SOLUTION OF 2D TRANSIENT DIFFUSION PROBLEM WITH INTERVAL SOURCE FUNCTION

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Abstract. In this paper an application of the interval boundary element method for solving 2D problems with interval heat source is presented. The numerical solution of the problem discussed has been obtained on the basis of the 1st scheme of the interval boundary element method. In the final part of the paper, results of numerical computations are shown.

Introduction

Transient temperature field in two-dimensional domain Ω is described by the following energy equation [1-4]

$$x \in \Omega: \quad c\frac{\partial T(x,t)}{\partial t} = \lambda \nabla^2 T(x,t) + \tilde{Q}(x,t) \tag{1}$$

where c is the volumetric specific heat, λ is the thermal conductivity, $\tilde{Q}(x,t)$ is the interval heat source, T is the temperature, $x = \{x_1, x_2\}$ are the spatial co-ordinates and t is the time.
The equation (1) must be supplemented by the following boundary-initial conditions

$$\begin{aligned} x \in \Gamma_1: &\quad T(x,\ t) = T_b \\ x \in \Gamma_2: &\quad q(x,\ t) = -\lambda \frac{\partial T(x,\ t)}{\partial n} = q_b \\ t\ =\ 0: &\quad T(x,\ t) = T_0(x) \end{aligned} \tag{2}$$

where T_b is the known boundary temperature, $\partial T(x,\ t)/\partial n$ is the normal derivative at the boundary point x, q_b is the given boundary heat flux, T_0 is the initial temperature.

1. Interval boundary element method

At first the time grid is introduced with a certain constant time step $\Delta t = t^f - t^{f-1}$ [1-3].

If the 1st scheme of the BEM is taken into account [5, 6] then the boundary integral equation corresponding to the transition $t^{f-1} \to t^f$ is of the following form

$$B(\xi)\tilde{T}(\xi, t^f) + \frac{1}{c\rho}\int_{t^{f-1}}^{t^f}\int_{\Gamma} T^*(\xi, x, t^f, t)\,\tilde{q}(x, t^f)\,\mathrm{d}\Gamma\mathrm{d}t =$$

$$\frac{1}{c\rho}\int_{t^{f-1}}^{t^f}\int_{\Gamma} q^*(\xi, x, t^f, t)\,\tilde{T}(x, t^f)\,\mathrm{d}\Gamma\mathrm{d}t + \iint_{\Omega} T^*(\xi, x, t^f, t^{f-1})\tilde{T}(x, t^{f-1})\,\mathrm{d}\Omega + \quad (3)$$

$$\frac{1}{c\rho}\int_{t^{f-1}}\iint_{\Omega}\tilde{Q}(x, t)T^*(\xi, x, t^f, t^{f-1})\,\mathrm{d}\Omega\mathrm{d}t$$

where ξ is the point where the concentrated heat source is applied, $T^*(\xi, x, t^f, t)$ is the fundamental solution, $q^*(\xi, x, t^f, t)$ is the heat flux corresponding to the fundamental solution, $\tilde{q}(x, t) = -\lambda\partial\tilde{T}(x, t)/\partial x$ is the interval boundary heat flux, $\tilde{T}(x, t)$ is the interval temperature value.

The numerical approximation of the equation (3) leads to the system of interval equations

$$\sum_{j=1}^{N} G_{ij}\,\tilde{q}_j^f = \sum_{j=1}^{N} H_{ij}\,\tilde{T}_j^f + \sum_{l=1}^{L} P_{il}\,\tilde{T}_l^{f-1} + \sum_{l=1}^{L} Z_{il}\,\tilde{Q}_l^{f-1} \quad (4)$$

where

$$G_{ij} = \int_{\Gamma_j} g(\xi^i, x)\,\mathrm{d}\Gamma_j \quad (5)$$

$$H_{ij} = \begin{cases} \int_{\Gamma_j} h(\xi^i, x)\,\mathrm{d}\Gamma_j, & i \neq j \\ -0.5, & i = j \end{cases} \quad (6)$$

$$P_{il} = \iint_{\Omega_l} T^*(\xi, x, t^f, t^{f-1})\,\mathrm{d}\Omega_l \quad (7)$$

$$Z_{il} = \iint_{\Omega_l} g(\xi^i, x)\,\mathrm{d}\Omega_l \quad (8)$$

The system of equations (4) can be written in the matrix form

$$\mathbf{G} \cdot \tilde{\mathbf{q}}^{f} = \mathbf{H} \cdot \tilde{\mathbf{T}}^{f} + \mathbf{P} \cdot \tilde{\mathbf{T}}^{f-1} + \mathbf{Z} \cdot \tilde{\mathbf{Q}}^{f-1} \tag{9}$$

After the determining the 'missing' boundary values of $\tilde{T}$ and $\tilde{q}$, the values of the temperature $\tilde{T}$ at the internal points ξ^i for time t^f can be calculated using the formula

$$\tilde{T}_i^{f} = \sum_{j=1}^{N} H_{ij} \tilde{T}_j^{f} - \sum_{j=1}^{N} G_{ij} \tilde{q}_j^{f} + \sum_{l=1}^{L} P_{il} \tilde{T}_l^{f-1} + \sum_{l=1}^{L} Z_{il} \tilde{Q}_l^{f-1} \tag{10}$$

2. Example of computations

As an numerical example the transient diffusion process proceeding in a steel bar of square section (0.2×0.2 m) has been analyzed. On the external boundary the Dirichlet condition (T_b = 100°C), has been assumed. The symmetrical fragment of the domain considered has been taken into account. The boundary has been divided into 40 constant boundary elements, the interior has been divided into 100 constant internal cells. The following input data have been introduced: initial temperature T_0 = 500°C, interval heat source depended on the time and spatial co-ordinates $\tilde{Q}(x,t) = \left\langle 0.8 \cdot 10^6, 1.2 \cdot 10^6 \right\rangle \cdot t \left(x_1^2 + x_2^2 \right)$ [W/m²], thermal conductivity λ = 35 W/(m·K), volumetric specific heat $c = 4.875 \cdot 10^6$ J/(K m³), time step Δt = 0.5 s.

Figure 1 illustrates the cooling curves obtained at the central node of the domain considered, where dashed and solid lines denote the lower and the upper bounds of the temperature intervals, respectively.

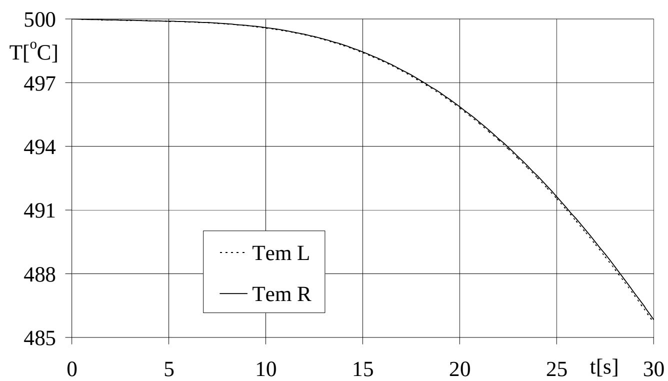

Fig. 1. Interval cooling curve at the central node

Figure 2 presents the course of the source function at the same node.

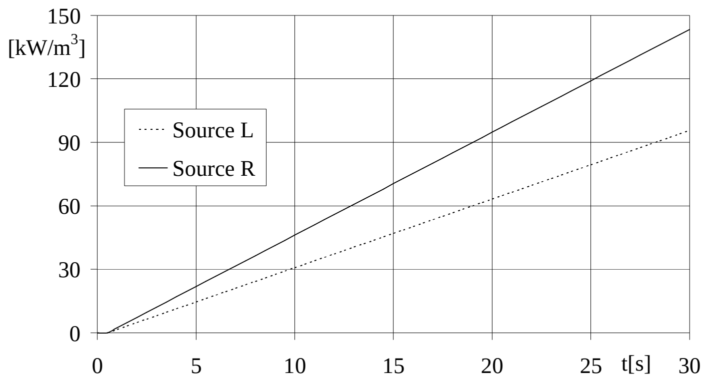

Fig. 2. The course of interval source function at the central node

The differences between upper and lower bounds of the temperature intervals are rather small for the interval source function defined like in this example.

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