=1}^{n}\lambda_{0i}^{-}z_{n+i}\right\}=\sum_{c_1=0}^{\infty}\ldots\sum_{c_n=0}^{\infty}\frac{t^{c_1+c_2+\ldots+c_n}}{c_1!c_2!\cdot\ldots\cdot c_n!}\left(\lambda_{01}^{-}\right)^{c_1}\cdot\ldots\cdot\left(\lambda_{0n}^{-}\right)^{c_n}z_{n+1}^{c_1}\cdot\ldots\cdot z_{2n}^{c_n},$$

$$a_3(z,t)=\exp\left\{t\sum_{i=1}^{n}\mu_i^{-}\frac{q_{i0}}{z_iz_{n+i}}\right\}=\prod_{i=1}^{n}\exp\left\{\mu_i^{-}t\frac{q_{i0}}{z_iz_{n+i}}\right\}=\sum_{d_1=0}^{\infty}\ldots\sum_{d_n=0}^{\infty}\prod_{i=1}^{n}\frac{\left[\mu_i^{-}q_{i0}tz_i^{-1}z_{n+i}^{-1}\right]^{d_i}}{d_i!}=$$

$$=\sum_{d_1=0}^{\infty}\ldots\sum_{d_n=0}^{\infty}\frac{t^{d_1+d_2+\ldots+d_n}}{d_1!d_2!\cdot\ldots\cdot d_n!}\left(\mu_1^{-}q_{10}\right)^{d_1}\cdot\ldots\cdot\left(\mu_n^{-}q_{n0}\right)^{d_n}z_1^{-d_1}\cdot\ldots\cdot z_n^{-d_n}z_{n+1}^{-d_1}\cdot\ldots\cdot z_{2n}^{-d_n},$$

$$a_4(z,t)=\exp\left\{t\sum_{i=1}^{n}\mu_i^{-}\frac{q_{ii}^{-}}{z_i}\right\}=\prod_{i=1}^{n}\exp\left\{\mu_i^{-}t\frac{q_{ii}^{-}}{z_i}\right\}=\sum_{g_1=0}^{\infty}\ldots\sum_{g_n=0}^{\infty}\prod_{i=1}^{n}\frac{\left[\mu_i^{-}q_{ii}^{-}tz_i^{-1}\right]^{g_i}}{g_i!}=$$

$$=\sum_{g_1=0}^{\infty}\ldots\sum_{g_n=0}^{\infty}\frac{t^{g_1+g_2+\ldots+g_n}}{g_1!g_2!\cdot\ldots\cdot g_n!}\left(\mu_1^{-}q_{11}\right)^{g_1}\cdot\ldots\cdot\left(\mu_n^{-}q_{nn}\right)^{g_n}z_1^{-g_1}\cdot\ldots\cdot z_n^{-g_n},$$

$$a_5(z,t)=\exp\left\{\sum_{i,j=1}^{n}t\mu_i^{+}p_{ij}^{+}\frac{z_j}{z_i}\right\}=\prod_{i=1}^{n}\prod_{j=1}^{n}\exp\left\{t\mu_i^{+}p_{ij}^{+}\frac{z_j}{z_i}\right\}=\prod_{i=1}^{n}\prod_{j=1}^{n}\sum_{h_i=0}^{\infty}\frac{\left[t\mu_i^{+}p_{ij}^{+}z_jz_i^{-1}\right]^{h_i}}{h_i!}=$$

$$=\sum_{h_1=0}^{\infty}\ldots\sum_{h_n=0}^{\infty}\prod_{i=1}^{n}\prod_{j=1}^{n}\frac{\left[t\mu_i^{+}p_{ij}^{+}z_jz_i^{-1}\right]^{h_i}}{h_i!}=$$

$$=\sum_{h_1=0}^{\infty}\ldots\sum_{h_n=0}^{\infty}t^{h_1}\cdot\ldots\cdot t^{h_n}\frac{\left(\prod_{j=1}^{n}\mu_1^{+}p_{1j}^{+}\right)^{h_1}\cdot\ldots\cdot\left(\prod_{j=1}^{n}\mu_n^{+}p_{nj}^{+}\right)^{h_n}}{h_1!\cdot\ldots\cdot h_n!}z_1^{H-h_1}\cdot\ldots\cdot z_n^{H-h_n},$$

$$a_6(z,t)=\exp\left\{\sum_{i,j=1}^{n}t\mu_i^{+}p_{ij}^{-}\frac{z_{n+j}}{z_i}\right\}=\prod_{i=1}^{n}\prod_{j=1}^{n}\exp\left\{t\mu_i^{+}p_{ij}^{-}\frac{z_{n+j}}{z_i}\right\}=\prod_{i=1}^{n}\prod_{j=1}^{n}\sum_{r_i=0}^{\infty}\frac{\left[t\mu_i^{+}p_{ij}^{-}z_{n+j}z_i^{-1}\right]^{r_i}}{r_i!}=$$

$$=\sum_{r_1=0}^{\infty}\ldots\sum_{r_n=0}^{\infty}\prod_{i=1}^{n}\prod_{j=1}^{n}\frac{\left[t\mu_i^{+}p_{ij}^{-}z_{n+j}z_i^{-1}\right]^{r_i}}{r_i!}=\sum_{r_1=0}^{\infty}\ldots\sum_{r_n=0}^{\infty}t^{r_1}\cdot\ldots\cdot t^{r_n}\frac{\left(\prod_{j=1}^{n}\mu_1^{+}p_{1j}^{-}\right)^{r_1}\cdot\ldots\cdot\left(\prod_{j=1}^{n}\mu_n^{+}p_{nj}^{-}\right)^{r_n}}{r_1!\cdot\ldots\cdot r_n!}\times$$

$$\times z_1^{-r_1}z_2^{-r_2}\ldots z_n^{-r_n}z_{n+1}^{r_1+r_2+\ldots+r_n}z_{n+2}^{r_1+r_2+\ldots+r_n}\cdot\ldots\cdot z_{2n}^{r_1+r_2+\ldots+r_n},$$

$$a_7(z,t)=\exp\left\{\sum_{i,j=1}^{n}t\mu_i^{-}q_{ij}^{+}\frac{z_j}{z_iz_{n+i}}\right\}=\prod_{i=1}^{n}\prod_{j=1}^{n}\exp\left\{t\mu_i^{-}q_{ij}^{+}\frac{z_j}{z_iz_{n+i}}\right\}=$$

$$=\sum_{u_1=0}^{\infty}\ldots\sum_{u_n=0}^{\infty}\prod_{i=1}^{n}\prod_{j=1}^{n}\frac{\left[t\mu_i^{-}q_{ij}^{+}z_jz_i^{-1}z_{n+i}^{-1}\right]^{u_i}}{u_i!}=$$

$$=\sum_{u_1=0}^{\infty}\ldots\sum_{u_n=0}^{\infty}t^{u_1}\cdot\ldots\cdot t^{u_n}\frac{\left(\prod_{j=1}^{n}\mu_1^{-}q_{1j}^{+}\right)^{u_1}\cdot\ldots\cdot\left(\prod_{j=1}^{n}\mu_n^{-}q_{nj}^{+}\right)^{u_n}}{u_1!\cdot\ldots\cdot u_n!}z_1^{U-u_1}z_2^{U-u_2}\ldots z_n^{U-u_n}z_{n+1}^{-u_1}z_{n+2}^{-u_2}\cdot\ldots\cdot z_{2n}^{-u_n},$$

$$a_8(z,t)=\exp\left\{\sum_{i,j=1}^{n} t\mu_i^- q_{ij}^- \frac{z_{n+j}}{z_i z_{n+i}}\right\}=\prod_{i=1}^{n}\prod_{j=1}^{n}\exp\left\{t\mu_i^- q_{ij}^- \frac{z_{n+j}}{z_i z_{n+i}}\right\}=$$

$$=\sum_{w_1=0}^{\infty}\cdots\sum_{w_n=0}^{\infty}\prod_{i=1}^{n}\prod_{j=1}^{n}\frac{\left[t\mu_i^- q_{ij}^- z_{n+j} z_i^{-1} z_{n+i}^{-1}\right]^{w_i}}{w_i!}=$$

$$=\sum_{w_1=0}^{\infty}\cdots\sum_{w_n=0}^{\infty} t^{w_1}\cdot\ldots\cdot t^{w_n}\,\frac{\left(\prod_{j=1}^{n}\mu_1^- q_{1j}^-\right)^{w_1}\cdot\ldots\cdot\left(\prod_{j=1}^{n}\mu_n^- q_{nj}^-\right)^{w_n}}{w_1!\cdot\ldots\cdot w_n!}\, z_1^{-w_1} z_2^{-w_2}\ldots z_n^{-w_n} z_{n+1}^{W-w_1} z_{n+2}^{W-w_2}\cdot\ldots\cdot z_{2n}^{W-w_n}.$$

Multiplying $a_0(t)$, $a_i(z,t)$, and $\prod_{m=1}^{n} z_m^{\alpha_m} z_{n+m}^{\alpha_{n+m}}$ we will obtain an expression (3), $i=\overline{1,8}$.

State probability of $P(k_1,k_2,..,k_n,l_1,l_2,...,l_n,t)$ is the coefficient of $z_1^{k_1} z_2^{k_2},...,z_n^{k_n} z_{n+1}^{l_1}, z_{n+2}^{l_2},...,z_{2n}^{l_n}$ in the expansion of $\Psi_{2n}(z,t)$ in multiple series (3), with the proviso that at the initial time the network is in a state $(\alpha_1,\alpha_2,...,\alpha_{2n},0)$.

5. Finding the average characteristics

As it is known, with the help of the generating function, a different average of network characteristics can be found in the transient behaviour. For example, for the average number of positive messages in the network system S_x will be used the relation:

$$N_x(t)=\left.\frac{\partial\Psi_{2n}(z,t)}{\partial z_x}\right|_{z=(1,1,\ldots,1)}=a_0(t)\sum_{\substack{b_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty}\sum_{\substack{c_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty}\sum_{\substack{d_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty}\sum_{\substack{g_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty}\times$$

$$\times\sum_{\substack{h_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty}\sum_{\substack{r_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty}\sum_{\substack{u_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty}\sum_{\substack{w_j=0\\ j=\overline{1,n},\,j\neq i}}^{\infty} t^{\sum_{i=1}^{n}(b_i+c_i+d_i+g_i+h_i+r_i+u_i+w_i)}\times$$

$$\times\left(\alpha_x+b_x-d_x-g_x+H-h_x-r_x+U-u_x-w_x\right)\times$$

$$\times\prod_{i=1}^{n}\left[\frac{\left(\lambda_{0i}^+\right)^{b_i}\left(\lambda_{0i}^-\right)^{c_i}\left(\mu_i^+\right)^{h_i+r_i}\left(\mu_i^-\right)^{d_i+g_i+u_i+w_i} q_{i0}^{d_i+g_i}\left(q_{ii}^-\right)^{g_i}}{b_i!c_i!d_i!g_i!h_i!r_i!u_i!w_i!}\times\right.$$

$$\left.\times\left(\prod_{j=1}^{n}p_{ij}^+\right)^{h_i}\left(\prod_{j=1}^{n}p_{ij}^-\right)^{r_i}\left(\prod_{j=1}^{n}q_{ij}^+\right)^{u_i}\left(\prod_{j=1}^{n}q_{ij}^-\right)^{w_i}\right],\ x=\overline{1,n}. \quad (4)$$

We will do in the expression (4) the change of variables

$$k_x = \alpha_x + b_x - d_x - g_x + H - h_x - r_x + U - u_x - w_x,$$

then $b_x = k_x - \alpha_x + d_x + g_x - H + h_x + r_x - U + u_x + w_x$ and considering that all network QS function under high load conditions, we obtain

$$N_x(t) = a_0(t) \sum_{\substack{c_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{d_j=0 \\ j=\overline{1,n}, j\neq i}}^{\alpha_j - g_j - h_j - r_j - u_j - w_j + H + U - 1} \sum_{\substack{g_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{h_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{r_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{u_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{w_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{k_j=1 \\ j=\overline{1,n}}}^{\infty} k_x \times$$

$$\times\, t^{\sum_{i=1}^{n}(k_i - \alpha_i + 2d_i + c_i + 2g_i + 2h_i + 2r_i + 2u_i + 2w_i - H - U)} \times$$

$$\times \prod_{i=1}^{n} \left[\frac{\left(\lambda_{0i}^{+}\right)^{k_i - \alpha_i + d_i + g_i - H + h_i + r_i - U + u_i + w_i} \left(\lambda_{0i}^{-}\right)^{c_i} \left(\mu_i^{+}\right)^{h_i + r_i} \left(\mu_i^{-}\right)^{d_i + g_i + u_i + w_i} q_{i0}^{d_i + g_i} \left(q_{ii}^{-}\right)^{g_i}}{(k_i - \alpha_i + d_i + g_i - H + h_i + r_i - U + u_i + w_i)!\, c_i!\, d_i!\, g_i!\, h_i!\, r_i!\, u_i!\, w_i!} \times \right.$$

$$\left. \times \left(\prod_{j=1}^{n} p_{ij}^{+}\right)^{h_i} \left(\prod_{j=1}^{n} p_{ij}^{-}\right)^{r_i} \left(\prod_{j=1}^{n} q_{ij}^{+}\right)^{u_i} \left(\prod_{j=1}^{n} q_{ij}^{-}\right)^{w_i} \right], \quad x = \overline{1,n}. \tag{5}$$

Similarly, we can find the relation for the average number of nonactivated signals in the system S_x:

$$N_x^{sign}(t) = a_0(t) \sum_{\substack{b_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{d_j=0 \\ j=\overline{1,n}, j\neq i}}^{\alpha_{n+j} - u_j - w_j + R + W - 1} \sum_{\substack{g_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{h_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{r_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{u_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{w_j=0 \\ j=\overline{1,n}, j\neq i}}^{\infty} \sum_{\substack{l_j=1 \\ j=\overline{1,n}}}^{\infty} l_x \times$$

$$\times\, t^{\sum_{i=1}^{n}(l_i - \alpha_{n+i} + b_i + 2d_i + g_i + h_i + r_i + 2u_i + 2w_i - R - W)} \times$$

$$\times \prod_{i=1}^{n} \frac{\left(\lambda_{0i}^{+}\right)^{b_i} \left(\lambda_{0i}^{-}\right)^{l_i - \alpha_{n+i} + d_i - R + u_i - W + w_i} \left(\mu_i^{+}\right)^{h_i + r_i} \left(\mu_i^{-}\right)^{d_i + g_i + u_i + w_i} q_{i0}^{d_i + g_i} \left(q_{ii}^{-}\right)^{g_i}}{b_i!\,(l_i - \alpha_{n+i} + d_i - R + u_i - W + w_i)!\, d_i!\, g_i!\, h_i!\, r_i!\, u_i!\, w_i!} \times$$

$$\times \left(\prod_{j=1}^{n} p_{ij}^{+}\right)^{h_i} \left(\prod_{j=1}^{n} p_{ij}^{-}\right)^{r_i} \left(\prod_{j=1}^{n} q_{ij}^{+}\right)^{u_i} \left(\prod_{j=1}^{n} q_{ij}^{-}\right)^{w_i}. \tag{6}$$

Example. Let the number of QS in QN equal $n = 3$. The intensity of the input stream of positive messages and signals λ_{0i}^{+} and λ_{0i}^{-} are respectively equal to: $\lambda_{01}^{+} = 1.8$, $\lambda_{02}^{+} = 1$, $\lambda_{03}^{+} = 2.14$, $\lambda_{01}^{-} = 2$, $\lambda_{02}^{-} = 3$, $\lambda_{03}^{-} = 3.11$. The intensity of service of positive messages and signals in the QS S_i set equals respectively: $\mu_1^{+} = 2$, $\mu_2^{+} = 4$, $\mu_3^{+} = 3$, $\mu_1^{-} = 1$, $\mu_2^{-} = 0.5$, $\mu_3^{-} = 3$. We assume that the transition probability of positive messages p_{ij}^{+} has the form: $p_{12}^{+} = 1/5$, $p_{13}^{+} = 1/5$, $p_{21}^{+} = 1/6$, $p_{23}^{+} = 1/6$,

$p_{31}^{+} = 1/7$, $p_{32}^{+} = 1/7$; signals transition probability p_{ij}^{-} are equal to: $p_{12}^{-} = 1/4$, $p_{13}^{-} = 1/4$, $p_{21}^{-} = 1/8$, $p_{23}^{-} = 1/8$, $p_{31}^{-} = 1/9$, $p_{32}^{-} = 1/9$; then the probabilities of leaving the network messages to the external environment p_{i0} will be equal respectively to: $p_{10} = 1/10$, $p_{20} = 5/12$, $p_{30} = 31/63$. We also define the probabilities q_{ij}^{+} and q_{ij}^{-}, when the signal is triggered by a trigger. Let $q_{12}^{+} = 1/10$, $q_{13}^{+} = 1/10$, $q_{21}^{+} = 1/8$, $q_{23}^{+} = 1/8$, $q_{31}^{+} = 1/9$, $q_{32}^{+} = 1/9$; $q_{12}^{-} = 1/9$, $q_{13}^{-} = 1/9$, $q_{21}^{-} = 1/11$, $q_{23}^{-} = 1/11$, $q_{31}^{-} = 1/12$, $q_{32}^{-} = 1/12$. The signal is triggered by a negative message that destroys one positive message in the QS S_i, and leaves the network with probabilities $q_{10} = 26/45$, $q_{20} = 25/44$, $q_{30} = 11/18$. In this case $a_0(t) = e^{-\frac{289t}{20}}$.

Find, for example, the probability of state $P(7,7,7,7,7,7,t)$. It is the coefficient of $z_1^7 z_2^7 z_3^7 z_4^7 z_5^7 z_6^7$ in the expansion of $\Psi_{2n}(z,t)$ in multiple series (3), so that when power at z_i and z_{n+i} must satisfy the relations

$$\alpha_i + b_i - d_i - g_i + H - h_i - r_i + U - u_i - w_i = 7,$$
$$\alpha_{n+i} + c_i - d_i + R - u_i + W - w_i = 7, \ i = \overline{1,3}.$$

Then, using (3), we obtain

$$P(7,7,7,7,7,7,t) =$$
$$= e^{-\frac{289t}{20}} \sum_{\substack{b_j=0 \\ j=\overline{1,3}, j\neq i}}^{\infty} \sum_{\substack{c_j=0 \\ j=\overline{1,3}, j\neq i}}^{\infty} \sum_{\substack{g_j=0 \\ j=\overline{1,3}, j\neq i}}^{\infty} \sum_{\substack{h_j=0 \\ j=\overline{1,3}, j\neq i}}^{\infty} \sum_{\substack{r_j=0 \\ j=\overline{1,3}, j\neq i}}^{\infty} \sum_{\substack{u_j=0 \\ j=\overline{1,3}, j\neq i}}^{\infty} \sum_{\substack{w_j=0 \\ j=\overline{1,3}, j\neq i}}^{\infty} t^{\sum_{i=1}^{3}(2\alpha_i - \alpha_{n+i} + 3b_i - g_i - h_i - r_i - 7 + 2H - R + 2U - W)} \times$$
$$\times \prod_{i=1}^{n} \left[\frac{(\lambda_{0i}^{+})^{b_i} (\lambda_{0i}^{-})^{c_i} (\mu_i^{+})^{h_i + r_i} (\mu_i^{-})^{\alpha_i + b_i + H - h_i - r_i + U - 7} q_{i0}^{\alpha_i + b_i - h_i - r_i - u_i - w_i + H + U - 7} (q_{ii}^{-})^{g_i}}{b_i! c_i! (\alpha_i + b_i - g_i + H - h_i - r_i + U - u_i - w_i - 7)! g_i! h_i! r_i! u_i! w_i!} \times \right.$$
$$\left. \times \left(\prod_{j=1}^{n} p_{ij}^{+} \right)^{h_i} \left(\prod_{j=1}^{n} p_{ij}^{-} \right)^{r_i} \left(\prod_{j=1}^{n} q_{ij}^{+} \right)^{u_i} \left(\prod_{j=1}^{n} q_{ij}^{-} \right)^{w_i} \right].$$

Figure 1 shows plots of the probability of the state $P(7,7,7,7,7,7,t)$ and on condition that at the initial time network is at one of three states: a) $\alpha_i = 3$, $i = \overline{1,6}$, b) $\alpha_i = 1$, $i = \overline{1,6}$, c) $\alpha_i = 0$, $i = \overline{1,6}$.

The average number of messages in network systems (in the queue and in servicing), on condition that $N_m(0) = 0$, $m = \overline{1,n}$, can be found by the formula (5), and the average number of signals in the system network (not in the nonactivated state) may be found by the formula (6).

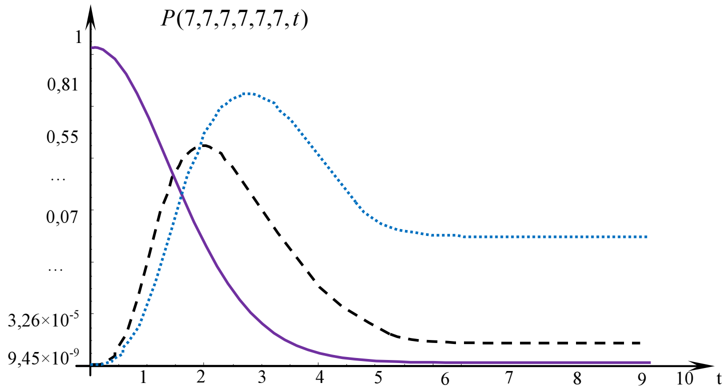

Fig. 1. The chart of the probability of the state $P(7,7,7,7,7,7,t)$
(The continuous line - case a), the dashed - case b), the dotted - c))

Figure 2 shows a graph of change in the average number of messages in the QS S_3 and a graph of change in the average number of signals in the QS S_1 respectively.

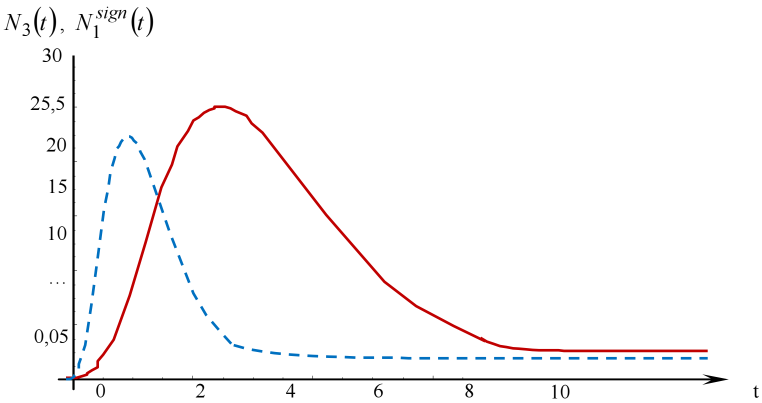

Fig. 2. The average number of messages $N_3(t)$ in QS S_3 and the average number of signals $N_1^{sign}(t)$ in QS S_1 (the dashed line)

Conclusion

We propose a method of finding nonstationary state probabilities of the G-network with a one-line QS and also with signals with random delay, based on the use of multivariate generating functions. The proposed approach can serve as a basis for the development of the approximate method of research specified network. Approximate expressions were obtained for time-dependent probabilities of states and the average characteristics of the network, provided that the network is operating under a high load. As described, the application of the results obtained on the model computer system attacks and virus penetration effect into a computer network. Further studies in this direction are connected to yield similar results for networks with incomes.

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