SHAPE CONTROL OF THE COLUMN WITH IMPERFECTIONS OF LOADING HEADS BY MEANS OF THE STIFFENING ROD

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Abstract. This paper concerns an influence of the imprecise manufacturing or improper assembling processes of supporting and loading heads on transversal displacements, bending moments and axial force distribution of a column consisted of two rods, where one of rods is a stiffening element. The problem has been formulated by means of the principle of stationary total potential energy, the von Karman theory and linear constitutive equations for column material. The results of numerical calculation have shown how important it is to provide the axial load of the beams and columns.

Introduction

The imprecise manufacturing or improper assembling process of supporting and loading heads is a great problem in every type of construction. This lack of precision leads to increase of transversal displacements and bending moments in construction elements. In paper [1, 2] the geometrically nonlinear column has been investigated. The external load was unintentionally applied to the column with an eccentricity. In paper [1] an influence of the eccentricity on longitudinal forces distribution, transversal displacements and bending moments of each rod of the structure have been presented. The natural vibration of the system has also been discussed. In paper [2] the single - rod column, supported on the free end by a spring has been presented. The line of action of the spring was overlapping with undeformed axis of the column. The theoretical and numerical studies were performed and as a result of this investigation the magnitude of the internal force in rod and vibration frequency of the system as a function of external load for different spring stiffness were obtained. The different value of the eccentricity of appliance of the external load was also investigated. Because there is no ideal axial loading, the main purpose of this work is to investigate the influence of the eccentrically applied load on transversal displacements, bending moments and axial force distribution. The localization of the stiffening element is also taken into account in order to achieve the shape control.

1. Formulation of the problem

The real structures have geometrical imperfections of shape and supporting and loading heads as a result of inaccurate assembling or manufacturing process. It has to be noted that it is almost impossible to achieve the one hundred percent axial loading of any system.

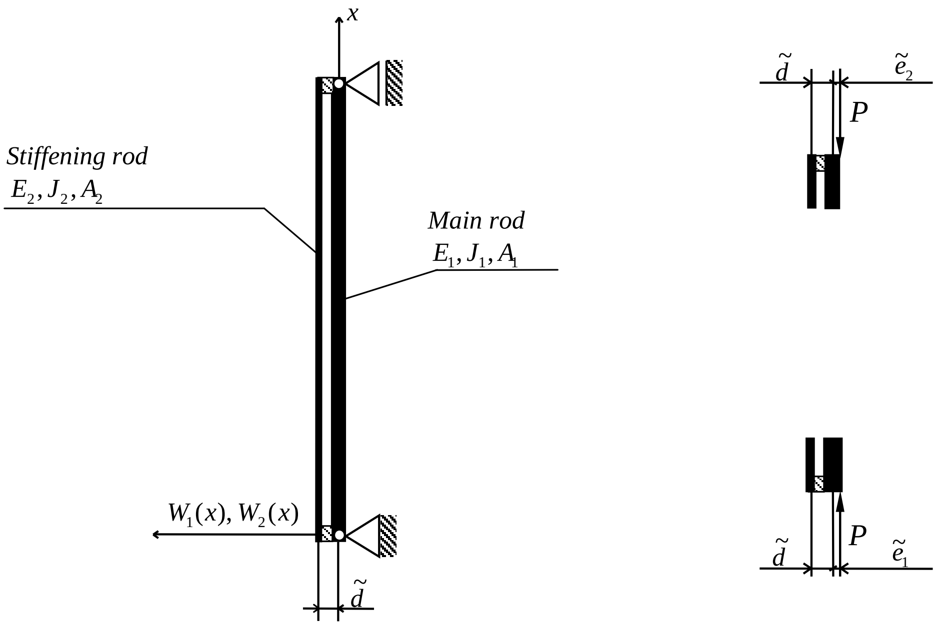

Fig. 1. Scheme of the considered column

Figure 1 shows a simply supported column with additional stiffening rod. The structure is loaded by external load *P*. In this investigation the main rod of the column is eccentrically loaded with eccentricities $\tilde{e}_1$ and $\tilde{e}_2$ (the line of action of external load is shifted in regard to the main rod axis). This results in the increase of transversal displacements while the external load value is getting greater. The additional rod is discretely attached on the offset distances $\tilde{d}$ and acts as reinforcing element.

The problem presented in this paper has been formulated on the basis of the principle of stationary total potential energy. The total potential energy of the system is as follows:

$$\Pi = \frac{1}{2}\sum_{i=1}^{2}\int_{\Omega_i}\sigma_{xi}\varepsilon_i(x)d\Omega_i + PU_1(l) \pm P\tilde{e}_1\frac{dW_1(x)}{dx}\bigg|_{x=0} \pm P\tilde{e}_2\frac{dW_1(x)}{dx}\bigg|_{x=l} \tag{1}$$

where:

– the normal stress in the elements are defined as

$$\sigma_{xi} = E_i \varepsilon_i(x), \qquad (i = 1,2) \tag{2}$$

– the non-linear von Karman strain – displacement relation is as follows

$$\varepsilon_i(x) = \frac{dU_i(x)}{dx} - z\frac{d^2W_i(x)}{dx^2} + \frac{1}{2}\left(\frac{dW_i(x)}{dx}\right)^2, \qquad (i = 1,2) \tag{3}$$

In equations (1)-(3) E_i is the Young's modulus, $W_i(x)$ and $U_i(x)$ are the transversal and longitudinal displacements of the i-th rod, respectively. Inserting equations (2)-(3) into (1) the following formula was obtained:

$$\begin{aligned}\Pi = \sum_{i=1}^{2} \Bigg\{ & \frac{1}{2}\int_{\Omega_i}\left[\frac{dU_i(x)}{dx} + \frac{1}{2}\left(\frac{dW_i(x)}{dx}\right)^2\right]^2 E_i d\Omega_i - \int_{\Omega_i} z\frac{d^2W_i(x)}{dx^2}\left[\frac{dU_i(x)}{dx} + \frac{1}{2}\left(\frac{dW_i(x)}{dx}\right)^2\right] E_i d\Omega_i + \\ & + \frac{1}{2}\int_{\Omega_i} z^2\left(\frac{d^2W_i(x)}{dx^2}\right)^2 E_i d\Omega_i \Bigg\} + PU_1(l) \pm P\tilde{e}_1 \left.\frac{dW_1(x)}{dx}\right|_{x=0} \pm P\tilde{e}_2 \left.\frac{dW_1(x)}{dx}\right|_{x=l} = \\ = \sum_{i=1}^{2} \Bigg\{ & \frac{b_i}{2}\overset{(i)}{A}_{11}\int_0^l \left[\frac{dU_i(x)}{dx} + \frac{1}{2}\left(\frac{dW_i(x)}{dx}\right)^2\right]^2 dx - b_i \overset{(i)}{B}_{11}\int_0^l \frac{d^2W_i(x)}{dx^2}\left[\frac{dU_i(x)}{dx} + \frac{1}{2}\left(\frac{dW_i(x)}{dx_i}\right)^2\right] dx + \\ & + \frac{b_i}{2}\overset{(i)}{D}_{11}\int_0^l \left[\frac{d^2W_i(x)}{dx^2}\right]^2 dx \Bigg\} + PU_1(l) \pm P\tilde{e}_1 \left.\frac{dW_1(x)}{dx}\right|_{x=0} \pm P\tilde{e}_2 \left.\frac{dW_1(x)}{dx}\right|_{x=l}\end{aligned} \tag{4}$$

where: l is the length, b_i is the width and h_i is the height of the of the element, $\overset{(i)}{A}_{11}, \overset{(i)}{B}_{11}, \overset{(i)}{D}_{11}$ denote the extensional, coupled bending-extensional and bending stiffness, respectively defined as

$$\overset{(i)}{A}_{11}, \overset{(i)}{B}_{11}, \overset{(i)}{D}_{11} = \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \left(1, z, z^2\right) E_i dz \tag{5}$$

Equating the first variation of the potential energy to zero and performing the integration and variational operations, knowing that virtual displacement longitudinal $\delta U_i(x)$ and transversal $\delta W_i(x)$ are arbitrary and independent for $0 < x < l$, the following governing equations were obtained:

$$E_i J_i \frac{d^4 W_i(x)}{dx^4} - E_i A_i \frac{d}{dx}\left\{\left[\frac{dU_i(x)}{dx} + \frac{1}{2}\left(\frac{dW_i(x)}{dx}\right)^2\right]\frac{dW_i(x)}{dx}\right\} = 0 \quad (i = 1,2) \qquad (6)$$

where $b_i \overset{(i)}{A}_{11} = E_i A_i$, $b_i \overset{(i)}{D}_{11} = E_i J_i$.

The strain in each rod is independent form the space variable, and can be expressed by the following formula:

$$\frac{d}{dx}\left[\frac{dU_i(x)}{dx} + \frac{1}{2}\left(\frac{dW_i(x)}{dx}\right)^2\right] = 0 \quad (i = 1,2) \qquad (7)$$

On the basis of equation (7) the axial force has been defined as follows:

$$S_i = -E_i A_i \left[\frac{dU_i(x)}{dx} + \frac{1}{2}\left(\frac{dW_i(x)}{dx}\right)^2\right] \quad (i = 1,2) \qquad (8)$$

The axial displacement after mathematical operations on equation (8) is expressed by the following formula:

$$U_i(x) = -\frac{S_i}{E_i A_i} x - \frac{1}{2}\int\left(\frac{dW_i(x)}{dx}\right)^2 dx + K_i \quad (i = 1,2) \qquad (9)$$

where K_i is the integration constant.

The equation (6) after introducing (8) has the following form:

$$E_i J_i \frac{d^4 W_i(x)}{dx^4} + S_i \frac{d^2 W_i(x)}{dx^2} = 0 \quad (i = 1,2) \qquad (10)$$

The geometrical boundary conditions for the considered system are as follows:

$$W_1(0) = W_2(0) = W_1(l) = W_2(l) = 0 \qquad \left.\frac{dW_1(x)}{dx}\right|_{x=0,\,l} = \left.\frac{dW_2(x)}{dx}\right|_{x=0,\,l}$$

$$U_1(0) = 0, \quad U_2(0) = -\tilde{d}\left.\frac{dW_1(x)}{dx}\right|_{x=0} \qquad U_1(l) = U_2(l) + \tilde{d}\left.\frac{dW_1(x)}{dx}\right|_{x=l} \qquad (11a\text{-}i)$$

Introducing geometrical boundary conditions into variational equation leads to the following natural boundary conditions:

$$E_1 J_1 \left. \frac{d^2 W_1(x)}{dx^2} \right|_{x=0} + E_2 J_2 \left. \frac{d^2 W_2(x)}{dx^2} \right|_{x=0} = S_2 \tilde{d} \mp P \tilde{e}_1 \tag{12a}$$

$$E_1 J_1 \left. \frac{d^2 W_1(x)}{dx^2} \right|_{x=l} + E_2 J_2 \left. \frac{d^2 W_2(x)}{dx^2} \right|_{x=l} = S_2 \tilde{d} \mp P \tilde{e}_2 \tag{12b}$$

$$S_1 + S_2 - P = 0 \tag{12c}$$

The results of numerical calculations have been presented in the non-dimensional form on the basis of the following relations:

$$\xi = \frac{x}{l}, \qquad w_i(\xi) = \frac{W_i(x)}{l}, \qquad u_i(\xi) = \frac{U_i(x)}{l}, \qquad \lambda_i = \frac{A_i l^2}{J_i},$$

$$r_m = \frac{E_2 J_2}{E_1 J_1}, \qquad k_i = \frac{S_i}{E_i J_i} l^2, \qquad e_i = \frac{\tilde{e}_i}{l}, \qquad d = \frac{\tilde{d}}{l}, \qquad p = \frac{P l^2}{E_1 J_1 + E_2 J_2}$$

(13a-i)

2. Results of numerical calculation

The first step in numerical calculation project was to investigate the influence of applied external load on longitudinal forces distribution. The results are presented in Figures 3 and 4.

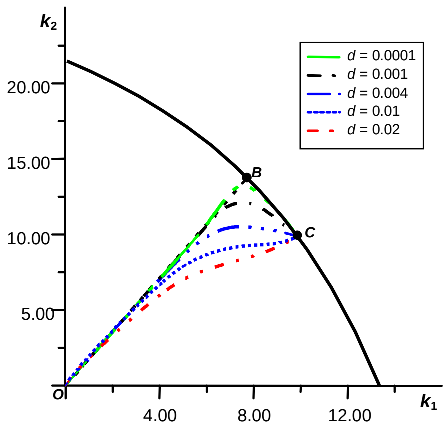

Fig. 2. Relation between axial forces under centrically applied external load and with different distance *d* value. Other parameters: $r_m = 0.48$, $e_1 = e_2 = 0$

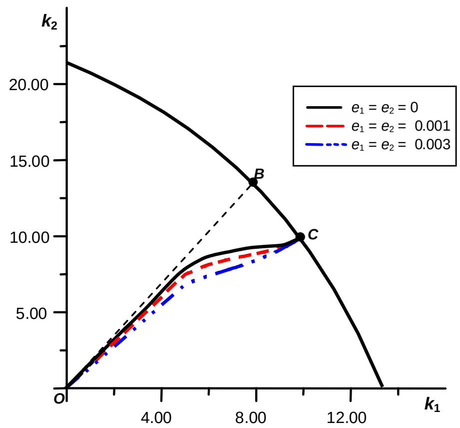

Fig. 3. Relation between axial forces under eccentrically applied external load. Other parameters: $r_m = 0.48$, $d = 0.01$

Figure 2 presents the case when external load is applied centrically. When the distance *d* is equal to zero the longitudinal forces in rods are changing along the *OB* line. In the *B* point (the bifurcation point) column losses the rectilinear form of static equilibrium and the relation between longitudinal forces changes along the buckling envelope up to critical load point *C*. Changing the distance *d* value causes deviation of internal forces relation curve from *OB* line and *BC* curve. The greater *d* value, the shorter linear relation between longitudinal forces. Independently from the distance between axes of each rod the critical load value is constant.

Results presented in Figure 3 describes the case when column is loaded by eccentrically applied load. When the distance between axes of rods is constant and value of eccentricity is increasing the internal forces relation curve deviates from *OB* line. The greater value of e_1, e_2 the major deviation of internal forces relation curve and reduction of force in second rod takes place. It was concluded that control of axial force value in each rod can be determined by eccentricity and distance *d* value, simultaneously the way that column transfers load can be controlled. During numerical calculations results analysis the statement was made that independently from the configuration of the column the critical load is constant.

In Figures 4-7 an influence of eccentrically applied external load and different distance *d* value between axes of rods on transversal displacements and bending moments distribution have been presented. The illustrated results of numerical calculation have been obtained for given value of external load *p*.

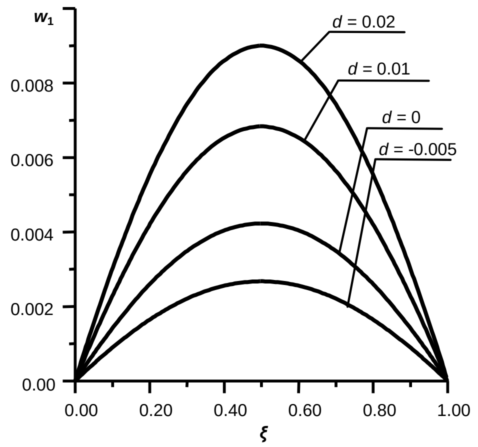

Fig. 4. Influence of distance *d* on transversal displacement of main rod. Other parameters: $r_m = 0.48$, $e_1 = e_2 = 0.003$, $p = 0.5\ p_{cr}$

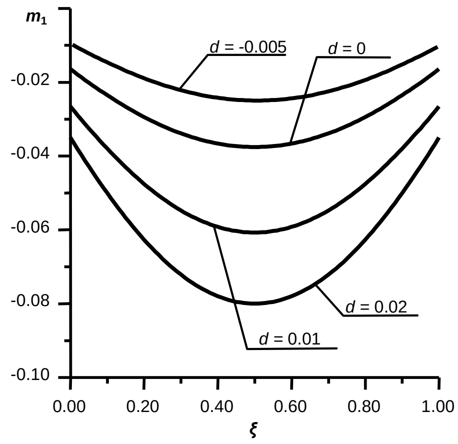

Fig. 5. Influence of distance *d* on bending moments of main rod. Other parameters: $r_m = 0.48$, $e_1 = e_2 = 0.003$, $p = 0.5\ p_{cr}$

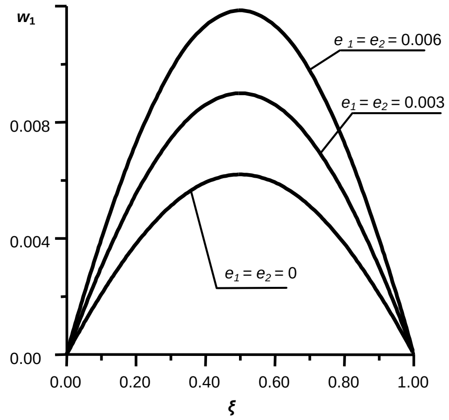

Fig. 6. Influence of eccentricity of external load on transversal displacement of main rod. Other parameters: $r_m = 0.48$, $d = 0.02$, $p = 0.5\ p_{cr}$

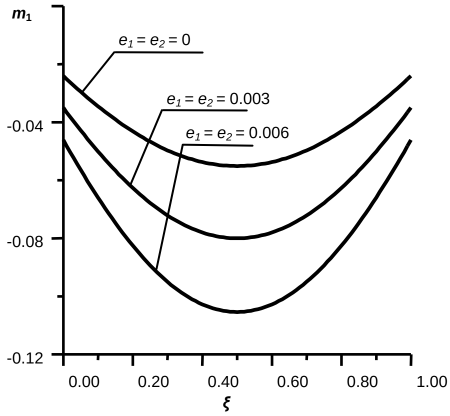

Fig. 7. Influence of eccentricity of external load on bending moments of main rod. Other parameters: $r_m = 0.48$, $d = 0.02$, $p = 0.5\ p_{cr}$

When the column is loaded with eccentricities and the distance *d* is varying (Figs. 4 and 5) it has been concluded that by the change of *d* parameter the control of transversal displacements and bending moments may be obtained. The localization of the stiffening element integrated with the main rod in order to reduce investigated parameters highly depends on eccentricity value and its direction. The statement has been made that there exists a parameter *d* value at which transversal displacements and bending moments may be reduced to zero.

In Figures 6 and 7 the influence of eccentrically applied load on transversal displacement and bending moments have been presented. After analysis of the results of numerical calculation it has been concluded that regardless from distance *d* value, eccentrically applied load causes the increase of investigated parameters. The greater eccentricity the greater value of transversal displacements and bending moments.

Conclusions

In this paper the influence of eccentrically applied external load with constant line of action on transversal displacements, bending moments and longitudinal force distribution of a column consisted of two rods have been presented. The eccentricity may result from imprecise manufacturing or improper assembling processes of supporting and loading heads. After series of numerical calculations and analysis of the results, it has been concluded that regardless of eccentricity by the change of value of the distance *d* between axes of rods the column may be straighten up (smaller value of transversal displacements and bending moments).

If the external load is applied on eccentricities e_1, e_2 the displacements and bending moments are changing (increase or decrease of value of the investigated parameters) depending on eccentricity value. It has been concluded that regardless of distance *d* and eccentricity value the critical load of the column is constant. The longitudinal force in each rod, transversal displacement and bending moment value may be controlled by the distance *d* regardless from eccentricity value.

References

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