

HEATING OF TISSUE BY MEANS OF THE ELECTRIC FIELD - NUMERICAL MODEL BASING ON THE BEM

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Abstract. The domain of tissue is subjected to the action of electrodes located on the skin surface. External electric field causes the heat generation in tissue domain. The distribution of electric potential in domain considered is described by the Laplace equation, while the temperature field is described by the Pennes equation. These problems are coupled by source function being the additional component in Pennes equation and resulting from the electric field action. The coupled problem is solved using the boundary element method. In the final part of the paper the examples of computations are shown.

1. Governing equations

In Figure 1 a typical radio frequency (RF) hyperthermia system is shown [1]. The mathematical model of the process analyzed consists of two parts [1, 2]. The electric part concerns the Laplace equation to obtain the electric field distribution. The thermal part is connected with the bioheat transfer equation to obtain the temperature distribution. In the bioheat transfer equation the additional source term associated with the heat generation caused by electric field distribution appears.

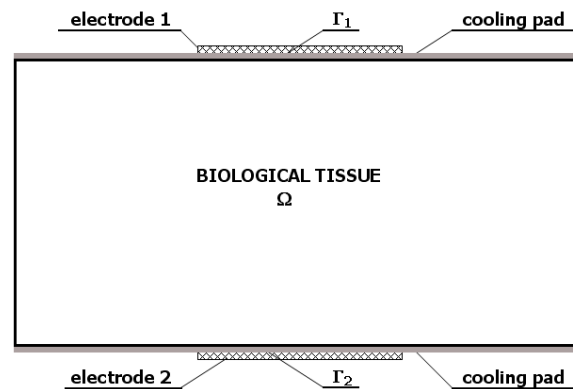


Fig. 1. Action of electric field on biological tissue

The potential inside the tissue is described by the Laplace equation

$$(x, y) \in \Omega: \nabla [\varepsilon(x, y)\varphi(x, y)] = 0 \quad (1)$$

where $\varepsilon(x, y)$ [$\text{C}^2/(\text{Nm}^2)$] is the dielectric permittivity of tissue.

On the external surface of tissue being in a contact with the electrodes the following condition is accepted:

$$\begin{aligned} (x, y) \in \Gamma_1: \varphi(x, y) &= U \\ (x, y) \in \Gamma_2: \varphi(x, y) &= -U \end{aligned} \quad (2)$$

where U [V] is the electric potential of the electrode relative to the ground.

On the remaining external boundary of the tissue the ideal electric isolation is assumed:

$$-\varepsilon \frac{\partial \varphi(x, y)}{\partial n} = 0 \quad (3)$$

The electric field inside the tissue is described by equation

$$\mathbf{E}(x, y) = -\nabla \varphi(x, y) = - \begin{bmatrix} \frac{\partial \varphi(x, y)}{\partial x} \\ \frac{\partial \varphi(x, y)}{\partial y} \end{bmatrix} \quad (4)$$

Heat generation Q [W/m^3] due to the electromagnetic dissipated power in tissue depends on the conductivity σ [S/m] and the electric field \mathbf{E} [2]

$$Q(x, y) = \frac{\sigma |\mathbf{E}(x, y)|^2}{2} = \frac{\sigma}{2} \left[\left(\frac{\partial \varphi(x, y)}{\partial x} \right)^2 + \left(\frac{\partial \varphi(x, y)}{\partial y} \right)^2 \right] \quad (5)$$

The temperature field in the domain considered is described by the Pennes equation [1, 2]

$$(x, y) \in \Omega: \lambda \nabla^2 T(x, y) + G_B c_B [T_B - T(x, y)] + Q_{met} + Q_e(x, y) = 0 \quad (6)$$

where T denotes the temperature, λ [$\text{W}/(\text{mK})$] is the thermal conductivity, G_B [1/s] is the perfusion rate, c_B [$\text{J}/(\text{m}^3\text{K})$] is the volumetric specific heat of blood, T_B is the supplying arterial blood temperature which is treated as a constant, Q_{met} is the metabolic heat source.

On the upper and lower surfaces of tissue domain (skin surface) the Dirichlet condition is assumed $T(x, y) = T_b$, where T_b is known temperature, on the remaining internal boundaries of tissue the adiabatic condition can be taken into account: $-\lambda \partial T(x, y) / \partial n = 0$.

2. Boundary element method - electric field

The boundary integral equation corresponding to the equation (1) is following [3]

$$B(\xi, \eta)\varphi(\xi, \eta) + \int_{\Gamma} \psi(x, y)\varphi^*(\xi, \eta, x, y)d\Gamma = \int_{\Gamma} \varphi(x, y)\psi^*(\xi, \eta, x, y)d\Gamma \quad (7)$$

where (ξ, η) is the observation point, the coefficient $B(\xi, \eta)$ is dependent on the location of source point (ξ, η) , $\psi(x, y) = -\varepsilon\partial\varphi(x, y)/\partial n$.

Fundamental solution of the problem discussed has the following form

$$\varphi^*(\xi, \eta, x, y) = \frac{1}{2\pi\varepsilon} \ln \frac{1}{r} \quad (8)$$

where r is the distance between points (ξ, η) and (x, y) . Differentiating the function $\varphi^*(\xi, \eta, x, y)$ with respect to the outward normal $\mathbf{n} = [\cos\alpha, \cos\beta]$ the function $\psi^*(\xi, \eta, x, y)$ is obtained

$$\psi^*(\xi, \eta, x, y) = -\varepsilon \frac{\partial\varphi^*(\xi, \eta, x, y)}{\partial n} = \frac{d}{4\pi r^2} \quad (9)$$

where

$$d = (x - \xi)\cos\alpha + (y - \eta)\cos\beta \quad (10)$$

The boundary of the domain is divided into N boundary elements. For constant boundary elements it is assumed that

$$(x, y) \in \Gamma_j : \begin{cases} \varphi(x, y) = \varphi(x_j, y_j) = \varphi_j \\ \psi(x, y) = \psi(x_j, y_j) = \psi_j \end{cases} \quad (11)$$

and then one obtains the following approximation of Equation (7)

$$\sum_{j=1}^N G_{ij}\psi_j = \sum_{j=1}^N H_{ij}\varphi_j, \quad i = 1, 2, \dots, N \quad (12)$$

where

$$G_{ij} = \int_{\Gamma_j} \varphi^*(\xi_i, \eta_i, x, y)d\Gamma_j \quad (13)$$

and $(i \neq j)$

$$H_{ij} = \int_{\Gamma_j} \psi^*(\xi_i, \eta_i, x, y) d\Gamma_j \quad (14)$$

while $H_{ii} = -1/2$.

The system of Equations (12) can be written in the matrix form

$$\mathbf{G}\boldsymbol{\psi} = \mathbf{H}\boldsymbol{\varphi} \quad (15)$$

This system allows to determine the 'missing' boundary values of functions φ_j, ψ_j .

Next, the values of function φ at the internal points (ξ_i, η_i) can be determined using the formula

$$\varphi_i = \sum_{j=1}^N H_{ij} \varphi_j - \sum_{j=1}^N G_{ij} \psi_j, \quad i = N+1, N+2, \dots, N+L \quad (16)$$

It should be pointed out that in order to determine the electric field inside tissue (Equation (5)) the partial derivatives $\partial\varphi_e(x, y)/\partial x, \partial\psi_e(x, y)/\partial y$ must be known.

One of the possibilities is application of equation (7) for internal nodes (ξ, η) ($B(\xi, \eta) = 1$) and then

$$\frac{\partial\varphi(\xi, \eta)}{\partial\xi} = \int_{\Gamma} \varphi(x, y) \frac{\partial\psi^*(\xi, \eta, x, y)}{\partial\xi} d\Gamma - \int_{\Gamma} \psi(x, y) \frac{\partial\varphi^*(\xi, \eta, x, y)}{\partial\eta} d\Gamma \quad (17)$$

and

$$\frac{\partial\varphi(\xi, \eta)}{\partial\eta} = \int_{\Gamma} \varphi(x, y) \frac{\partial\psi^*(\xi, \eta, x, y)}{\partial\eta} d\Gamma - \int_{\Gamma} \psi(x, y) \frac{\partial\varphi^*(\xi, \eta, x, y)}{\partial\xi} d\Gamma \quad (18)$$

where

$$\frac{\partial\varphi^*}{\partial\xi} = \frac{x-\xi}{2\pi\epsilon r^2}, \quad \frac{\partial\varphi^*}{\partial\eta} = \frac{y-\eta}{2\pi\epsilon r^2} \quad (19)$$

and

$$\frac{\partial\psi^*}{\partial\xi} = \frac{1}{2\pi} \left[\frac{2(x-\xi)d}{r^4} - \frac{\cos\alpha}{r^2} \right], \quad \frac{\partial\psi^*}{\partial\eta} = \frac{1}{2\pi} \left[\frac{2(y-\eta)d}{r^4} - \frac{\cos\beta}{r^2} \right] \quad (20)$$

Applying previously presented discretization of the boundary of domain, numerical calculations of partial derivatives are not difficult to obtain. These derivatives are determined at the internal nodes.

3. Dual reciprocity boundary element method - temperature field

The Pennes equation (6) can be written in the form

$$(x, y) \in \Omega: \lambda \nabla^2 T(x, y) - gT(x, y) + Q(x, y) = 0 \quad (21)$$

where

$$g = G_B c_B, \quad Q(x, y) = gT_B + Q_{met} + Q_e(x, y) \quad (22)$$

The standard boundary element method algorithm leads to the following integral equation [2, 3]

$$\begin{aligned} B(\xi, \eta)T(\xi, \eta) + \int_{\Gamma} q(x, y)T^*(\xi, \eta, x, y) d\Gamma = \\ \int_{\Gamma} T(x, y)q^*(\xi, \eta, x, y) d\Gamma + \int_{\Omega} [Q(x, y) - gT(x, y)]T^*(\xi, \eta, x, y) d\Omega \end{aligned} \quad (23)$$

where

$$T^*(\xi, \eta, x, y) = \frac{1}{2\pi\lambda} \ln \frac{1}{r} \quad (24)$$

and

$$q^*(\xi, \eta, x, y) = -\lambda \frac{\partial T^*(\xi, \eta, x, y)}{\partial n} = \frac{d}{4\pi r^2} \quad (25)$$

while $q(x, y) = -\lambda \partial T(x, y) / \partial n$.

It should be pointed out that the function $T^*(\xi, \eta, x, y)$ fulfills the equation

$$\lambda \nabla^2 T^*(\xi, \eta, x, y) = -\delta(\xi, \eta, x, y) \quad (26)$$

where $\delta(\xi, \eta, x, y)$ is the Dirac function.

The solution of Pennes equation (21) can be written as a sum

$$T(x, y) = \hat{T}(x, y) + U(x, y) \quad (27)$$

where the first function is the solution of Laplace's equation

$$\lambda \nabla^2 \hat{T}(x, y) = 0 \quad (28)$$

and $U(x, y)$ is a particular solution of equation

$$\lambda \nabla^2 U(x, y) - gU(x, y) + Q(x, y) = 0 \quad (29)$$

From Equations (27), (28), (29) results that

$$T(x, y) = U(x, y) \quad (30)$$

In the dual reciprocity method the following approximation is proposed [4]

$$Q(x, y) - gU(x, y) = - \sum_{k=1}^{N+L} a_k f_k(x, y) \quad (31)$$

where a_k are unknown coefficients and $f_k(x, y)$ are approximating functions fulfilling the equations (c.f. Equation (29))

$$\lambda \nabla^2 U_k(x, y) = f_k(x, y) \quad (32)$$

Putting (32) into (31) one obtains

$$Q(x, y) - gU(x, y) = -\lambda \sum_{k=1}^{N+L} a_k \nabla^2 U_k(x, y) \quad (33)$$

In Equations (31), (33) $N+L$ corresponds to the total number of nodes, where N is the number of boundary nodes and L is the number of internal nodes.

We consider the last integral in equation (23)

$$D = \int_{\Omega} [Q(x, y) - gT(x, y)] T^*(\xi, \eta, x, y) d\Omega \quad (33)$$

Taking into account the dependences (30), (33) one obtains

$$D = - \sum_{k=1}^{N+L} \int_{\Omega} [a_k \lambda \nabla^2 U_k(x, y)] T^*(\xi, \eta, x, y) d\Omega \quad (34)$$

Using the second Green formula [3] one has

$$\begin{aligned} D = & - \sum_{k=1}^{N+L} a_k \int_{\Omega} [\lambda \nabla^2 T^*(\xi, \eta, x, y)] U_k(x, y) d\Omega - \\ & \sum_{k=1}^{N+L} a_k \int_{\Gamma} \lambda T^*(\xi, \eta, x, y) \frac{\partial U_k(x, y)}{\partial n} d\Gamma + \\ & \sum_{k=1}^{N+L} a_k \int_{\Gamma} \lambda U_k(x, y) \frac{\partial T^*(\xi, \eta, x, y)}{\partial n} d\Gamma \end{aligned} \quad (35)$$

Because (c.f. formula (26))

$$\begin{aligned} & \int_{\Omega} [\lambda \nabla^2 T^*(\xi, \eta, x, y)] U_k(x, y) d\Omega = \\ & - \int_{\Omega} \delta(\xi, \eta, x, y) U_k(x, y) d\Omega = -B(\xi, \eta) U_k(\xi, \eta) \end{aligned} \quad (36)$$

so

$$\begin{aligned} D &= \sum_{k=1}^{N+L} a_k B(\xi, \eta) U_k(\xi, \eta) + \\ & \sum_{k=1}^{N+L} a_k \int_{\Gamma} [T^*(\xi, \eta, x, y) W_k(x, y) - U_k(x, y) q^*(\xi, \eta, x, y)] d\Gamma \end{aligned} \quad (37)$$

where

$$W_k(x, y) = -\lambda \frac{\partial U_k(x, y)}{\partial n} \quad (38)$$

Taking into account the formula (38) the Equation (23) can be written in the form

$$\begin{aligned} & B(\xi, \eta) T(\xi, \eta) + \int_{\Gamma} T^*(\xi, \eta, x, y) q(x, y) d\Gamma = \\ & \int_{\Gamma} q^*(\xi, \eta, x, y) T(x, y) d\Gamma + \sum_{k=1}^{N+L} a_k B(\xi, \eta) U_k(\xi, \eta) + \\ & \sum_{k=1}^{N+L} a_k \left[\int_{\Gamma} T^*(\xi, \eta, x, y) W_k(x, y) d\Gamma - \int_{\Gamma} q^*(\xi, \eta, x, y) U_k(x, y) d\Gamma \right] \end{aligned} \quad (39)$$

For constant boundary elements the following approximation of equation (40) can be taken into account ($i = 1, 2, \dots, N, N+1, \dots, N+L$)

$$B_i T_i + \sum_{j=1}^N P_{ij} q_j = \sum_{j=1}^N \hat{R}_{ij} T_j + \sum_{k=1}^{N+L} a_k \left(B_i U_{ik} + \sum_{j=1}^N P_{ij} W_{jk} - \sum_{j=1}^N \hat{R}_{ij} U_{jk} \right) \quad (40)$$

where

$$P_{ij} = \int_{\Gamma_j} T^*(\xi_i, x) d\Gamma_j \quad (41)$$

and

$$\hat{R}_{ij} = \int_{\Gamma_j} q^*(\xi_i, x) d\Gamma_j \quad (42)$$

while $B_i = B(\xi_i, \eta_i)$.

We define [5, 6]

$$U_{jk} = \frac{r_{jk}^2}{4} + \frac{r_{jk}^3}{9} \quad (43)$$

where

$$r_{jk}^2 = (x_k - x_j)^2 + (y_k - y_j)^2 \quad (44)$$

Using the formula (39) one obtains

$$W_{jk} = -\lambda \begin{bmatrix} \cos \alpha_j & \cos \beta_j \end{bmatrix} \begin{bmatrix} \frac{\partial U_{jk}}{\partial x_j} \\ \frac{\partial U_{jk}}{\partial y_j} \end{bmatrix} = \lambda d_{jk} \left(\frac{1}{2} + \frac{1}{3} r_{jk} \right) \quad (45)$$

where

$$d_{jk} = (x_k - x_j) \cos \alpha_j + (y_k - y_j) \cos \beta_j \quad (46)$$

Because

$$\nabla^2 U_{sk} = \frac{\partial^2 U_{sk}}{\partial x_{1k}^2} + \frac{\partial^2 U_{sk}}{\partial x_{2k}^2} = 1 + r_{sk} \quad (47)$$

so on the basis of equation (32) one has

$$f_{sk} = f_k(x_s, y_s) = \lambda(1 + r_{sk}) \quad (48)$$

Taking into account the dependencies (30), (31) one obtains

$$gT_s - Q_s = \sum_{k=1}^{N+L} a_k f_{sk} \quad (49)$$

where $T_s = T(x_s, y_s)$ and $Q_s = Q(x_s, y_s)$. The system of Equations (50) can be written in the matrix form

$$\begin{bmatrix} gT_1 - Q_1 \\ \dots \\ gT_N - Q_N \\ \dots \\ gT_{N+L} - Q_{N+L} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & \dots & f_{1,N+L} \\ \dots & \dots & \dots & \dots \\ f_{N,1} & f_{N,2} & \dots & f_{N,N+L} \\ \dots & \dots & \dots & \dots \\ f_{N+L,1} & f_{N+L,L} & \dots & f_{N+L,N+L} \end{bmatrix} \begin{bmatrix} a_1 \\ \dots \\ a_N \\ \dots \\ a_{N+L} \end{bmatrix} \quad (50)$$

or

$$g\mathbf{T} - \mathbf{Q} = \mathbf{f}\mathbf{a} \rightarrow \mathbf{a} = \mathbf{f}^{-1}(g\mathbf{T} - \mathbf{Q}) \quad (51)$$

The following matrices of dimensions $N+L \times N+L$ can be defined

$$\mathbf{P} = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1,N} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ P_{N,1} & P_{N,2} & \dots & P_{N,N} & 0 & \dots & 0 \\ P_{N+1,1} & P_{N+1,2} & \dots & P_{N+1,N} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ P_{N+L,1} & P_{N+L,2} & \dots & P_{N+L,N} & 0 & \dots & 0 \end{bmatrix} \quad (52)$$

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1,N} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ R_{N,1} & R_{N,2} & \dots & R_{N,N} & 0 & \dots & 0 \\ R_{N+1,1} & R_{N+1,2} & \dots & R_{N+1,N} & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ R_{N+L,1} & R_{N+L,2} & \dots & R_{N+L,N} & 0 & \dots & -1 \end{bmatrix} \quad (53)$$

where

$$R_{ij} = \begin{cases} \hat{R}_{ij}, & i \neq j \\ \hat{R}_{ij} - 0.5, & i = j \end{cases} \quad (54)$$

and

$$\mathbf{U} = \begin{bmatrix} U_{11} & U_{12} & \dots & U_{1,N} & U_{1,N+1} & \dots & U_{1,N+L} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ U_{N,1} & U_{N,2} & \dots & U_{N,N} & U_{N,N+1} & \dots & U_{N,N+L} \\ U_{N+1,1} & U_{N+1,2} & \dots & U_{N+1,N} & U_{N+1,N+1} & \dots & U_{N+1,N+L} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ U_{N+L,1} & U_{N+L,2} & \dots & U_{N+L,N} & U_{N+L,N+1} & \dots & U_{N+L,N+L} \end{bmatrix} \quad (55)$$

$$\mathbf{W} = \begin{bmatrix} W_{11} & W_{12} & \dots & W_{1,N} & W_{1,N+1} & \dots & W_{1,N+L} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ W_{N,1} & W_{N,2} & \dots & W_{N,N} & W_{N,N+1} & \dots & W_{N,N+L} \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix} \quad (56)$$

So, the system of Equations (41) can be written in the matrix form

$$\mathbf{Pq} = \mathbf{RT} + (\mathbf{PW} - \mathbf{RU})\mathbf{a} \quad (57)$$

or (c.f. formula (52))

$$\mathbf{Pq} = \mathbf{RT} + (\mathbf{PW} - \mathbf{RU})\mathbf{f}^{-1}(\mathbf{gT} - \mathbf{Q}) \quad (58)$$

where

$$\mathbf{q} = \begin{bmatrix} q_1 \\ \dots \\ q_N \\ 0 \\ \dots \\ 0 \end{bmatrix} \quad (59)$$

This system of equations allows to find, among others, the temperatures at the boundary and internal nodes.

4. Results of computations

The rectangular domain of dimensions $0.08 \text{ m} \times 0.04 \text{ m}$ has been considered. The heating area is described as $\{0.032 \leq x \leq 0.048, y = 0 \text{ m}\}$, $\{0.032 \leq x \leq 0.048, y = 0.04 \text{ m}\}$ and the voltage applied on these surfaces is 10V and -10V , respectively. For biological tissue the following parameters have been assumed: thermal conductivity $\lambda = 0.5 \text{ W/(mK)}$, perfusion rate $G_B = 0.0005 \text{ 1/s}$, metabolic heat source $Q_{met} = 420 \text{ W/m}^3$, blood temperature $T_B = 37^\circ\text{C}$, volumetric specific heat of blood $c_B = 4.2 \text{ MJ/(m}^3\text{K)}$ [2]. On the skin surface the temperature $T = 32.5^\circ\text{C}$ has been accepted. At first, the temperature distribution in the tissue without electric field influence under the assumption that on the external surface 60 constant boundary elements have been distinguished (Fig. 2).

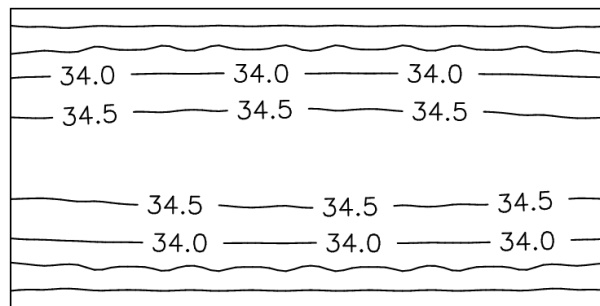


Fig. 2. Temperature distribution without electric field

The distribution of electric field is shown in Figure 3, while Figure 4 illustrates the temperature field in the tissue subjected to electric field. The source function Q_e under the assumption that $\sigma = 0.4 \text{ S/m}$ (Equation (5)) is shown in Figure 5.

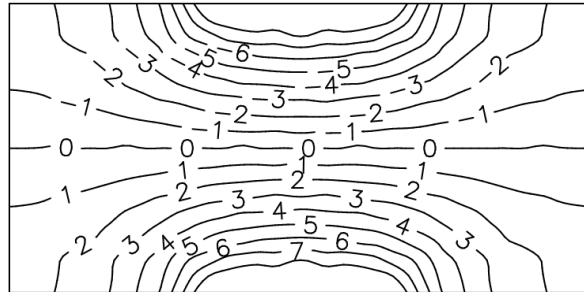


Fig. 3. Electric field distribution

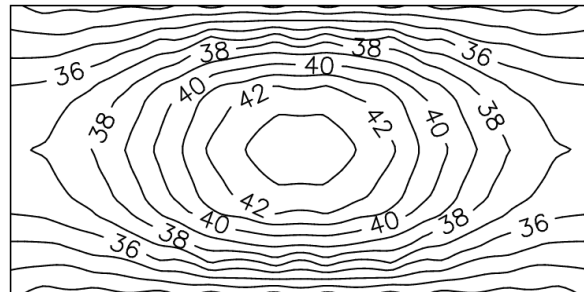


Fig. 4. Temperature distribution

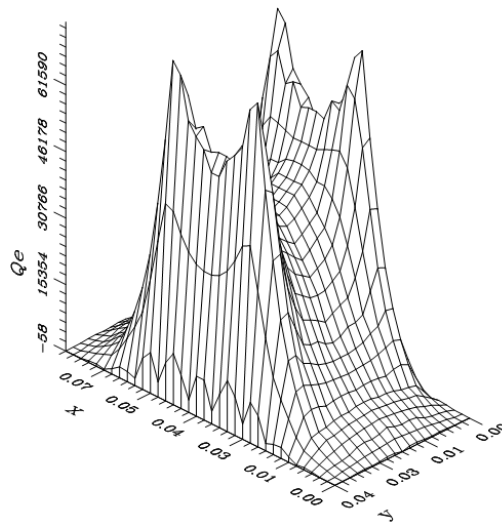


Fig. 5. Source function due to the electric field

Summing up, the boundary element method has been applied to solve the coupled problem connected with the biological tissue heating. The simplified 2D mathematical model based on the Pennes equation supplemented with an equation determining the electric field due to the external electrodes action has been considered.

Acknowledgement

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