Please cite this article as:
Bohdan Mochnacki, Jarosław Siedlecki, Sensitivity of tissue freezing process on changes of time-dependent cryoprobe tip cooling rate, Scientific Research of the Institute of Mathematics and Computer Science, 2011, Volume 10, Issue 2, pages 191-197.
The website: http://www.amcm.pcz.pl/
Scientific Research of the Institute of Mathematics and Computer Science, 2(10) 2011, 191-197

# SENSITIVITY OF TISSUE FREEZING PROCESS ON CHANGES OF TIME-DEPENDENT CRYOPROBE TIP COOLING RATE

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**Abstract.** In the paper, the problems of thermal interactions between skin tissue and a cylindrical cryoprobe tip are discussed. The cryoprobe tip temperature is time-dependent and its changes from the initial temperature to the final one and back are assumed in the form of a broken line (successive sectors correspond to cooling and heating processes or cooling and heating ones divided by a horizontal sector (a constant freezing temperature)). The aim of the considerations resolves itself in the analysis of dependences between the tip cooling (heating) rate and the course of the freezing process. To solve the problem, the direct approach of sensitivity analysis is used. The mathematical model of freezing is based on the energy equation corresponding to the 'fixed domain method' in which a parameter called 'the substitute thermal capacity' appears. The sensitivity model results from the differentiation of energy equation and boundary-initial conditions with respect to the cooling (heating) rate. At the stage of computations, the finite differences method (FDM) is used. In the final part of paper, examples of computations are presented.

## Introduction

Sensitivity analysis can be used to observe the dependence between the cryoprobe tip cooling rate and the course of biological tissue freezing. The process discussed is described by the transient diffusion equation (Pennes equation) with an additional term (source function) controlling the evolution of freezing latent heat. Mathematical manipulations allow one to transform the basic equation to a form containing a parameter called 'the substitute thermal capacity' [1-3]. This approach leads to a model called 'the fixed domain approach' [1, 2]. The energy equation is supplemented by boundary conditions, in particular on the contact surface between the cryoprobe tip and skin tissue the Dirichlet one is given and the time-dependent boundary temperature results from the control mode of the cryoprobe tip. On the remaining surfaces limiting the system, no-flux conditions can be taken into account. The initial tissue temperature (the initial condition) is also known, namely $t = 0:\ T(x, 0) = T_0(x)$.

The first version of numerical modeling concerns the case for which the changes of temperature from its initial value to final one $T_{min}$ and back are assumed in the form of a broken line, cooling (heating) rates $v$ and $-v$ correspond to the slopes of successive segments (Fig. 1). In the second version, the 'hold time' between these

two stages is introduced (Fig. 2). The aim of the considerations presented in this paper is the analysis of common interactions between the parameters determining the Dirichlet condition and the kinetics of the freezing process. The sensitivity model is constructed using the direct approach, this means by the differentiation of energy equation and boundary-initial conditions with respect to the parameter considered [4-6]. All the parameters appearing in the energy equation are assumed to be temperature-dependent. It should be pointed out that the basic model of the freezing process and the sensitivity one are coupled because the solution of the sensitivity model requires the knowledge of the time and spatial dependent temperature field resulting from the basic one.

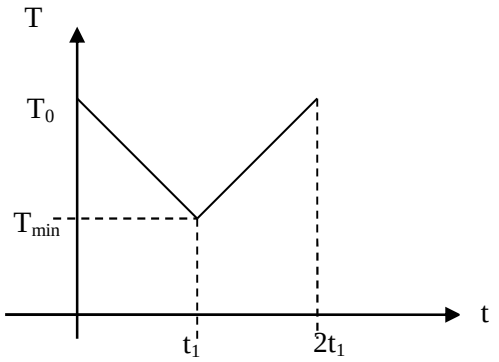


Fig. 1. First variant of cooling (heating) process

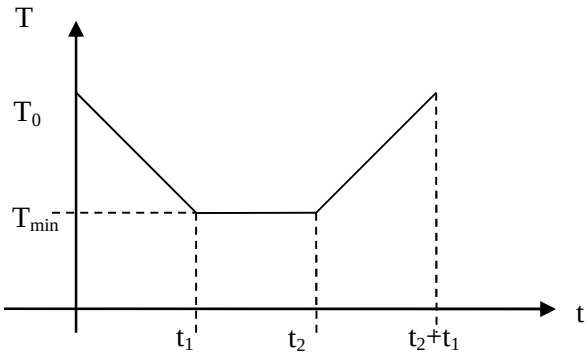


Fig. 2. Second variant of cooling (heating) process

At the stage of numerical modeling both in the case of basic and sensitivity models, the finite difference method (an explicit scheme) in the version presented in detail in [3, 7] is used. The solution to the sensitivity problem gives information concerning the changes of the local and temporary temperature field connected with the perturbations of the parameter considered.

## 1. Mathematical description of freezing process

The equation describing biological tissue freezing can be written as follows [1, 2, 6]

$$x \in \Omega : c(T)\frac{\partial T(x,t)}{\partial t} = \mathrm{div}\left[\lambda(T)\mathrm{grad}\,T(x,t)\right] + Q_P(T) + Q_M(T) + L_V \frac{\partial f_S(x,t)}{\partial t} \quad (1)$$

where $c$ is the tissue volumetric specific heat, $\lambda$ is the tissue thermal conductivity, $L_V$ is the volumetric latent heat, $f_S$ is the frozen state fraction at the point considered, $Q_p$ is the perfusion heat source, $Q_m$ is the metabolic heat source, $T$, $x$, $t$ denote the temperature, spatial co-ordinates and time.

The perfusion heat source is given by the formula

$$Q_p(T) = k(T)\left[T_b - T(x,t)\right] \quad (2)$$

where $k(T) = G_b(T)\, c_b$, $G_b$ is the tissue perfusion [m$^3$ blood/s/m$^3$ tissue], $c_b$ is the specific heat of blood per unit of volume, $T_b$ is the arterial blood temperature.

The boundary condition given on the contact surface between the tissue and cryoprobe tip is of the form

$$x \in \Gamma_c : T(x,t) = T_0 - vt,\ t < t_1, \quad T(x,t) = T_{\min} + v(t - t_m),\ t_1 \le t \le 2t_1 \quad (3)$$

or

$$x \in \Gamma_c : \quad T(x,t) = T_0 - vt \qquad t < t_1$$
$$T(x,t) = T_{\min} \qquad t_1 \le t \le t_2 \quad (4)$$
$$T(x,t) = T_{\min} + v(t - t_2) \quad t_2 \le t \le t_2 + t_1$$

where $T_0$ is the initial temperature of tissue, $v$ is the cooling (heating) rate, $t_0$ is the cooling time, $T_{\min}$ denotes the final temperature of the cryoprobe tip, $t_2 - t_1$ corresponds to the hold time. For the other parts of the external boundary (conventionally assumed cylindrical domain), no-flux conditions are taken into account, this means

$$x \in \Gamma_0 : \quad \frac{\partial T(x,t)}{\partial n} = 0 \quad (5)$$

where $n$ denotes a normal direction.

For $t = 0$, the initial temperature field is known: $t = 0 : T(x,0) = T_0$.

The freezing (melting) rate in equation (1) can be transformed in the following way:

$$\frac{\partial f_S(x,t)}{\partial t} = \frac{\mathrm{d}f_S(T)}{\mathrm{d}T}\frac{\partial T(x,t)}{\partial t} \quad (6)$$

and then equation (1) takes the form

$$x \in \Omega: \quad c(T)\frac{\partial T(x,t)}{\partial t} = \mathrm{div}\left[\lambda(T)\mathrm{grad}T(x,t)\right] + Q_P(T) + Q_M(T) \tag{7}$$

where $C(T)$ is the substitute thermal capacity. The introduction of this parameter leads to a model called 'the one domain approach' (e.g. [2]). Let $T_1$ and $T_2$ denote the temperatures corresponding to the beginning and the end of tissue freezing i.e. [–8°C, –1°C]. Then for $T > T_1$: $f_S(T) = 0$, while for $T < T_2$: $f_S(T) = 1$ and $C(T) \rightarrow c(T)$. Summing up, equation (7) describes the heat transfer processes in the whole conventionally homogenous domain. The problem is strongly non-linear - both parameters $C(T)$, $\lambda(T)$, $k(T)$ and $Q_m(T)$ are temperature-dependent. Taking into account the cryoprobe geometry (Fig. 3), energy equation (7) should be written in the form corresponding to the cylindrical co-ordinate system $x = \{r, z\}$ (axially-symmetrical problem) this means

$$\begin{aligned} C(T)\frac{\partial T(r,z,t)}{\partial t} &= \frac{1}{\mathrm{r}}\frac{\partial}{\partial r}\left[r\lambda(T)\frac{\partial T(r,z,t)}{\partial r}\right] + \\ &\frac{\partial}{\partial z}\left[\lambda(T)\frac{\partial T(r,z,t)}{\partial z}\right] + Q_P(r,z,t) + Q_M(r,z,t) \end{aligned} \tag{8}$$

In Figures 4 and 5 the assumed courses of substitute thermal capacity and thermal conductivity of tissue [3] are shown.

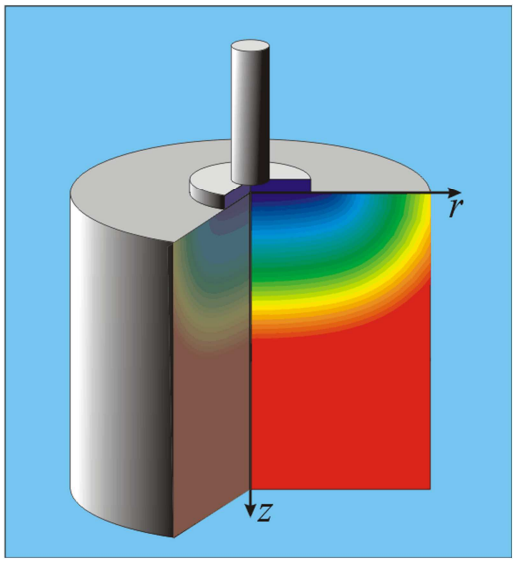


Fig. 3. Cryoprobe geometry [3]

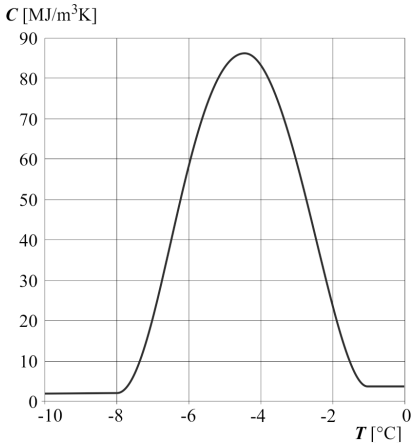


Fig. 4. Substitute thermal capacity $C(T)$

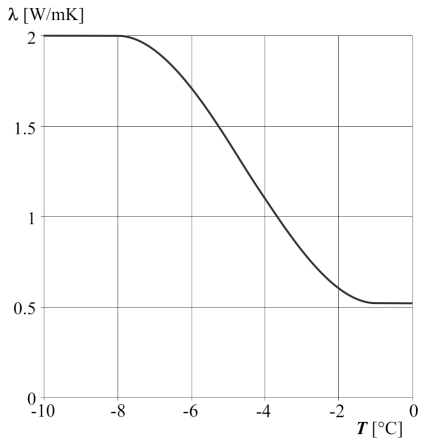


Fig. 5. Thermal conductivity

## 2. Sensitivity analysis

To analyze the common interactions between the cryoprobe tip cooling rate and course of the freezing process, the basic equations creating the model of the process discussed should be differentiated with respect to parameter $v$ (a direct variant of sensitivity analysis [5, 6]). Therefore, after the mathematical manipulations (the Schwarz theorem must be also taken into account) one obtains

$$\begin{aligned}&\frac{\mathrm{d}C(T)}{\mathrm{d}T}V\frac{\partial T}{\partial t}+C(T)\frac{\partial V}{\partial t}=\frac{1}{\mathrm{r}}\frac{\partial}{\partial r}\left[r\frac{\mathrm{d}\lambda(T)}{\mathrm{d}T}V\frac{\partial T}{\partial r}+r\lambda(T)\frac{\partial V}{\partial r}\right]+\\&\frac{\partial}{\partial z}\left[\frac{\mathrm{d}\lambda(T)}{\mathrm{d}T}V\frac{\partial T}{\partial z}+\lambda(T)\frac{\partial V}{\partial z}\right]+\frac{\mathrm{d}k(T)}{\mathrm{d}T}V(T_b-T)-k(T)V+\frac{\partial Q_m(T)}{\partial T}V\end{aligned} \tag{9}$$

where $V = V(r, z, t)$ is the sensitivity of the temperature field with respect to parameter $v$ ($V = \partial T / \partial v$). The boundary conditions take the form

– first variant of control mode

$$x \in \Gamma_c : V(x,t) = -t, \quad t < t_1, \qquad V(x,t) = t - t_m, \quad t_1 \le t \le 2t_1 \tag{10}$$

– second variant

$$\begin{aligned} x \in \Gamma_c : \quad & V(x,t) = -t \qquad t < t_1 \\ & V(x,t) = 0 \qquad t_1 \le t \le t_2 \\ & V(x,t) = v \qquad t_{21} \le t \le t_2 + t_1 \end{aligned} \tag{11}$$

The initial one is the following

$$t = 0: \qquad V(x,0) = 0 \tag{12}$$

while the adiabatic conditions are in force.

One can see that the basic problem and the sensitivity one are coupled - the identification of function $V(r, z, t)$ requires the knowledge of temperature field $T(r, z, t)$.

## 3. Selected results of numerical simulations

At the stage of numerical computations, the explicit scheme of the finite difference method described in detail by Mochnacki, Suchy [3] and Mochnacki, Majchrzak [7] has been used. The cylindrical cryoprobe (tip diameter $d$ = 10 mm) has been considered and $v$ = 10 K/min, cooling time $t_1$ = 10 min, at the same time different values (from 2 to 6 minutes) of hold time have been taken into account. The data concerning the mean values of skin tissue parameters can be found in [1]. As an example in Figure 6 the temperature field in the tissue domain for time 10 min is shown (first variant of control mode).

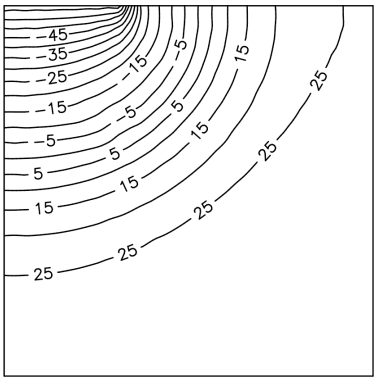


Fig. 6. Temperature field after 10 min

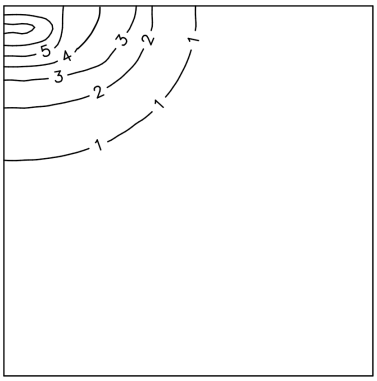


Fig. 7. Temperature differences after 5 min

In Figure 7, the temperature differences due to the 1% perturbation of cooling (heating) rate are presented. The other results of computations, numerical algorithm and the conclusions will be discussed in a wider version of the paper, but in this place it should be pointed out that the obtained results give interesting information from both the practical and theoretical points of view.

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