NUMERICAL MODELLING OF TISSUE HEATING BY MEANS OF THE ELECTROMAGNETIC FIELD

***Ewa Majchrzak*[1, 2], *Marek Paruch*[1]**

[1] *Department of Strength of Materials and Computational Mechanics*
Silesian University of Technology, Poland
[2] *Institute of Mathematics,Czestochowa University of Technology*
ewa.majchrzak@polsl.pl, marek.paruch@polsl.pl

Abstract. Electromagnetic field induced by two external electrodes and temperature field resulting from electrodes action in 3D domain of biological tissue is considered. External electric field causes the heat generation in tissue domain. The distribution of electric potential in domain considered is described by the Laplace equation, while the temperature field is described by the Pennes equation. These problems are coupled by source function being the additional component in Pennes equation and resulting from the electric field action. The boundary element method is applied to solve the coupled problem connected with the biological tissue heating. In the final part of the paper the examples of computations are shown.

1. Governing equations

In Figure 1 a typical radio frequency (RF) hyperthermia system is shown [1]. The mathematical model of the process analyzed consists of two parts [1-3]. The electric part concerns the Laplace equation to obtain the electric field distribution. The thermal part is connected with the bioheat transfer equation to obtain the temperature distribution. In the bioheat transfer equation the additional source term associated with the heat generation caused by electric field distribution appears.

The potential inside the tissue is described by the Laplace equation

$$(x, y, z) \in \Omega: \quad \nabla\left[\varepsilon(x, y, z)\nabla\varphi(x, y, z)\right] = 0 \tag{1}$$

where $\varepsilon(x, y, z)$ [$C^2/(Nm^2)$] is the dielectric permittivity of tissue.
On the external surface of tissue being in a contact with the electrodes the following condition is given

$$\begin{aligned} (x, y, z) \in \Gamma_1: &\quad \varphi(x, y, z) = U \\ (x, y, z) \in \Gamma_2: &\quad \varphi(x, y, z) = -U \end{aligned} \tag{2}$$

where U [V] is the electric potential of the electrode relative to the ground.

On the remaining external boundary of the tissue the ideal electric isolation is assumed:

$$-\varepsilon \frac{\partial \varphi(x, y, z)}{\partial n} = 0 \tag{3}$$

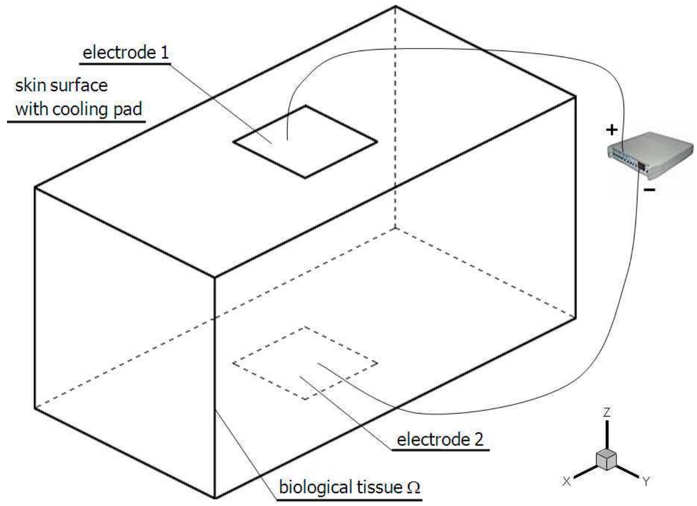

Fig. 1. Action of electric field on biological tissue

The electric field inside the tissue is described by equation

$$\mathbf{E}(x, y, z) = -\nabla \varphi(x, y, z) = -\begin{bmatrix} \dfrac{\partial \varphi(x, y, z)}{\partial x} \\ \dfrac{\partial \varphi(x, y, z)}{\partial y} \\ \dfrac{\partial \varphi(x, y, z)}{\partial z} \end{bmatrix} \tag{4}$$

Heat generation Q [W/m^3] due to the electromagnetic dissipated power in tissue depends on the conductivity σ [S/m] and the electric field **E** [2]

$$Q(x, y, z) = \frac{\sigma \left| \mathbf{E}(x, y, z) \right|^2}{2} = \frac{\sigma}{2}\left[\left(\frac{\partial \varphi(x, y, z)}{\partial x} \right)^2 + \left(\frac{\partial \varphi(x, y, z)}{\partial y} \right)^2 + \left(\frac{\partial \varphi(x, y, z)}{\partial z} \right)^2 \right] \tag{5}$$

The temperature field in the domain considered is described by the Pennes equation [1, 4, 5]

$$(x,\,y,\,z)\in\Omega:\;\;\lambda\nabla^2 T(x,\,y,\,z)+G_B c_B\left[T_B-T(x,\,y,\,z)\right]+Q_{met}+Q_e(x,\,y,\,z)=0 \tag{6}$$

where T denotes the temperature, λ [W/(mK)] is the thermal conductivity, G_B [1/s] is the perfusion rate, c_B [J/(m^{3}K)] is the volumetric specific heat of blood, T_B is the supplying arterial blood temperature which is treated as a constant, Q_{met} is the metabolic heat source.

At the $\{x, y\}$ and $\{x, z\}$ surfaces (skin surface - c.f. Figure 1) of tissue domain the convection condition is assumed $q(x,\,y,\,z)=\alpha\left[T(x,\,y,\,z)-T_w\right]$, where α_w [W/(m^{2}K)] is the heat transfer coefficient between the skin surface and the cooling water, T_w is the cooling water temperature. On the remaining surfaces of tissue the adiabatic condition can be taken into account: $-\lambda\partial T(x, y, z)/\partial n = 0$.

2. Boundary element method - electric field

The boundary integral equation corresponding to the equation (1) can be expressed as [6-8]

$$\begin{gathered}B(\xi_1,\,\xi_2,\,\xi_3)\varphi(\xi_1,\,\xi_2,\,\xi_3)+\iint_\Gamma \psi(x,\,y,\,z)\varphi^*(\xi_1,\,\xi_2,\,\xi_3,\,x,\,y,\,z)\,\mathrm{d}\Gamma=\\ \iint_\Gamma \varphi(x,\,y,\,z)\psi^*(\xi_1,\,\xi_2,\,\xi_3,\,x,\,y,\,z)\,\mathrm{d}\Gamma\end{gathered} \tag{7}$$

where (ξ_1, ξ_2, ξ_3) is the observation point, the coefficient $B(\xi_1, \xi_2, \xi_3)$ is dependent on the location of source point (ξ_1, ξ_2, ξ_3), $\psi(x,y,\,z)=-\varepsilon\partial\varphi(x,y,\,z)/\partial n$.

Fundamental solution of the problem discussed has the following form

$$\varphi^*(\xi_1,\,\xi_2,\,\xi_3,x,\,y,\,z)=\frac{1}{4\pi\varepsilon r} \tag{8}$$

where r is the distance between points $(\xi_1,\,\xi_2,\,\xi_3)$ and $(x,\,y,\,z)$. Differentiating the function $\varphi^*(\xi_1,\,\xi_2,\,\xi_3,\,x,\,y,\,z)$ with respect to the outward normal $\mathbf{n}=[\cos\alpha,\,\cos\beta,\,\cos\gamma]$ the function $\psi^*(\xi_1,\,\xi_2,\,\xi_3,\,x,\,y,\,z)$ is obtained

$$\psi^*(\xi_1,\,\xi_2,\,\xi_3,\,x,\,y,\,z)=-\varepsilon\frac{\partial\varphi^*(\xi_1,\,\xi_2,\,\xi_3,\,x,\,y,\,z)}{\partial n}=\frac{d}{4\pi r^3} \tag{9}$$

where

$$d=(x-\xi_1)\cos\alpha+(y-\xi_2)\cos\beta+(z-\xi_3)\cos\gamma \tag{10}$$

The boundary of the domain is divided into N boundary elements. For constant boundary elements it is assumed that

$$(x,\,y,\,z)\in\Gamma_j:\begin{cases}\varphi(x,\,y,\,z)=\varphi(x,\,y,\,z)=\varphi_j\\ \psi(x,\,y,\,z)=\psi(x,\,y,\,z)=\psi_j\end{cases} \tag{11}$$

and then one obtains the following approximation of equation (7)

$$\sum_{j=1}^{N}G_{ij}\psi_j=\sum_{j=1}^{N}H_{ij}\varphi_j,\quad i=1,2,...,N \tag{12}$$

where

$$G_{ij}=\iint_{\Gamma_j}\varphi^*(\xi_{1i},\,\xi_{2i},\,\xi_{3i},\,x,\,y,\,z)\,\mathrm{d}\Gamma_j \tag{13}$$

and (for $i\neq j$)

$$\hat{H}_{ij}=\iint_{\Gamma_j}\psi^*(\xi_{1i},\,\xi_{2i},\,\xi_{3i},\,x,\,y,\,z)\,\mathrm{d}\Gamma_j \tag{14}$$

while $H_{ii}=-\sum_{\substack{j=1\\ j\neq i}}^{N}H_{ij},\quad i=1,\,2,\,...N$.

The system of equations (12) can be written in the matrix form

$$\mathbf{G}\boldsymbol{\psi}=\mathbf{H}\boldsymbol{\varphi} \tag{15}$$

This system allows to determine the 'missing' boundary values of functions $\varphi_j,\,\psi_j$. Next, the values of function φ at the internal points $(\xi_{1i},\,\xi_{2i},\,\xi_{3i})$ can be determined using the formula

$$\varphi_i=\sum_{j=1}^{N}H_{ij}\varphi_j-\sum_{j=1}^{N}G_{ij}\psi_j=,\quad i=N+1,N+2,...,N+L \tag{16}$$

It should be pointed out that in order to determine the electric field inside tissue (equation (5)) the partial derivatives $\partial\varphi_e(x,\,y,\,z)/\partial x$, $\partial\varphi_e(x,\,y,\,z)/\partial y$, $\partial\varphi_e(x,\,y,\,z)/\partial z$ must be known. One of the possibilities is the application of equation (7) for internal nodes $(\xi_1,\,\xi_2,\,\xi_3)$ $\left(B(\xi_1,\,\xi_2,\,\xi_3)=1\right)$ and then

$$\frac{\partial\varphi(\xi,\eta)}{\partial\xi_1}=\iint_\Gamma \varphi(x,y,z)\frac{\partial\psi^*(\xi_1,\xi_2,\xi_3,x,y,z)}{\partial\xi_1}\mathrm{d}\Gamma - \iint_\Gamma \psi(x,y,z)\frac{\partial\varphi^*(\xi_1,\xi_2,\xi_3,x,y,z)}{\partial\xi_1}\mathrm{d}\Gamma \tag{17}$$

$$\frac{\partial\varphi(\xi_1,\xi_2,\xi_3)}{\partial\xi_2}=\iint_\Gamma \varphi(x,y,z)\frac{\partial\psi^*(\xi_1,\xi_2,\xi_3,x,y,z)}{\partial\xi_2}\mathrm{d}\Gamma - \iint_\Gamma \psi(x,y,z)\frac{\partial\varphi^*(\xi_1,\xi_2,\xi_3,x,y,z)}{\partial\xi_2}\mathrm{d}\Gamma \tag{18}$$

and

$$\frac{\partial\varphi(\xi_1,\xi_2,\xi_3)}{\partial\xi_3}=\iint_\Gamma \varphi(x,y,z)\frac{\partial\psi^*(\xi_1,\xi_2,\xi_3,x,y,z)}{\partial\xi_3}\mathrm{d}\Gamma - \iint_\Gamma \psi(x,y,z)\frac{\partial\varphi^*(\xi_1,\xi_2,\xi_3,x,y,z)}{\partial\xi_3}\mathrm{d}\Gamma \tag{19}$$

where

$$\frac{\partial\varphi^*}{\partial\xi_1}=\frac{x-\xi_1}{4\pi\varepsilon r^3},\quad \frac{\partial\varphi^*}{\partial\xi_2}=\frac{y-\xi_2}{4\pi\varepsilon r^3},\quad \frac{\partial\varphi^*}{\partial\xi_3}=\frac{z-\xi_3}{4\pi\varepsilon r^3} \tag{20}$$

and

$$\begin{aligned}\frac{\partial\psi^*}{\partial\xi_1}&=\frac{1}{4\pi}\left[\frac{3(x-\xi_1)d}{r^5}-\frac{\cos\alpha}{r^3}\right],\\ \frac{\partial\psi^*}{\partial\xi_2}&=\frac{1}{4\pi}\left[\frac{3(y-\xi_2)d}{r^5}-\frac{\cos\beta}{r^3}\right],\\ \frac{\partial\psi^*}{\partial\xi_3}&=\frac{1}{4\pi}\left[\frac{3(z-\xi_3)d}{r^5}-\frac{\cos\gamma}{r^3}\right]\end{aligned} \tag{21}$$

Applying previously presented discretization of the boundary of domain, numerical calculations of partial derivatives are not difficult to obtain. These derivatives are determined at the internal nodes.

3. Boundary element method - temperature field

The Pennes equation (6) can be written in the form

$$(x,y,z)\in\Omega:\quad \lambda\nabla^2 T(x,y,z)+Q(x,y,z)=0 \tag{22}$$

where

$$Q(x, y, z) = Q_{perf} + Q_{met} + Q_e(x, y, z) \tag{23}$$

The boundary integral equations corresponding to the equations (22) can be expressed as follows [6-8]

$$\begin{aligned} B(\xi_1, \xi_2, \xi_3) T(\xi_1, \xi_2, \xi_3) + \iint_\Gamma q(x, y, z) T^*(\xi_1, \xi_2, \xi_3, x, y, z) \mathrm{d}\Gamma = \\ \iint_\Gamma T(x, y, z) q^*(\xi_1, \xi_2, \xi_3, x, y, z) \mathrm{d}\Gamma + \\ \iiint_\Omega Q(x, y, z) T^*(\xi_1, \xi_2, \xi_3, x, y, z) \mathrm{d}\Omega \end{aligned} \tag{24}$$

where

$$T^*(\xi_1, \xi_2, \xi_3, x, y, z) = \frac{1}{4\pi\lambda r} \tag{25}$$

and

$$q^*(\xi_1, \xi_2, \xi_3, x, y, z) = -\lambda \frac{\partial T^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial n} = \frac{d}{4\pi r^3} \tag{26}$$

while $q(x, y, z) = -\lambda \partial T(x, y, z) / \partial n$.

To solve the equations (24), not only the boundary but also the interior of the domains considered should be discretized.

For constant boundary elements and constant internal cells one obtains the following systems of equations

$$\sum_{j=1}^{N} W_{ij} q_j = \sum_{j=1}^{N} Z_{ij} T_j + \sum_{l=1}^{L} P_{il} Q_l, \quad i = 1, 2, ..., N \tag{27}$$

where

$$W_{ij} = \iint_{\Gamma_j} T^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z) \mathrm{d}\Gamma_j \tag{28}$$

and

$$\begin{aligned} Z_{ij} &= \iint_{\Gamma_j} q^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z) \mathrm{d}\Gamma_j, \quad i \neq j \\ Z_{ii} &= -\sum_{\substack{j=1 \\ j \neq i}}^{N} Z_{ij}, \quad i = 1, 2, ...N \end{aligned} \tag{29}$$

while

$$P_{il} = \iiint_{\Omega_l} T^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z) \mathrm{d}\Omega_l \tag{30}$$

So, the system of equations (27) can be written in the matrix form

$$\mathbf{Wq} = \mathbf{ZT} + \mathbf{PQ} \tag{31}$$

The remaining boundary conditions should be introduced to the system of equations (31). The solution of (31) allows one to calculate the 'missing' boundary temperatures T_j and heat fluxes q_j. Next, the temperatures at the internal nodes are calculated by means of formula

$$T_i = \sum_{j=1}^{N} Z_{ij} T_j - \sum_{j=1}^{N} W_{ij} q_j + \sum_{l=1}^{L} P_{il} Q_l \tag{32}$$

In the paper the external boundary of the tissue has been divided into 832 constant boundary elements. To solve the Pennes equation in the interior Ω, the L = 1024 internal cells have been distinguished.

4. Results of computations

The 3D domain of dimensions 0.04×0.04×0.08 m has been considered. The heating area is described as $\{0.032 \le x \le 0.048, 0.012 \le y \le 0.028, z = 0 \text{ m}\}$, $\{0.032 \le x \le 0.048, 0.012 \le y \le 0.028, z = 0.04 \text{ m}\}$ and the voltage applied on these surfaces is 15 V and −15 V, respectively. The following parameters have been assumed: thermal conductivity of tissue λ = 0.5 W/(mK), perfusion heat source Q_{perf} = −200 W/m^3, metabolic heat source Q_{met} = 420 W/m^3 [2]. On the skin surface the Robin condition has been accepted (α_w = 45 W/m^{2}K, T_w = 20°C). The electric conductivity equals to σ = 0.4 S/m, dielectric permittivity: $\varepsilon = 2000\varepsilon_0$ (ε_0 = = $8.85 \cdot 10^{-12}$ C^2/(Nm^2)).

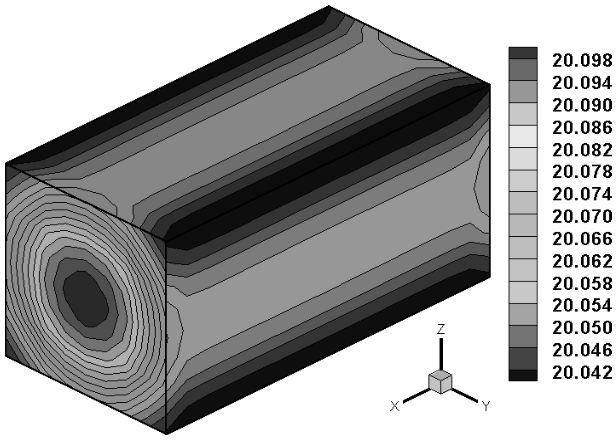

Fig. 2. Temperature distribution in tissue without electric field

Figure 2 illustrates the temperature distribution in the tissue without electric field influence. The distribution of electric field is shown in Figure 3, while Figure 4 illustrates the temperature field in the tissue subjected to the electric field.

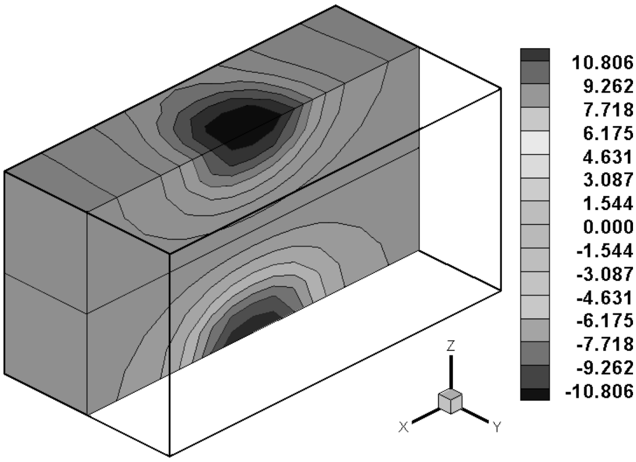

Fig. 3. Electric field distribution

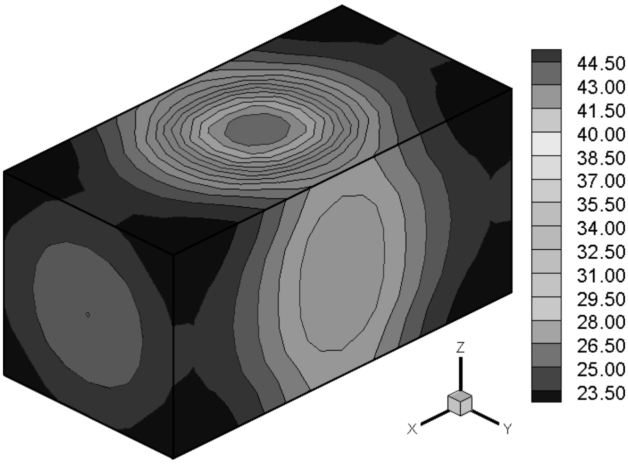

Fig. 4. Temperature distribution subjected to the electric field

Conclusions

Boundary element method has been applied to solve the coupled problem connected with the biological tissue heating. The 3D mathematical model basing on the Pennes equation supplemented by the equation determining the electric field due to the external electrodes action has been considered.

The next step of investigations will concern the temperature field determination in the tissue with a tumor subjected to the action of external electrodes (3D problem) and it will be the extension of problems presented in [9], where 2D model has been considered.

Acknowledgement

This work was supported by Grant No N N501 3667 34 from the Polish Ministry of Science and Higher Education.

References

[1] Lv Y.G., Deng Z.S., Liu J., 3D numerical study on the induced heating effects of embedded micro/nanoparticles on human body subject to external medical electromagnetic field, IEEE Transactions on Nanobioscience 2005, 4, 4, 284-294.

[2] Majchrzak E., Dziatkiewicz G., Paruch M., The modelling of heating a tissue subjected to external electromagnetic field, Acta of Bioengineering and Biomechanics 2008, 10, 2, 29-37.

[3] Majchrzak E., Drozdek J., Paruch M., Heating of tissue by means of the electric field - numerical model basing of the BEM, Scientific Research of the Institute of Mathematics and Computer Science of Czestochowa University of Technology 2008, 1(7), 99-110.

[4] Liu J., Xu L.X., Boundary information based diagnostics on the thermal states of biological bodies, Int. Journal of Heat and Mass Transfer 2000, 43, 2827-2839.

[5] Majchrzak E., Numerical modelling of bio-heat transfer using the boundary element method, Journal of Theoretical and Applied Mechanics 1998, 2, 36, 437-455.

[6] Majchrzak E., Boundary element method in heat transfer, Publ. of the Czestochowa University of Technology, Czestochowa 2001 (in Polish).

[7] Brebbia C.A., Dominguez J., Boundary Elements, An Introductory Course, Computational Mechanics Publications, McGraw-Hill Book Company, London 1992.

[8] Jabłoński P., Boundary element method in the analysis of electromagnetic field, Publ. of the Czestochowa University of Technology, Czestochowa 2003 (in Polish).

[9] Majchrzak E., Paruch M., Numerical modelling of temperature field in the tissue with a tumor subjected to the action of two external electrodes, Scientific Research of the Institute of Mathematics and Computer Science, Czestochowa University of Technology 2009, 1(8), 137-146.