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THE PROPERTIES OF METHOD BALANCING THE UNSUSTAINABLE PRODUCTION AND CONSUMPTION MODEL

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Abstract. The article presents some properties of method used for balancing the unsustainable production and consumption model, defined in our previous article. This model assumes that the total demand exceeds supply at a fixed period of time.

When we analyzed the consumption model we didn't find the case in which the supply exceeds the demand. Unsustainable production and consumption model has been defined in the previous articles [1, 2].

This model contains $n \geq 2$ contractors, in which there are $m \geq 1$ there are producers and $n - m \geq 1$ consumers. Demand and supply for a specified product at a fixed period of time are presented in a form of vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$, meeting the following requirements:

a) $i \leq m \Rightarrow p_i \rangle 0$ for each $i \leq n$,

b) $m + 1 \leq i \leq n \Rightarrow p_i \langle 0$ for each $i \leq n$,

c) $p_1 + p_2 + \ldots + p_n \langle 0.$

In the article [3] an iterative algorithm balancing this model has been presented (model (1-5)).

In this article, certain properties of this algorithm will be presented.

Statement 1. For each $k \in N$ $(s,\ 0)$ there are equalities

a) $\left(p^k - p^{k-1}, e_n\right) = S^k,$

b) $\left(p^k - p, e_n\right) = \overline{S}^k,$

where
$\left(\left(x, e_n\right). = x_1 + x_2 + \ldots + x_n\right),$
$p^o = p,$

$N(s) = \{s \in N; r^s \neq 0\}$,

$s_o = \min\limits_s (r^s = 0)$,

$N(s,o) = N(s) \cup \{s_o\}$,

$S^k \left(\overline{S}^k \right)$ local (global) correction of k row for the unadjusted $\mathbf{p}$ vector. The proof of this statement was given in the previous article [2].

Statement 2. For each $k \in N(s,o)$ there is equality $\overline{S}^k + \left(r^{k+1}a^k, e_n\right) = r^1$

Proof. From the statement 1 and algorithm definition we get

$$\overline{S}^k + \left(r^{k+1}a^k, e_n\right) = \left(p^k - p, e_n\right) + \left(r^{k+1}a^k, e_n\right) = \left(p^k, e_n\right) - \left(p, e_n\right) + r^{k+1}\left(a^k, e_n\right) =$$
$$= -r^{k+1} + r^1 + r^{k+1} = r^1. \quad \text{Q.E.D.}$$

The above statement informs us, that the total number of concession by all contractors, measured from the first iteration, increased by the proposed change in demand or supply in the next iterative step, is constant. This size is equal to the difference left to balance the model at the starting position.

The algorithm could, therefore, lead to balance the model in the first iteration, if the contractors had adequate possibilities to make concessions.

Statement 3. For each $k \in N(s,o)$ there is inequality

$$p_i^k \leq p_i + u_i \quad \text{for } i = 1,2,\ldots,n$$

Proof (induction due to k)

a) $k = 1$. From the algorithm's definition we get

$$p_i^1 = \begin{cases} p_i^o + r^1 a_i^o & \text{for } i \in N^1 \\ p_i^o + u_i & \text{for } i \notin N^1. \end{cases}$$

For $i \notin N^1$ the thesis is clear. For $i \in N^1$ defined from set N^s results $r^1 a_i^o \leq u_i$ that $p_i^1 = p_i^o + r^1 a_i^o \leq p_i + u_i$

b) Assume that the statement is true for $k = m$ that $p_i^m \leq p_i + u_i$ for $i = 1,2,\ldots,n$.

c) Let $k = m+1$

$$p_i^{m+1} = \begin{cases} p_i^m + r^m a_i^{m-1} & \text{for } i \in N^{m+1} \\ p_i + u_i & \text{for } i \in \overline{N}^{m+1} \\ p_i^m & \text{for } i \in \bigcup_{j \le m} \overline{N}^j. \end{cases}$$

For $i \in \bigcup_{j \le m} \overline{N}^j$ the statement is true from the induction assumption because $\bigcup_{j \le m} \overline{N}^j \subset N^o$ (see [3] Lemma 1).

For $i \in \overline{N}^{m+1}$ the thesis is clear.

Case when $i \in N^{m+1}$ has been left to prove. From the algorithm's definition the $N^{m+1} \subset N^m \subset \ldots \subset N^1$ inclusion is shown together with the following equalities:

$$p_i^1 = p_i + r^1 a_i^o,$$

$$p_i^2 = p_i^1 + r^2 a_i^1,$$

$$p_i^m = p_i^{m-1} + r^m a_i^{m-1},$$

hence

$$p_i^m = p_i + \sum_{j=1}^{m} r^j a_i^{j-1}.$$

From the set's N^s definition it results that for each $i \in N^{m+1}$ there is inequality $\sum_{j \le m} r^j a_i^{j-1} \le u_i$.

Thus, from the above equality we get $p_i^m \le p_i + u_i$ Q.E.D.

This statement informs us that at every iteration step, each contractor did not exceed the limit of his production and consumption capacity.

Statement 4. For each $k \in N(s,o)$ there is $r^k \ge 0$.

Proof: From the algorithm's definition (see [3]) results that $r^k = -\left(p^{k-1}, e_n\right)$.

For $i \in \overline{N}^j, \left(j\langle k-1\right), a_i^{k-2} = 0.$ Then $p_i^{k-1} = p_i^{k-2} = p_i^{k-2} + r^{k-1} \cdot a_i^{k-2}$.

For $i \in N^{k-1}, p_i^{k-1} = p_i^{k-2} + r^{k-1} a_i^{k-2}$.

Therefore, for $i = 1,2,\dots,n$ there is inequality $p_i^{k-1} \le p_i^{k-2} + r^{k-1} a_i^{k-2}$. Summing we get

$$\left(p^{k-1}, e_n\right) \le \left(p_i^{k-2} + r^{k-1} a_i^{k-2}, e_n\right) = \left(p^{k-2}, e_n\right) + r^{k-1}\left(a_i^{k-2}, e_n\right) = -r^{k-1} + r^{k-1} = 0$$

Hence $r^k = -\left(p^{k-1}, e_n\right)$

The above statement shows that at every iteration step, supply of the certain good does not exceed demand.

From the a_i^k definition (see [3]) results that for $i = 1,2,\dots,n$ and for each $k \in N(s,o)$ there is $a_i^k \ge 0$. Hence, for the statement 4 we get

Statement 5. For each $k \in N(s,o)$, $r^k a_i^{k-1} \ge 0$ for $i = 1,2,\dots,n.$

It means that at every iteration step, each contractor, in fact, makes a concession, the producer does not reduce the production and the consumer does not reduce his demand.

Statement 6. For any $k \in N(s,o)$ there is inequality $\overline{S}^k \le r^1$.

Proof. From statement 2 results, that for any $k \in N(s,o)$ there is $\overline{S}^k + \left(r^k a^{k-1}, e_n\right) = r^1$ while from statement 5 results that $\left(r^k a^{k-1}, e_n\right) \ge 0$. Therefore $\overline{S}^k = r^1 - \left(r^k a^{k-1}, e_n\right) \le r^1$. Q.E.D.

The above statement means that at every iteration step the total amount of concessions made by all contractors, from the 1st to k-th iteration does not exceed the amount needed to balance the model.

Statement 7. For each $k \in N(s)$ there is $0 \langle r^{k+1} \langle r^k$.

Proof. From the statement 5 results that for each $i \le n$ and for each $k \in N(s)$ there is $r^k a_i^{k-1} \ge 0$. Because for $k \in N(s)$ $r^k \ne 0$ therefore, for any $i \in N^k$ $r^k a_i^{k-1} \ne 0$ that for any $k \in N(s)$ and for any $i \in N^k$ $r^k a_i^{k-1} \rangle 0$. Then, for any $k \in N(s)$ and $i \in N^k$ there is $p_i^k = p_i^{k-1} + r^k a_i^{k-1} \rangle p_i^{k-1}$. It follows that, for any $k \in N(s)$ and any $i \in N^k$ there is $p_i^k = p^{k-1} + r^k a_i^{k-1} \rangle p_i^{k-1}$. Because for any $i \in N^o$ and any $k \in N(s)$, $p_i^k \le p_i + u_i$ (statement 3) then, for any $i \in \overline{N}^k$ there is inequality $p_i^k \le p_i^{k-1}$.

In a sum

$$p_i^k = \begin{cases} p_i^{k-1} + r^k a_i^{k-1} \rangle\, p_i^{k-1} & \text{for } i \in N^k \\ p_i + u_i \geq p_i^{k-1} & \text{for } i \in \overline{N}^k \\ p_i^{k-1} & \text{for } i \in \bigcup\limits_{j \langle k} \overline{N}^j. \end{cases}$$

Summing up we get

$$-r^{k+1} = \sum_{i \in N^o} p_i^k \rangle \sum_{i \in N^o} p_i^{k-1} = -r^k$$

then

$$r^{k+1} \langle r^k.$$

Because $r^k \geq 0$ (statement 4) and $k \in N(s)$ then $r^k \neq 0$. Therefore $r^k \rangle 0$. Q.E.D.

The above statement shows that at every iteration step the mount left to balance the model is smaller than the previous one.

References

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