ON SOME PROPERTY OF THE MODIFIED POWER OF AN ALGEBRAIC NUCLEUS

Grażyna Ciecierska

University of Warmia and Mazury in Olsztyn, grac@matman.uwm.edu.pl

Abstract. Given two pairs $(\varXi, X)$, (Ω, Y) of conjugate linear spaces, we show that the modified power of an algebraic nucleus preserves (Ω, X)- weak continuity of multilinear functionals. An application of the result in the determinant theory is also considered.

Introduction

Algebraic nuclei play an essential role in the theory of determinant systems [1--4]. They allow to construct determinant systems for nuclear perturbations of Fredholm operators, i.e. if $(D_n)_{n\in N\cup\{0\}}$ is a determinant system for Fredholm operator $A \in op(\Omega \to \varXi, X \to Y)$, then one can obtain effective formulae for determinant system for $A + T_F$, where $F \in an(\Omega \to \varXi, X \to Y)$. The terms D_n, $n \in N \cup \{0\}$, are, in particular, bi-skew symmetric multilinear (Ω, X)- weakly continuous functionals. It is well known that functionals $F^{pk} D_{n+k}$, $k \in N$, are bi-skew symmetric. We shall prove that the modified power of an algebraic nucleus transforms (Ω, X)- weakly continuous functionals into (Ω, X)- weakly continuous functionals. Therefore, in view of the result, $F^{pk} D_{n+k}$ are also (Ω, X)- weakly continuous functionals.

1. Terminology and notation

Let $(\varXi, X)$, (Ω, Y) denote pairs of conjugate linear spaces over the same real or complex field K. A bilinear functional $A: \Omega \times X \to K$, whose value at a point $(\omega, x) \in \Omega \times X$ is denoted by $\omega A x$, satisfying the condition $\omega A x = \omega(Ax) = (\omega A)x$, where $Ax \in Y$ and $\omega A \in \varXi$, is called $(\varXi, Y)$- *operator on* $\Omega \times X$. Let $op(\Omega \to \varXi, X \to Y)$ denotes the space of $(\varXi, Y)$- operators on $\Omega \times X$. Each $A \in op(\Omega \to \varXi, X \to Y)$ can simultaneously be interpreted as a linear operator $A: X \to Y$ and as a linear operator $A: \Omega \to \varXi$. For fixed non-zero elements

$x_0 \in X$, $\omega_0 \in \Xi$, $x_0 \cdot \omega_0$ denotes the bilinear functional on $\Xi \times Y$, defined by $\xi(x_0 \cdot \omega_0)y = \xi x_0 \cdot \omega_0 y$ for $(\xi, y) \in \Xi \times Y$. A linear functional $F : op(\Xi \to \Omega, Y \to X) \to K$ is called an *algebraic nucleus*, if there exists $T_F \in op(\Omega \to \Xi, X \to Y)$ such that $F(x \cdot \omega) = \omega T_F x$ for $(\omega, x) \in \Omega \times X$. The operator T_F is called a *nuclear operator determined by F.* The space of all algebraic nuclei on $op(\Xi \to \Omega, Y \to X)$ is denoted by $an(\Omega \to \Xi, X \to Y)$. The value of a $(\mu + m)$-linear functional $D : \Xi^{\mu} \times Y^{m} \to K$, $\mu, m \in N \cup \{0\}$, at a point $(\xi_1, \ldots, \xi_{\mu}, y_1, \ldots, y_m) \in \Xi^{\mu} \times Y^{m}$ is denoted by $D\begin{pmatrix} \xi_1, \ldots, & \xi_{\mu} \\ y_1, \ldots, & y_m \end{pmatrix}$. A $(\mu + m)$-linear functional D on $\Xi^{\mu} \times Y^{m}$ is said to be *bi-skew symmetric* if it is skew symmetric in variables from both Ξ, and Y. A $(\mu + m)$-linear functional $D : \Xi^{\mu} \times Y^{m} \to \mathrm{K}$ is said to be (Ω, X)- *weakly continuous functional on* $\Xi^{\mu} \times Y^{m}$, if for any fixed elements $\xi_1, \ldots, \xi_{i-1}, \xi_{i+1}, \ldots, \xi_{\mu} \in \Xi$ $(i = 1, \ldots, \mu)$, $y_1, \ldots, y_m \in Y$ there exists an element $x_i \in X$ such that $\xi x_i = D\begin{pmatrix} \xi_1, \ldots, & \xi_{i-1}, \xi, \xi_{i+1}, \ldots, & \xi_{\mu} \\ y_1, & \ldots, & y_m \end{pmatrix}$ for every $\xi \in \Xi$ and for any fixed elements $\xi_1, \ldots, \xi_{\mu} \in \Xi$, $y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_m \in Y$ $(j = 1, \ldots, m)$ there exists an element $\omega_j \in \Omega$ such that $\omega_j y = D\begin{pmatrix} \xi_1, & \ldots, & \xi_{\mu} \\ y_1, \ldots, & y_{j-1}, y, y_{j+1}, \ldots, & y_m \end{pmatrix}$ for every $y \in Y$.

For $F \in an(\Omega \to \Xi, X \to Y)$ and a bi-skew symmetric (Ω, X)-weakly continuous functional D on $\Xi^{\mu} \times Y^{m}$, interpreted as a function of variables ξ_1, y_1 only, we define a $(\mu + m - 2)$-linear functional $F_{\xi_1 y_1} D$ on $\Xi^{\mu-1} \times Y^{m-1}$ by the formula

$$\left(F_{\xi_1 y_1} D\right)\begin{pmatrix} \xi_2, \ldots, & \xi_{\mu} \\ y_2, \ldots, & y_m \end{pmatrix} = F(A_1), \text{ where } \xi_1 A_1 y_1 = D\begin{pmatrix} \xi_1, & \xi_2, \ldots, & \xi_{\mu} \\ y_1, & y_2, \ldots, & y_m \end{pmatrix}$$

for $\xi_1 \in \Xi$, $y_1 \in Y$.

If $k = min\{\mu, m\}$, then assuming that $F_{\xi_{k-1} y_{k-1}} F_{\xi_{k-2} y_{k-2}} \ldots F_{\xi_1 y_1} D$ is (Ω, X)-weakly continuous functional and interpreting it as a function of variables ξ_k, y_k only, we define a $(\mu + m - 2k)$-linear functional $F_{\xi_k y_k} F_{\xi_{k-1} y_{k-1}} \ldots F_{\xi_1 y_1} D$ on $\Xi^{\mu-k} \times Y^{m-k}$ by $\left(F_{\xi_k y_k} F_{\xi_{k-1} y_{k-1}} \ldots F_{\xi_1 y_1} D\right)\begin{pmatrix} \xi_{k+1}, \ldots, & \xi_{\mu} \\ y_{k+1}, \ldots, & y_m \end{pmatrix} = F(A_k)$,

where $\xi_k A_k y_k = \left(F_{\xi_{k-1} y_{k-1}} F_{\xi_{k-2} y_{k-2}} \ldots F_{\xi_1 y_1} D\right)\begin{pmatrix} \xi_k, & \xi_{k+1}, \ldots, & \xi_{\mu} \\ y_k, & y_{k+1}, \ldots, & y_m \end{pmatrix}$ for $\xi_k \in \Xi$, $y_k \in Y$.

Since for fixed $F \in an(\Omega \to \varXi, X \to Y)$ and every permutation τ of integers $1,\dots,k$, $F_{\xi_{\tau_k} y_{\tau_k}} \dots F_{\xi_{\tau_1} y_{\tau_1}} = F_{\xi_k y_k} \dots F_{\xi_1 y_1}$ [3], the common value of all $F_{\xi_{\tau_k} y_{\tau_k}} \dots F_{\xi_{\tau_1} y_{\tau_1}}$ is denoted by $\underbrace{F_{\square} \dots F_{\square}}_{k-times}$ [2]. Moreover, $F^{\square k}$ denotes the *modified k-th power of a nucleus F*, i.e. $F^{\square k} = \frac{1}{k!} \underbrace{F_{\square} \dots F_{\square}}_{k-times}$.

2. Main theorem

Given $F \in an(\Omega \to \varXi, X \to Y)$, $B \in op(\varXi \to \Omega, Y \to X)$, let $T = T_F$ and $(BT)^{(m)} = B(TB)^m$ for $m \in N \cup \{0\}$. Obviously, $(BT)^{(m)} \in op(\varXi \to \Omega, Y \to X)$. If $m = m_1 + m_2 + 1$, $m_1, m_2 \in N \cup \{0\}$, then

$$\xi(BT)^{(m)} y = F_{\xi' y'}\left(\xi'(BT)^{(m_1)} y \cdot \xi(BT)^{(m_2)} y'\right) \text{ for } (\xi, y) \in \varXi \times Y \tag{1}$$

$$\xi(BT)^{m} x = F_{\xi' x'}\left(\xi'(BT)^{m_1} x \cdot \xi(BT)^{(m_2)} x'\right) \text{ for } (\xi, x) \in \varXi \times X \tag{2}$$

$$\omega(TB)^{m} y = F_{\omega' y'}\left(\omega'(BT)^{(m_1)} y \cdot \omega(TB)^{m_2} y'\right) \text{ for } (\omega, y) \in \Omega \times Y \tag{3}$$

Lemma. Let $F \in an(\Omega \to \varXi, X \to Y)$, $B \in op(\varXi \to \Omega, Y \to X)$, $r = min\{n', m'\}$, $n', m' \in N \cup \{0\}$, $z_1, \dots, z_{n'} \in X$, $\varsigma_1, \dots, \varsigma_{m'} \in \Omega$. *If* $n \in N$ *and* $p = (p_1, \dots, p_{n+n'-r})$, $q = (q_1, \dots, q_{n+m'-r})$ *are permutations of the integers* $1, \dots, n+n'-r$ *and* $1, \dots, n+m'-r$, *respectively, then for every integer* $0 \le k \le \min\{n+n'-r, n+m'-r\}$

$$F_{\xi_1 y_1} \dots F_{\xi_k y_k}\left(\prod_{i=1}^{n-r} \xi_{p_i} B y_{q_i} \prod_{i=1}^{n'} \xi_{p_{n-r+i}} z_i \prod_{i=1}^{m'} \varsigma_i y_{q_{n-r+i}}\right) = \tag{4}$$

$$= c_k \prod_{i=1}^{n_k} \xi_{\sigma_i} (BT)^{(m_i)} y_{\tau_i} \prod_{i=1}^{n+n'-r-k-n_k} \xi_{\sigma_{n_k+i}} (BT)^{k_i} z_i \prod_{i=1}^{n+m'-r-k-n_k} \varsigma_i (TB)^{l_i} y_{\tau_{n_k+i}}$$

for every $(\xi_1, \dots, \xi_{n+n'-r}, y_1, \dots, y_{n+m'-r}) \in \varXi^{n+n'-r} \times Y^{n+m'-r}$, *where* $c_k \in K$, $n_k \le \min\{n+n'-r-k, n+m'-r-k\}$, $(m_i)_{i=1}^{n_k}$, $(k_i)_{i=1}^{n+n'-r-k-n_k}$, $(l_i)_{i=1}^{n+m'-r-k-n_k}$ *are finite sequences of non-negative integers,* $\sigma = (\sigma_1, \dots, \sigma_{n+n'-r-k})$, $\tau = (\tau_1, \dots, \tau_{n+m'-r-k})$ *are permutations of integers* $k+1, \dots, n+n'-r$ *and* $k+1, \dots, n+m'-r$, *respectively.*

Proof. Induction on k $\left(k=0,\ldots,\min\{n+n'-r,n+m'-r\}\right)$. Let $\left(\xi_1,\ldots,\xi_{n+n'-r},y_1,\ldots,y_{n+m'-r}\right)\in \Xi^{n+n'-r}\times Y^{n+m'-r}$. If $k=0$, then (4) holds for $c_0=1$, $n_0=n-r$, $m_i=0$ $\left(i=1,\ldots,n-r\right)$, $k_i=0$ $\left(i=1,\ldots,n'\right)$, $l_i=0$ $\left(i=1,\ldots,m'\right)$, $\sigma=p,\tau=q$. Suppose that (4) holds for k $\left(0\le k<\min\{n+n'-r,n+m'-r\}\right)$. Then

$$F_{\xi_1 y_1}\ldots F_{\xi_{k+1}y_{k+1}}\left(\prod_{i=1}^{n-r}\xi_{p_i}By_{q_i}\prod_{i=1}^{n'}\xi_{p_{n-r+i}}z_i\prod_{i=1}^{m'}\varsigma_i y_{q_{n-r+i}}\right)=$$

$$=F_{\xi_{k+1}y_{k+1}}\left(c_k\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}\right)$$

In the case: $\sigma_{i_0}=k+1$, $\tau_{i_0}=k+1$, $1\le i_0\le n_k$, we obtain

$$F_{\xi_{k+1}y_{k+1}}\left(c_k\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}\right)=$$

$$=c_kF_{\xi_{k+1}y_{k+1}}\left(\xi_{k+1}(BT)^{(m_{i_0})}y_{k+1}\right)\prod_{\substack{i=1\\ i\ne i_0}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}$$

$$\cdot\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}=$$

$$=c_{k+1}\prod_{\substack{i=1\\ i\ne i_0}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}$$

where $c_{k+1}=c_kF\left[(BT)^{(m_{i_0})}\right]$

If $\sigma_{i_1}=k+1$, $\tau_{i_2}=k+1$, $1\le i_1,i_2\le n_k$, $i_1\ne i_2$, then according to (1)

$$F_{\xi_{k+1}y_{k+1}}\left(c_k\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}\right)=$$

$$=c_kF_{\xi_{k+1}y_{k+1}}\left(\xi_{k+1}(BT)^{(m_{i_1})}y_{\tau_{i_1}}\cdot\xi_{\sigma_{i_2}}(BT)^{(m_{i_2})}y_{k+1}\right)\prod_{\substack{i=1\\ i\ne i_1,i_2}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}$$

$$\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}=$$

$$=c_{k+1}\prod_{\substack{i=1\\ i\ne i_1,i_2}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}},$$

where $c_{k+1}=c_k\xi_{\sigma_{i_2}}(BT)^{(m_{i_1}+m_{i_2}+1)}y_{\tau_{i_1}}$.

If $\sigma_{i_1}=k+1$, $\tau_{n_k+i_2}=k+1$, $1\le i_1\le n_k$, $1\le i_2\le n+m'-r-k-n_k$, then by (3)

$$F_{\xi_{k+1}y_{k+1}}\left(c_k\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}\right)=$$

$$=c_kF_{\xi_{k+1}y_{k+1}}\left(\xi_{k+1}(BT)^{(m_{i_1})}y_{\tau_{i_1}}\cdot\varsigma_{i_2}(TB)^{l_{i_2}}y_{k+1}\right)\prod_{\substack{i=1\\i\neq i_1}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}$$

$$\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{\substack{i=1\\i\neq i_2}}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}=$$

$$=c_k\varsigma_{i_2}(TB)^{m_{i_1}+l_{i_2}+1}y_{\tau_{i_1}}\prod_{\substack{i=1\\i\neq i_1}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{\substack{i=1\\i\neq i_2}}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}.$$

If $\sigma_{n_k+i_1}=k+1$, $\tau_{i_2}=k+1$, $1\le i_1\le n+n'-r-k-n_k$, $1\le i_2\le n_k$, then by (2)

$$F_{\xi_{k+1}y_{k+1}}\left(c_k\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}\right)=$$

$$=c_kF_{\xi_{k+1}y_{k+1}}\left(\xi_{k+1}(BT)^{k_{i_1}}z_{i_1}\cdot\xi_{\sigma_{i_2}}(BT)^{(m_{i_2})}y_{k+1}\right)\prod_{\substack{i=1\\i\neq i_2}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}$$

$$\prod_{\substack{i=1\\i\neq i_1}}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}=$$

$$=c_{k+1}\prod_{\substack{i=1\\i\neq i_1}}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{\substack{i=1\\i\neq i_2}}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}},$$

where $c_{k+1}=c_k\varsigma_{i_2}(TB)^{m_{i_1}+l_{i_2}+1}y_{\tau_{i_1}}$.

If $\sigma_{n_k+i_1}=k+1$, $\tau_{n_k+i_2}=k+1$, $1\le i_1\le n+n'-r-k-n_k$, $1\le i_2\le n+m'-r-k-n_k$, then we obtain

$$F_{\xi_{k+1}y_{k+1}}\left(c_k\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}\right)=$$

$$=c_kF_{\xi_{k+1}y_{k+1}}\left(\xi_{k+1}(BT)^{k_{i_1}}z_{i_1}\cdot\varsigma_{i_2}(TB)^{l_{i_2}}y_{k+1}\right)\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}$$

$$\prod_{\substack{i=1\\i\neq i_1}}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{\substack{i=1\\i\neq i_2}}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}}=$$

$$= c_{k+1}\prod_{i=1}^{n_k}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{\substack{i=1\\ i\neq i_1}}^{n+n'-r-k-n_k}\xi_{\sigma_{n_k+i}}(BT)^{k_i}z_i\prod_{\substack{i=1\\ i\neq i_2}}^{n+m'-r-k-n_k}\varsigma_i(TB)^{l_i}y_{\tau_{n_k+i}},$$

where $c_{k+1} = c_k\varsigma_{i_2}(TB)^{l_{i_2}+k_{i_1}}Tz_{i_1}$.

Let $F \in an(\Omega \to \Xi, X \to Y)$. It follows from Lemma that if $(D_n)_{n\in N\cup\{0\}}$ is a determinant system for Fredholm operator $A \in op(\Omega \to \Xi, X \to Y)$ of order $r = min\{n', m'\}$, $n', m' \in N \cup \{0\}$, $B \in op(\Xi \to \Omega, Y \to X)$ is a generalized inverse of A, $\{z_1, \ldots, z_{n'}\}$, $\{\varsigma_1, \ldots, \varsigma_{m'}\}$are complete systems of solutions of equations, $Ax = 0$ and $\omega A = 0$, respectively, $n, k \in N \cup \{0\}$, then for every $0 \le l \le k$ and $(\xi_1,\ldots,\xi_{n+n'-r},\xi_{n+n'-r+l+1},\ldots,\xi_{n+n'-r+k},y_1,\ldots,y_{n+m'-r},y_{n+m'-r+l+1}\ldots,y_{n+m'-r+k}) \in \Xi^{n+n'-r+k-l}\times Y^{n+m'-r+k-l}$,
$\left(F_{\xi_{n+n'-r+1}y_{n+m'-r+1}}\ldots F_{\xi_{n+n'-r+l}y_{n+m'-r+l}}D_{n+k}\right)\begin{pmatrix}\xi_1,\ldots, & \xi_{n+n'-r},\xi_{n+n'-r+l+1},\ldots, & \xi_{n+n'-r+k}\\ y_1,\ldots, & y_{n+m'-r},y_{n+m'-r+l+1},\ldots, & y_{n+m'-r+k}\end{pmatrix}$ is a finite sum of expressions of the form

$$c_l\prod_{i=1}^{n_l}\xi_{\sigma_i}(BT)^{(m_i)}y_{\tau_i}\prod_{i=1}^{n+n'-r+k-l-n_l}\xi_{\sigma_{n_l+i}}(BT)^{k_i}z_i\prod_{i=1}^{n+m'-r+k-l-n_l}\varsigma_i(TB)^{l_i}y_{\tau_{n_l+i}}$$

where c_l is a constant, $n_l \le \min\{n+n'-r+k-l, n+m'-r+k-l\}$, $(m_i)_{i=1}^{n_l}$, $(k_i)_{i=1}^{n+n'-r+k-l-n_l}$, $(l_i)_{i=1}^{n+m'-r+k-l-n_l}$ are sequences of non-negative integers, σ, τ are permutations of integers, $1,\ldots,n+n'-r$, $n+n-r+l+1,\ldots,n+n'-r+k$ and $1,\ldots,n+m'-r, n+m'-r+l+1,\ldots,n+m'-r+k$, respectively.

Thus, in view of the above considerations, we obtain the following

Theorem. *If $F \in an(\Omega \to \Xi, X \to Y)$, $A \in op(\Omega \to \Xi, X \to Y)$ is a Fredholm operator of order $r = min\{n', m'\}$, $n', m' \in N \cup \{0\}$, and $(D_n)_{n\in N\cup\{0\}}$ is a determinant system for A, then $F^{\square k}D_{n+k}$, where $n, k \in N \cup \{0\}$, is (Ω, X)-weakly continuous functional on $\Xi^{n+n'-r}\times Y^{n+m'-r}$.*

Conclusions

We have shown that the modified power $F^{\square k}$ of an algebraic nucleus $F \in an(\Omega \to \Xi, X \to Y)$ preserves (Ω, X)-weak continuity of terms of a determinant system for a given Fredholm operator. The result can be applied to a construction of determinant systems for nuclear perturbations of Fredholm operators.

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