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NUMERICAL MODELLING OF COOLING PROCESS USING FUZZY BOUNDARY ELEMENT METHOD WITH α-CUTS

Alicja Piasecka Belkhayat

Department of Strength of Materials and Computational Mechanics
Silesian University of Technology, Poland
alicja.piasecka@polsl.pl

Abstract. In the paper, the description of an unsteady heat transfer for a two-dimensional problem is presented. It is assumed that all the thermophysical parameters appearing in the mathematical model of the problem analyzed are given as fuzzy numbers. The problem discussed has been solved by means of the 1st scheme of the fuzzy boundary element method using α-cuts. The application of α-cuts allows one to avoid complicated arithmetical operations in the fuzzy numbers set. The interval Gauss elimination method with the decomposition procedure has been applied to solve the obtained fuzzy system of equations. In the final part of the paper, the results of numerical computations are shown.

1. Formulation of the problem

Let us consider a two-dimensional homogeneous domain. The transient temperature field is described by the following fuzzy Fourier equation [1-3]

$$x \in \Omega: \quad \tilde{c}\,\tilde{\rho}\,\frac{\partial T(x,\ t)}{\partial t} = \tilde{\lambda}\,\nabla^2 T(x,\ t) + Q(x,\ t) \tag{1}$$

where $\tilde{c}$, $\tilde{\rho}$, $\tilde{\lambda}$ denote fuzzy values of the specific heat, the mass density and the thermal conductivity, T is the temperature, Q is the heat source, $x = \{x_1,\ x_2\}$ are the spatial co-ordinates and t is the time.

Equation (1) can be expressed as

$$x \in \Omega: \quad \frac{\partial T(x,\ t)}{\partial t} = \tilde{a}\,\nabla^2 T(x,\ t) + \frac{1}{\tilde{c}\,\tilde{\rho}}\,Q(x,\ t) \tag{2}$$

where $\tilde{a}$ is the fuzzy diffusion coefficient.

The fuzzy energy equation must be supplemented by the following initial condition

$$t = 0: \quad T(x,\ t) = T_0(x) \tag{3}$$

and the boundary conditions

$$\begin{cases} x \in \Gamma_1: & T(x,\ t) = T_b \\ x \in \Gamma_2: & \tilde{q}(x,\ t) = -\tilde{\lambda}\dfrac{\partial T(x,\ t)}{\partial n} = \tilde{q}_b \\ x \in \Gamma_3: & \tilde{q}(x,\ t) = -\tilde{\lambda}\dfrac{\partial T(x,\ t)}{\partial n} = \alpha\left(\tilde{T} - T^{\infty}\right) \end{cases} \tag{4}$$

where T_0 is the initial temperature, T_b is the known boundary temperature, $\partial T(x,\ t)/\partial n$ is the normal derivative at boundary point x, $\tilde{q}_b$ is the given boundary heat flux, α is the heat transfer coefficient and T^{∞} is the ambient temperature.

2. Fuzzy boundary element method

In the paper, the 1st scheme of the fuzzy boundary element method is used [3, 4]. The criterion of the weighted residual method (WRM) is of the following form [1, 2]

$$\int_{t^{f-1}}^{t^f} \iint_{\Omega} \left[\tilde{a}\nabla^2 T(x,\ t) - \frac{\partial T(x,t)}{\partial t} + \frac{1}{\tilde{c}}Q(x,t) \right] \tilde{T}^*(\xi,\ x,\ t^f,\ t)\,\mathrm{d}\Omega\,\mathrm{d}t = 0 \tag{5}$$

where $\tilde{T}^*(\xi,\ x,\ t^f,\ t)$ is the fuzzy fundamental solution, ξ is the point where the concentrated heat source is applied. Fuzzy function $\tilde{T}^*(\xi,\ x,\ t^f,\ t)$ is expressed as [1, 5, 6]

$$\tilde{T}^*(\xi,\ x,\ t^f,\ t) = \frac{1}{4\pi\tilde{a}(t^f - t)} \exp\left[-\frac{r^2}{4\tilde{a}(t^f - t)} \right] \tag{6}$$

where r is the distance from the point under consideration x to observation point ξ

$$r = \sqrt{\left(x_1 - \xi_1\right)^2 + \left(x_2 - \xi_2\right)^2} \tag{7}$$

The fuzzy boundary integral equation for transition $t^{f-1} \rightarrow t^f$ corresponding to equation (5) is of the following form [1, 3, 5, 6]

$$B(\xi)\tilde{T}(\xi,\ t^{f})+\frac{1}{\tilde{c}\tilde{\rho}}\int_{t^{f-1}}^{t^{f}}\int_{\Gamma}\tilde{T}^{*}(\xi,\ x,\ t^{f},\ t)\,\tilde{q}(x,\ t)\,\mathrm{d}\Gamma\,\mathrm{d}t=$$

$$\frac{1}{\tilde{c}\tilde{\rho}}\int_{t^{f-1}}^{t^{f}}\int_{\Gamma}\tilde{q}^{*}(\xi,\ x,\ t^{f},\ t)\,\tilde{T}(x,\ t)\,\mathrm{d}\Gamma\,\mathrm{d}t+\iint_{\Omega}\tilde{T}^{*}(\xi,\ x,\ t^{f},\ t^{f-1})\,\tilde{T}(x,\ t^{f-1})\,\mathrm{d}\Omega \tag{8}$$

$$+\frac{1}{\tilde{c}\tilde{\rho}}\int_{t^{f-1}}^{t^{f}}\iint_{\Omega}Q(x,\ t)\,\tilde{T}^{*}(\xi,\ x,\ t^{f},\ t)\,\mathrm{d}\Omega\,\mathrm{d}t$$

where $\tilde{T}(x, t)$ is the fuzzy function of the temperature, $\tilde{q}(x, t)$ is the fuzzy heat flux, $B(\xi)$ is the coefficient from interval (0, 1).

3. Numerical realization

Let us consider the constant elements with respect to time, which can be defined as follows [1]

$$t\in\left[t^{f-1},\ t^{f}\right]:\ \begin{cases}\tilde{T}(x,\ t)=\tilde{T}(x,\ t^{f})\\ \tilde{q}(x,\ t)=\tilde{q}(x,\ t^{f})\end{cases} \tag{9}$$

Dividing boundary Γ of the domain considered into N constant boundary elements and the interior of this area into L constant internal cells we obtain the following fuzzy system of equations ($i=1,\ 2,\ \ldots,\ N$) [1, 3]

$$\sum_{j=1}^{N}\tilde{G}_{ij}\,\tilde{q}_{j}^{f}=\sum_{j=1}^{N}\tilde{H}_{ij}\,\tilde{T}_{j}^{f}+\sum_{l=1}^{L}\tilde{P}_{il}\,\tilde{T}_{l}^{f-1}+\sum_{l=1}^{L}\tilde{Z}_{il}\,Q_{l}^{f-1} \tag{10}$$

After determining the 'missing' boundary values, fuzzy functions $\tilde{T}(x, t)$ at the internal nodes of the domain considered are calculated using the formula

$$\tilde{T}_{i}^{f}=\sum_{j=1}^{N}\tilde{H}_{ij}\,\tilde{T}_{j}^{f}-\sum_{j=1}^{N}\tilde{G}_{ij}\,\tilde{q}_{j}^{f}+\sum_{l=1}^{L}\tilde{P}_{il}\,\tilde{T}_{l}^{f-1}+\sum_{l=1}^{L}\tilde{Z}_{il}\,Q_{l}^{f-1} \tag{11}$$

where $i=N+1,\ N+2,\ \ldots,\ N+L$.

It should be pointed out that one of the ideas to avoid very complicated arithmetical operations in the fuzzy numbers set is to introduce α-cuts of the fuzzy numbers. In this case it is possible to apply interval arithmetic (classical or directed) for every α-cut because α-cuts are treated as interval numbers [3, 7]. Here the directed interval arithmetic has been applied [3, 6, 8].

Let us assume that the considered thermophysical parameters are triangle fuzzy numbers of the following form [3, 9]

$$\tilde{a} = \left(a^-,\, a_0,\, a^+\right) \tag{12}$$

where a_0 is the core and a^-, a^+ are the left and the right side of the fuzzy number respectively.

A membership function for triangular fuzzy numbers is expressed as

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a^- \\ \dfrac{x - a^-}{a_0 - a^-}, & a^- \le x \le a_0 \\ \dfrac{a^+ - x}{a^+ - a_0}, & a_0 \le x \le a^+ \\ 0, & x > a^+ \end{cases} \tag{13}$$

The α-cuts of the fuzzy numbers are computed using the formula

$$\forall \alpha \in [0, 1] \qquad \tilde{a}_\alpha = \left[\left(a_0 - a^-\right)\alpha + a^-,\, \left(a_0 - a^+\right)\alpha + a^+\right] \tag{14}$$

Applying α-cuts of the fuzzy numbers allows one to use directed interval arithmetic and the interval Gauss elimination method with the decomposition procedure to solve the obtained fuzzy system of equations (10) [10, 11].

4. Numerical examples

In numerical computations, a two dimensional domain (square) of dimensions d_1 = 0.1 m and d_2 = 0.1 m has been considered. The following input data have been introduced: $\tilde{\lambda} = (33.25,\, 35,\, 36.75)$ W/(m·K), $\tilde{c} = (655.5,\, 690,\, 724.5)$ J/(kg·K), $\tilde{\rho} = (7125,\, 7500,\, 7875)$ kg/m^3, $Q = 10000$ J/m^3, initial temperature T_0 = = 1200°C, the time step Δt = 1 s, the boundary is divided into 32 constant elements and the interior is divided into 64 constant square internal cells.

On the right side of the domain considered, the boundary condition of the second type is assumed: q_b = 10000 W/m^2. On the other sides, the boundary condition of the first type is assumed: T_b = 500°C - see Figure 1.

Figures 2-5 illustrate the cooling curves obtained at points 68 ($x_1 = 0.04375$ m, $x_2 = 0.05625$ m) and 90 ($x_1 = 0.01875$ m, $x_2 = 0.09375$ m) for chosen α-cuts.

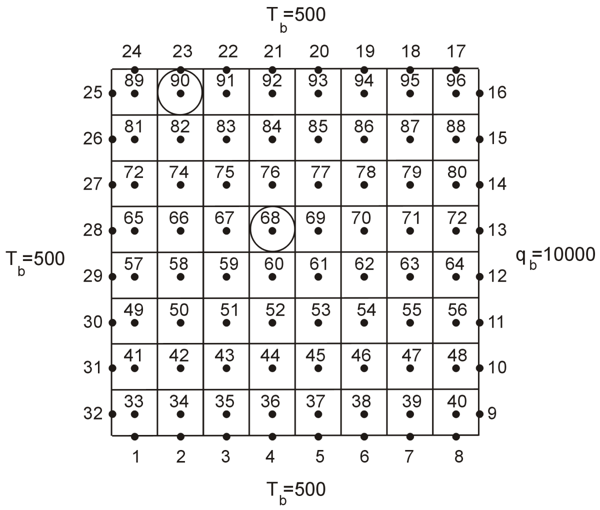

Fig. 1. Discretization of domain considered

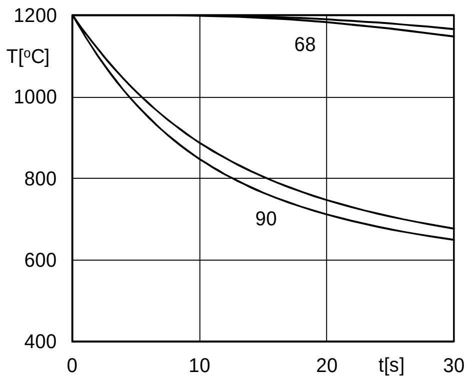

Fig. 2. Cooling curves for $\alpha = 0$

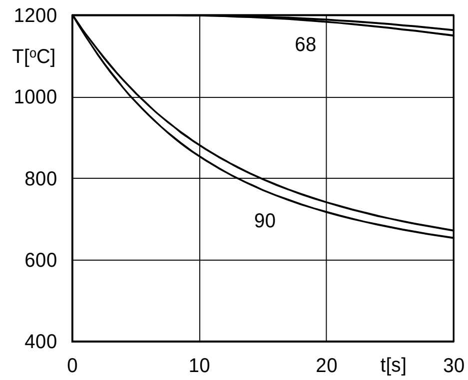

Fig. 3. Cooling curves for $\alpha = 0.3$

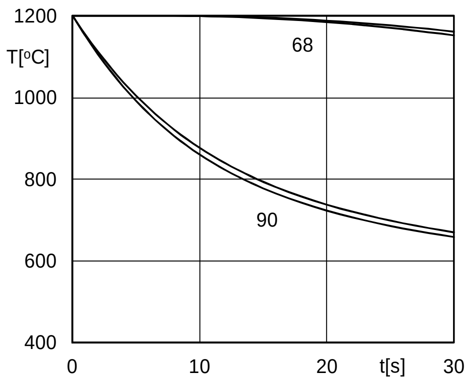

Fig. 4. Cooling curves for $\alpha = 0.6$

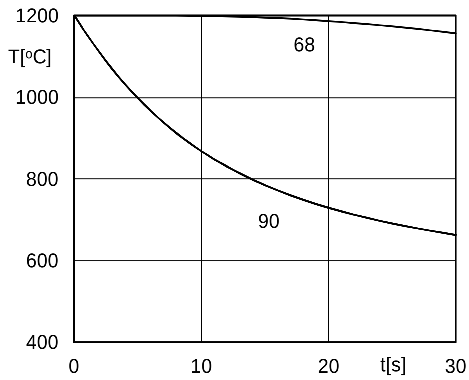

Fig. 5. Cooling curves for $\alpha = 1$

Conclusions

In the paper, the description of an unsteady heat transfer for a 2D problem has been presented. All the thermophysical parameters have been considered as fuzzy numbers. The problem analyzed has been solved using the 1st scheme of the fuzzy boundary element method with α-cuts.

Such an approach allows one to avoid complicated fuzzy arithmetic and treat the considered fuzzy numbers as interval numbers. For bigger values of α, the temperature interval is narrower. For $\alpha = 1$ the width of the temperature interval is equal to 0.

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