Scientific Research of the Institute of Mathematics and Computer Science, 1(10) 2011, 11-20

NUMERICAL MODELING OF BIOLOGICAL TISSUE HEATING. ADMISSIBLE THERMAL DOSE

Mariusz Ciesielski[1], Bohdan Mochnacki[2], Romuald Szopa[3]

[1]Institute of Computer and Information Sciences, [2]Institute of Mathematics
[3]Faculty of Civil Engineering
Czestochowa University of Technology, Poland
bohdan.mochnacki@im.pcz.pl

Abstract. The cylindrical domain of skin tissue subjected to an external heat flux is considered (shape of domain is determined by form of function describing Neumann boundary condition on external surface of the system). The first version of numerical simulations concerns the heterogeneous multi-layered skin tissue domain (epidermis, dermis, sub-cutaneous region). The thermophysical parameters of successive layers are assumed to be different, but constant. The second version of computations concerns the homogeneous domain, but the mean values of the thermophysical parameters are temperature-dependent (non-linear task). Knowledge of the spatial, time-dependent temperature field allows one to determine the so-called thermal dose and also the degree of tissue destruction. The algorithm presented can be useful in medical practice, among others, at the stage of the hyperthermia therapy scheme.

Introduction

The domain of skin tissue can be treated as a heterogeneous domain being the composition of layers corresponding to the epidermis, dermis and sub-cutaneous region - Figure 1.

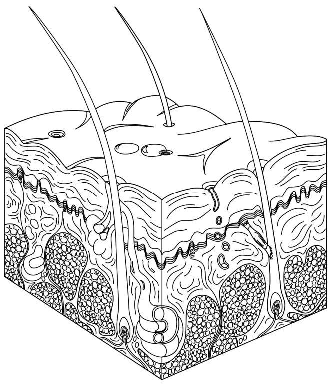

Fig. 1. Skin tissue

The thicknesses of layers and also their thermophysical parameters are individual personal traits, at the stage of numerical computations presented here, the mean values taken from [1] have been introduced.

The values of tissue thermophysical parameters can be treated as temperature--dependent ones. Such an approach is closer to the real model of the material considered. In literature, one can find information concerning the changes of tissue volumetric specific heat, thermal conductivity, volumetric perfusion coefficient and metabolic heat source [2, 3].

1. Governing equations

Thermal processes proceeding in the domain considered can be described by a system of partial differential equations (Pennes equations), in particular [4-7]

$$x \in \Omega_e: \quad c_e(T)\frac{\partial T_e(x,t)}{\partial t} = \operatorname{div}\left[\lambda_e(T)\operatorname{grad} T_e(x,t)\right] + Q_{pe}(T) + Q_{me}(T), \tag{1}$$

where $e = 1, 2, 3$ corresponds to the successive skin layers, c is the volumetric specific heat, λ is the thermal conductivity, Q_p, Q_m are the capacities of volumetric internal heat sources connected with the blood perfusion and metabolism, T, x, t denote the temperature, geometrical co-ordinates and time. The perfusion heat source is given by formula

$$Q_{pe}(T) = k_e(T)\left[T_b - T_e(x,t)\right], \tag{2}$$

where $k_e(T) = G_{be}(T)\cdot c_b$, G_b is the blood perfusion [m^3 blood/(s m^3 tissue)], c_b is the blood volumetric specific heat and T_b arterial blood temperature. The model presented concerns the tissue domain supplied by a large number of capillary vessels. When the presence of bigger vessels (arteries or veins) should be taken into account, then the basic Pennes equation must be supplemented by a additional equations concerning vessel domains (e.g. [6]), but this problem will not be discussed here.

Metabolic heat source Q_m can be treated both as a constant value and temperature-dependent function.

In this paper, an axially-symmetrical problem is analyzed. The choice of tissue geometry results from the form of function describing the action of external heat flux (2D Gauss-type function). Conventionally, the assumed tissue domain is characterized by radius R and high Z, at the same time the dimensions are determined in the way that one can introduce the no-flux boundary conditions on the bottom and lateral surfaces limiting the domain considered. The thicknesses of successive layers are equal to L_1, L_2, L_3 and $Z = L_1 + L_2 + L_3$.

The boundary conditions given on the contact surface between epidermis--dermis and dermis-sub-cutaneous region are assumed in the form of continuity ones, namely

$$x \in \Gamma_e: \quad \begin{cases} -\lambda_e \dfrac{\partial T_e(x,t)}{\partial z} = -\lambda_{e+1} \dfrac{\partial T_{e+1}(x,t)}{\partial z} \\ T_e(x,t) = T_{e+1}(x,t) \end{cases} . \tag{3}$$

At the same time, indexes e i $e + 1$ identify the sub-domains being in thermal contact.

On the external surface ($z = 0$), the boundary heat flux is given (the Neumann boundary condition)

$$x \in \Gamma_0: \quad -\lambda_1 \frac{\partial T_1(x,t)}{\partial n} = q_b(x,t), \tag{4}$$

in particular

$$x = \{r, z\} \in \Gamma_0: \quad q_b(x,t) = q_0 \exp\left[-\frac{r^2}{2(R/3)^2}\right], \quad r \in [0, R], \quad t \le t_p, \tag{5}$$

where $R/3 = \sigma$ is the standard deviation of a normal distribution of heat source, q_0 is the factor corresponding to the maximum incident heat flux and t_p is the exposure time. For t > t_p on the surface considered, the Robin condition should be taken into account

$$x \in \Gamma_0: \quad -\lambda_1 \frac{\partial T_1(x,t)}{\partial n} = \alpha\left[T_1(x,t) - T_a\right], \tag{6}$$

where α is the heat transfer coefficient, T_a is the ambient temperature.

On the conventionally assumed lateral and bottom surface of the cylinder, no-flux conditions can be taken into account

$$\{r, z\} \in \Gamma_\infty: \quad -\lambda_e \frac{\partial T_e(x,t)}{\partial n} = 0, \quad n = r \cup z. \tag{7}$$

The initial condition is also given

$$t = 0: \quad T_e(x,t) = T_{e0}(x). \tag{8}$$

The above presented system of energy equations and boundary-initial conditions constitutes the mathematical model of the problem discussed - Figure 2.

It should be pointed out that for the epidermis domain, both Q_p and Q_m are equal to zero [7].

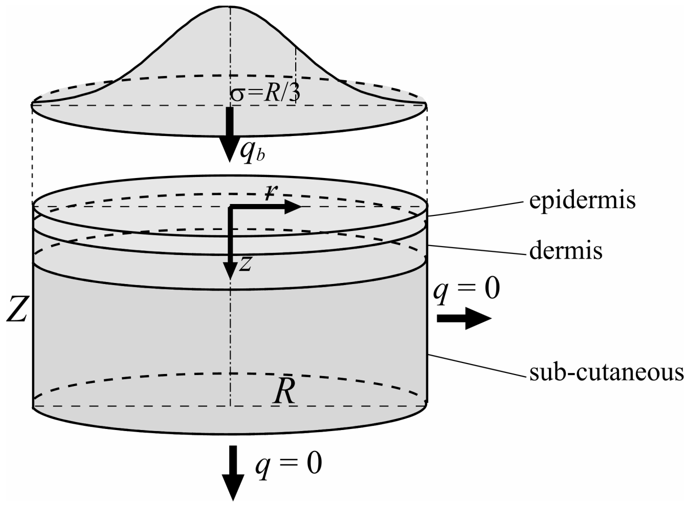

Fig. 2. Axially-symmetrical tissue domain

From the numerical point of view the same mathematical model can be a base for computer program construction in the case of a homogeneous tissue domain. It is sufficient to assume the same values of thermophysical parameters for successive sub-domains.

2. Modeling of tissue destruction (a burn degree)

Thermal damage of biological tissue can be treated as a certain chemical process and a burn degree can be found using the first order Arrhenius equation (Henriques burn integral [4, 5, 8, 9]). This integral determined the tissue damage on the basis of the protein denaturation rate and the local changes of tissue temperature, in particular

$$I(r,z,t) = \int_0^t P \exp\left(-\frac{\Delta E}{R_g \left(T(r,z,t) + 273 \right)} \right) \mathrm{d}t \,, \tag{9}$$

where P [1/s] is the pre-exponential factor, ΔE [J/mol] is the activation energy and R_g [kJ/(kmol K)] is the gas constant (R_g = 8.3114472). The values of P and ΔE can be found experimentally and they can be found in literature (e.g. [5, 7]). It is said that a I step burn degree appears when $I \geq 0.53$, II step - $I \geq 1$ and III step - $I \geq 10^4$.

Assuming the knowledge of the function describing the boundary heat flux, one can find the so-called thermal dose, this means

$$TD = \int_0^{t_p} \int_0^{2\pi} \int_0^R q_0 \exp\left[-\frac{r^2}{2(R/3)^2} \right] \cdot r \, \mathrm{d}r \, \mathrm{d}\varphi \, \mathrm{d}t, \tag{10}$$

in other words

$$TD = \frac{2\pi(1-\exp(-9/2))}{9} t_p \, q_0 \, R^2 \approx 0.69 \, t_p \, q_0 \, R^2. \tag{11}$$

3. Results of computations

The problem discussed has been solved using numerical methods, in particular the Control Volume Method has been applied. The details concerning the CVM algorithm can be found in [10, 11, 13].

The cylindrical domain of biological tissue (R = 20 mm, Z = 12.1 mm) has been considered. At the stage of a heterogeneous problem solution, the following thicknesses of layers have been taken into account: L_1 = 0.1 mm, L_2 = 2 mm, L_3 = = 10 mm. The mean values of the thermophysical parameters of successive layers: λ_1 = 0.235 W/(mK), λ_2 = 0.445 W/(mK), λ_3 = 0.185 W/(mK), c_1 = = 4.3068·10^6 J/(m^{3}K), c_2 = 3.96·10^6 J/(m^{3}K), c_3 = 2.674·10^6 J/(m^{3}K) and c_b = 3.9962·10^6 J/(m^{3}K), T_b = 37°C, G_{b1} = 0, G_{b2} = G_{b3} = 0.00125 (m^3 blood/s/m^3 tissue), Q_{m1} = 0, Q_{m2} = Q_{m3} = 245 W/m^3 (rest conditions). Initial conditions $T_{e\,0}(x)$ have been determined on the basis of the solution of steady state equation (1) with boundary conditions (6) and (7), ambient temperature T_a = 20°C, heat transfer coefficient α = 10 W/(m^{2}K). Parameters of burn integral: ΔE = 6.285·10^5 J/mol, P = 3.1·10^{98} 1/s.

The first example of computations has been solved under the assumption that the exposure time of the heat source equals t_p = 5 s, while the maximum incident heat flux q_0 = 20 kW/m^2. In Figure 3, the courses of isotherms for the time of 5 s (the end of heat source exposure time) and also for time t = 10 s are shown. In Figure 4, the changes of temperature and burn integral at points selected from the tissue domain: A (0, L_1), B (R/4, L_1), C (R/2, L_1), D (0, L_1+ L_2), E (R/4, L_1+ L_2), F (R/2, L_1+ L_2) are presented. One can see that at point A, a 2$^{\text{nd}}$ degree burn occurs. The results shown in Figure 5 concern the profiles of temperature and burn integral for the selected times at the symmetry axis (r = 0) of the domain considered. It should be pointed out that the values of burn integral grow even after termination of the external heat flux. If the tissue temperature decreases (as a result of heat dissipation to the environment or the process of blood perfusion), then the previous growth of burn integral stops.

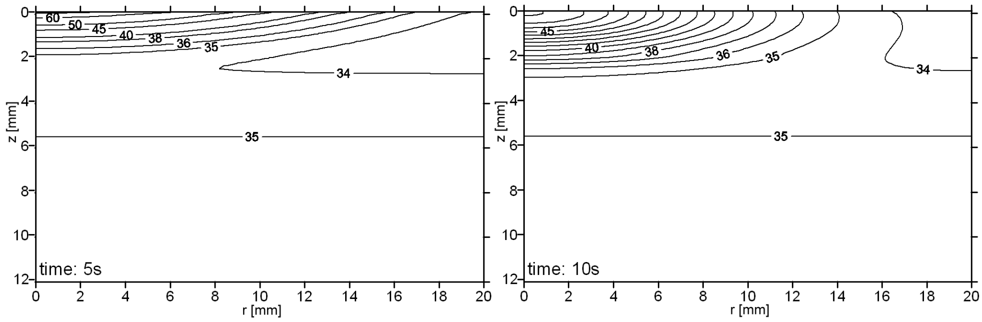

Fig. 3. Courses of isotherms in skin tissue at 5 s and 10 s
(parameters of external heat source: q_0 = 20 kW/m^2 and t_p = 5 s)

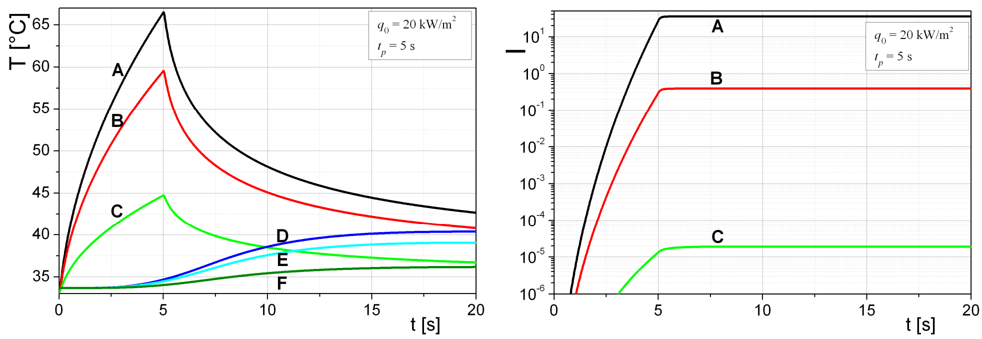

Fig. 4. Temperature histories (left side) and burn integral histories (right side)
at selected points in tissue domain: A (0, L_1), B (R/4, L_1), C (R/2, L_1), D (0, L_1+ L_2),
E (R/4, L_1+ L_2), F (R/2, L_1+ L_2)

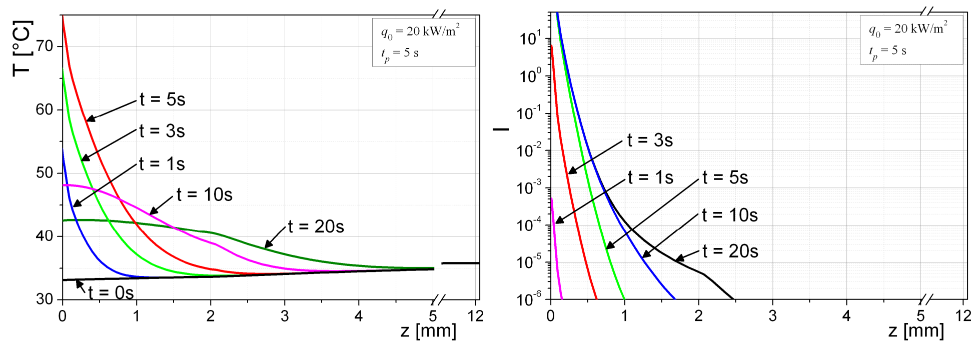

Fig. 5. Profiles of temperature (left-side) and burn integral (right-side)
at selected moments of time at symmetry axis of cylinder domain (r = 0)

The second example deals with determination of the relationship between the values of q_0 and t_p for which may occur a first-, second- or third-degree burn (i.e. when the value of burn integral on the surface between the skin layer and the dermis exceeds the specified value). At the stage of numerical simulations, the values q_0 and t_p from ranges $q_0 \in$ [10 kW/m^2, 30 kW/m^2] and $t_p \in$ [0 s, 15 s] have been taken into account. The computations for succesive cases were terminated when the temperature in the entire area of the tissue went below 50°C (then an increase of burn integral is not very significant). For each simulation, a value of burn integral at point (0, L_1) was observed. This point is located in the symmetry axis of the cylinder between the epidermis and dermis regions - and it is most sensitive to the impact of the heat flux. In addition, the thermal dose (11) for both parameters q_0 and t_p has been calculated. In Figure 6 the results obtained are presented in the form of isolines. It should be noted that the specified value of the thermal dose absorbed by the tissue does not always cause burns. For example, the value of thermal dose *TD* = 20 J for q_0 = 10 kW/m^2 does not cause burns, but for q_0 = 30 kW/m^2 a 2nd degree burn appears.

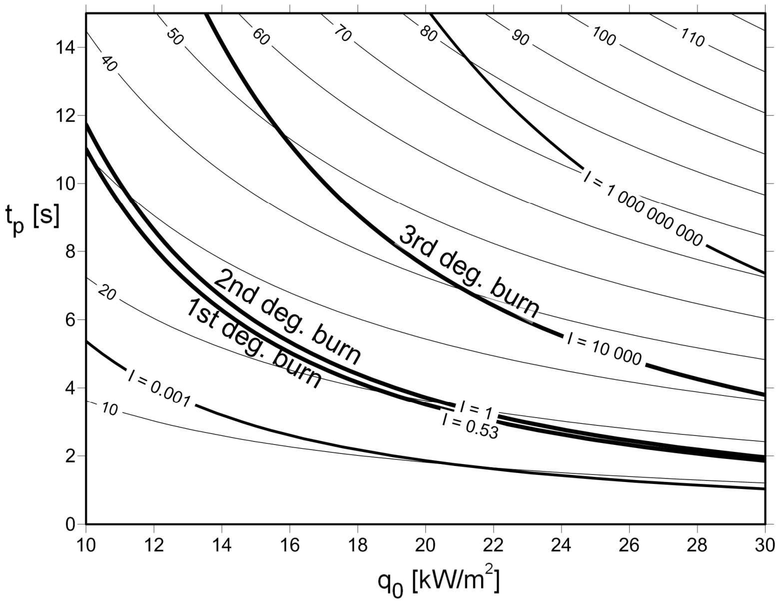

Fig. 6. Dependence of burn integral and thermal dose (thin lines - isolines of thermal dose, thick lines - selected isolines of burn integral)

The last example has been solved as a non-linear task concerning the homogeneous tissue domain. The temperature-dependent mean thermophysical parameters of tissue have been introduced in the form [2, 12]:

$$c(T) = 10^6 \begin{cases} 3.8, & T < 98 \\ 3.8 + 2.339515(T - 98), & 98 \le T \le 100 \\ 0.44 - 4.019515(T - 102), & 100 \le T \le 102 \\ 0.44, & T > 102 \end{cases} \tag{12}$$

while the thermal conductivity is defined as follows

$$\lambda(T)=\begin{cases}0.52, & T<98\\ 0.52-0.107(T-98), & 98\le T\le 102\\ 0.092, & T>102\end{cases} \tag{13}$$

The perfusion heat source (because of approachable numerical data, formula (2) was somewhat changed)

$$Q_b(T)=c_B W_B(T)\left[T_B-T(x,t)\right], \tag{14}$$

where c_B = 3900 J/(kgK) is the specific heat of blood, W_B [kg/(m^{3}s)] is the volumetric perfusion coefficient, T_B = 37°C is the blood temperature and

$$W_B=\begin{cases}1.159, & T\le 42.5\\ 1.159\left[1+9.6(T-42.5)\right], & 42.5<T<45\\ 28.975, & T\ge 45\end{cases} \tag{15}$$

Additionally

$$Q_{met}(T)=1091\left[1+0.1(T-37)\right]. \tag{16}$$

For example, if T = 45°C then the capacity of the metabolic heat source equals Q_{met} = 1963.8 W/m^3.

Figure 7 shows the changes of temperature and burn integral at the points selected from the tissue domain. The simulation was done for the same geometry of the domain and the same parameters determining the boundary conditions as previously. One can see that the tissue temperature after heating is several degrees lower in comparison to the first example.

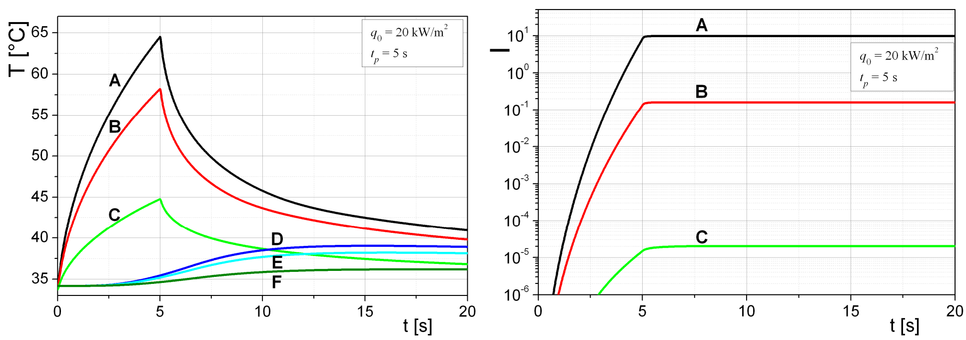

Fig. 7. Temperature histories (left side) and burn integral histories (right side) at selected points in tissue domain: A (0, L_1), B (R/4, L_1), C (R/2, L_1), D (0, L_1+ L_2), E (R/4, L_1+ L_2), F (R/2, L_1+ L_2) for homogeneous tissue domain

Conclusions

The study described in this paper investigated the thermal processes occurring in skin tissue. The tissue was subjected to the influence of an external heat source in the form of a 2D Gauss-type function. This source can cause burns of the skin tissue. At the stage of numerical simulation (the 2D axially-symmetric task), the control volume method was used. This method in a relatively simple way allows one to take into account the homogeneity and heterogeneity of the layers of skin tissue. The computer program worked out by the authors of this paper gives the possibility to estimate the impact of the external heat flux intensity and the exposure time to the possibility of burn formation, simultaneously it is possible to determine the degree of skin burn.

Acknowledgement

This work was supported by Grant No NR 13-0124-10.

References

[1] Torvi D.A., Dale J.D., A finite element model of skin subjected to a flash fire, Journal of Biomechanical Engineering 1994, 116, 250-255.

[2] Mochnacki B., Dziewonski M., Poteralska J., Thermal effects in domain of tissue subjected to an external heat source, Computational Modeling and Advanced Simulations, CMAS 2009, Bratislava 2009, 1, 10.

[3] Stańczyk M., Telega J.J., Modelling of heat transfer in biomechanics - a review, Part I: Soft tissues, Acta of Bioengineering and Biomechanics 2002, 4, 1, 31-61.

[4] Majchrzak E., Modelowanie i analiza zjawisk termicznych, [in:] Mechanika Techniczna, Biomechanika, Tom XII, cz. IV, IPPT PAN, 2011, 223-362.

[5] Majchrzak E., Jasiński M., Sensitivity analysis of burn integrals, Computer Assisted Mechanics and Engineering Sciences 2004, 11, 2/3, 125-136.

[6] Majchrzak E., Mochnacki B., Analysis of bio-heat transfer in the system of blood vessel-biological tissue, Kluwer Academic Publishers 2001, 201-211.

[7] Majchrzak E., Mochnacki B., Jasiński M., Numerical modelling of bioheat transfer in multilayer skin tissue domain subjected to a flash fire, Computational Fluid and Solid Mechanics, Elsevier 2003, Vol. II, 1766-1770.

[8] Ng E. Y-K., Chua L.T., Prediction of skin burn injury, Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine 2002, 216, 171-183.

[9] Xu F., Lu T.J., Seffen K.A., Biothermomechanics of skin tissues, Journal of the Mechanics and Physics of Solids 2008, 58, 1852-1884.

[10] Mochnacki B, Suchy J.S., Numerical methods in computations of foundry processes, PFTA, Cracow 1995.

[11] Mochnacki B., Ciesielski M., Modelowanie nagrzewania tkanki skórnej, dopuszczalna dawka termiczna, II Kongres Mechaniki Polskiej, Poznań 2011, 1-14.

[12] Abraham J.P., Sparrow E.M., A thermal-ablation bioheat model including liquid-to vapour phase change, pressure- and necrosis-dependent perfusion, and moisture-dependent properties, International Journal of Heat and Mass Transfer 2007, 50, 2537-2544.

[13] Majchrzak E., Mochnacki B., Metody numeryczne. Podstawy teoretyczne, aspekty praktyczne, algorytmy, Wydawnictwa Politechniki Śląskiej, Gliwice 2004.