THE INFLUENCE OF GLOBAL CHARACTERISTICS OF A FILTERING SYSTEM IN THE EFFICIENCY IN THE PROCESS OF DEEP BED FILTRATION

Bożena Baran, Zbigniew Domański

Institute of Mathematics and Computer Science, Czestochowa University of Technology, Poland
email: bozenab@imi.pcz.pl, zdomanski@imi.pcz.pl

Abstract. In the process of deep bed filtration there is an occurrence of clogging the filters bonds by particles of size comparable to the size of the bonds. In this paper it is examined how the structure of connections of a filter's bonds influences the time of its clogging as well as the efficiency of the filtering process. It is shown that the clogging time depends on two things: the network coordinating number - it means the average number of bonds with a common node and on a percolation threshold - it means the fraction of clogged bonds when the statistic lose of communication between the entrance and exit of the system occurs.

Introduction

The movement of particles in incompressible liquid which is transported through the network of different structure and geometry of bonds, is a continuous subject of researches, because of a very wide spectrum of use and constant development of biotechnology. In the last several years intense researches of micro-flow showed that the description of liquid transport based on the idea of the continuous centre is completely inadequate to the transport through pores of size of fraction of micrometer and it is necessary to use the discreet description [1, 2]. Modern filtering systems used in this context work in the way that the liquid transport is two-dimensional.

1. Formulation of the problem

Several two-dimensional networks representative for networks created from polymorphic divisions of a plane [3] which good model the structure of bonds in two-dimensional separating systems or in porous materials were used in this numerical experiment. Three networks created from a regular division of the plane, it means networks of triangular, square and hexagonal symmetry, were chosen another one was created from a semiregular division on the plane so-called Kagomé network and the dual to that one - Necker-Cube network (Fig. 1).

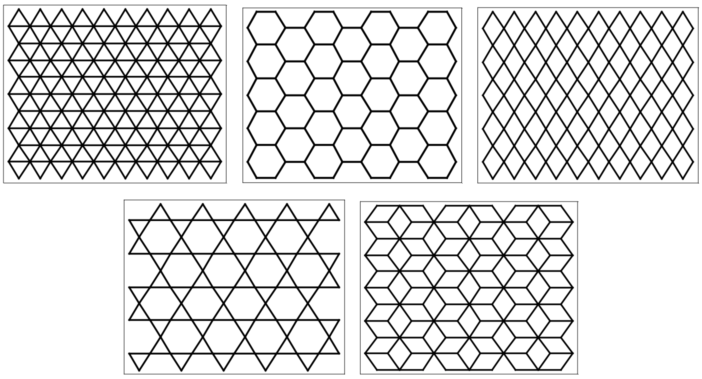

Fig. 1. The networks created from polymorphic divisions of a plane: networks of triangular, hexagonal, square, Kagomé and Necker-Cube symmetry

The network of reference was a part of a square network of dimensions X×X nodes joined by the bonds of identical length. Wide (W) and narrow (N) bonds were spaced randomly, and $p\in(0,1)$ is the concentration of the wide bonds. To the nodes on the left of the network there was inserted a mixture of large-sized (L) and small-sized (S) particles, when the fraction of particles L in the flow of particles given was maintained on the steady level $q = 0.1$. It was assumed that a big particle clogs a narrow bond and a small particle goes through each bond that is not stuck, provided that the consecutive accessible bonds are not stuck. While leaving the bond, it is equally probable that a particle chooses one of the consecutive accessible bonds, if at least one such a bond exists, otherwise, the bond is clogged. Particles are injected to each entrance node at regular intervals $\Delta t = 1$. The particles which reached the nodes on the right side of the system leave it. When injecting an additional particle to the network is impossible, the filter becomes clogged and the number of time steps t is the clogging time of the filter.

2. Calculations

The purpose of calculations was to examine in what way the effectiveness of filtering process depends on the symmetry of transporting network and on the local changes of its properties.

An analysis of an empirical function of the filter clogging time distribution let us build a functioning correlation between the clogging time and the concentration of wide bonds (Fig. 2).

On the basis of the shape and location of approximation function graphs, we can read three properties of the $\bar{t} = \bar{t}(p)$ relation:

1. The function graphs of clogging-time distribution for a filter of a square symmetry and Kagomé are situated very close to each other.

2. Vertical asymptotes for each function are different.
3. All the graphs represent one functioning dependence:

$$\bar{t} = c_1 \cdot tg(c_2 p) + const.$$

where c_1, c_2 - parameters dependent on the network symmetry.

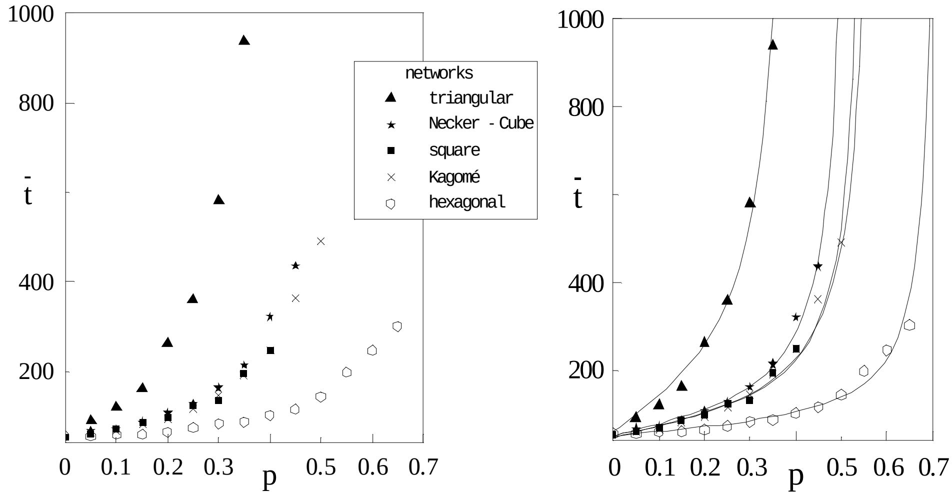

Fig. 2. Approximation of a function of average filter clogging time $\bar{t}$, with a changing W-type bonds concentration p in the range from p = 0.05 to a certain value $p = p_\infty$, with a step Δp, where p_∞ means the concentration where clogging was not observed, despite a large number of time steps

Table 1

The values of the network coordinating number and a percolation threshold for networks of triangular, hexagonal, square, Kagomé and Necker-Cube symmetry

coordinating number z	percolation threshold p_c	network
3	0.6527	hexagonal
4	0.5	square
4	0.5234	Kagomé
4.5	0.4755	Necker-Cube
6	0.372	triangular

The analysis of these properties explains the connection between clogging time and the network symmetry properties.

The property 1. and the order of graphs that can be seen in Figure 2 suggest that the clogging time depends on the network coordinating number z (Tab. 1).

The function $\bar{t}$ cannot be rescaled on itself when we change the value of only one of the parameters: c_1, c_2, what means that each of these parameters is the function of network coordinating number z.

The property 2. - the presence of vertical asymptotes - suggests that increasing the fractions of wide bonds over some critical concentration value causes a qualitative change consisting in losing the filtering properties. If the asymptotes position were connected with percolation thresholds for particular networks (Tab. 1), the loss of filtering values by the system would be the result of the transition between a filtering phase with finite clogging time and a phase without filtration.

It follows that the argument of the function $\bar{t}$ for $p = p_c$ must strive for the value corresponding to the vertical asymptotes of the tangens function, it means the value $\frac{\pi}{2}$. Therefore in the argument of tangens function the concentration of nodes S is found in the p/p_c combination.

The final analysis of properties 1. to 3. leads to a formula:

$$\bar{t} = c_1(z) \cdot tg\left[\frac{\pi}{2p_c(z)}p\right] + const.$$

Conclusions

We conclude that the clogging time depends on two things: the network coordinating number and on a percolation threshold. Therefore, between two filtering networks with the same coordinating number, the network with a higher percolation threshold is more resistant to clogging.

References

[1] Chou C-F., Bakajin O., Turner S.W.P., Duke T.A.J., Chan S.S., Cox E.C., Craighead H.G., Austin R.H., Sorting by diffusion: An asymmetric obstacle course for continuous molecular separation, Proc. Natl. Acad. Sci. USA 1999, 96(24), 13762-13765.

[2] Katz A.J., Thompson A.H., Quantitative prediction of permeability in porous rock, Phys. Rev. 1986, B34, 8179.

[3] Britton J., Symmetry and Tessellations: Investigating Patterns, Prentice-Hall, Englewood Cliffs 1999.

[4] Merck S.C., Percolation thresholds and universal formulas, Phys. Rev. E 1997, 55, 1514.