ESTIMATION OF EPIDERMIS AND DERMIS THICKNESSES ON THE BASIS OF SKIN SURFACE TEMPERATURE

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Abstract. Heat transfer in the skin tissue is treated as a multi-layer domain in which one can distinguish the epidermis, dermis and subcutaneous region. The thermophysical parameters of these sub-domains are equal to \( \lambda_e \) [W/(mK)] (thermal conductivity) and \( c_e \) [J/(m³K)] (specific heat per unit of volume), \( e = 1, 2, 3 \). The transient bioheat transfer in domain of skin is described by following system of equations [1, 2]

\[ \begin{align*}
\frac{\partial T_e(x,t)}{\partial t} &= \frac{\partial^2 T_e(x,t)}{\partial x^2} + k_e \left[ T_B - T_e(x,t) \right] + Q_{me} \\
\end{align*} \]

where \( k_e = G_e c_B \) (\( G_e \) [(m³ blood/s)/(m³ tissue)]) is the blood perfusion rate, \( c_B \) [J/(m³K)] is the volumetric specific heat of blood, \( T_B \) is the arterial blood temperature and \( Q_{me} \) [W/m³] is the metabolic heat source. It should be pointed out that for epidermis sub-domain \( (e = 1) \) \( G_1 = 0 \) and \( Q_{m1} = 0 \).

The system of equations (1) is supplemented by the following boundary conditions

- on the skin surface
  \[ \begin{align*}
x \in \Omega_0 : & \quad q_e(x,t) = -\lambda_e \frac{\partial T_e(x,t)}{\partial x} = q_s \\
\end{align*} \]

- on the contact surfaces between sub-domains considered \( (e = 1, 2) \)
Estimation of epidermis and dermis thicknesses on the basis of skin surface temperature

\( x \in \Gamma_{\varepsilon, \varepsilon} : \begin{cases} -\lambda_e \frac{\partial T_e(x,t)}{\partial x} = -\lambda_{\varepsilon, \varepsilon} \frac{\partial T_{\varepsilon, \varepsilon}(x,t)}{\partial x} \\ T_e(x,t) = T_{\varepsilon, \varepsilon}(x,t) \end{cases} \) \hspace{1cm} (3)

- on the conventionally assumed internal boundary limiting the system

\( x \in \Gamma_3 : q(x,t) = -\lambda_3 \frac{\partial T_3(x,t)}{\partial x} = 0 \) \hspace{1cm} (4)

A quadratic initial temperature distribution between \( T_p = 32.5^\circ C \) at the surface and \( T_c = 37^\circ C \) at the base of the subcutaneous region \( (x = L) \) is assumed [1, 2]

\[ t = 0 : T(x,0) = T_p + \frac{x(2L_3 - x)}{L_3^2}(T_c - T_p) \] \hspace{1cm} (5)

The direct problem described by equations (1)-(5) can be solved under the assumption that all thermal and geometrical parameters of skin tissue are given.

1. Shape sensitivity analysis

For 1D problem the following definition of material derivative is used [3, 4]

\[ \frac{D T_e}{Db} = \frac{\partial T_e}{\partial b} + \frac{\partial T_e}{\partial x} v \] \hspace{1cm} (6)

where \( v = v(x, b) \) is the velocity associated with design parameter \( b \).

If the direct approach of sensitivity analysis is applied [3, 4], then the governing equations should be differentiated with respect to design parameter \( b \).

Differentiation of equations (1) gives

\[ c_e \frac{D}{Db} \left( \frac{\partial T_e}{\partial t} \right) = \lambda_e \frac{D}{Db} \left( \frac{\partial^2 T_e}{\partial x^2} \right) - k_e \frac{DT_e}{Db} \] \hspace{1cm} (7)

Because [5]

\[ \frac{D}{Db} \left( \frac{\partial^2 T_e}{\partial x^2} \right) = \frac{\partial^2}{\partial x^2} \left( \frac{D T_e}{Db} \right) - 2 \frac{\partial^2 T_e}{\partial x^2} \frac{\partial v}{\partial x} - \frac{\partial T_e}{\partial x} \frac{\partial^2 v}{\partial x^2} \] \hspace{1cm} (8)

and

\[ \frac{D}{Db} \left( \frac{\partial T_e}{\partial t} \right) = \frac{\partial}{\partial t} \left( \frac{D T_e}{Db} \right) \] \hspace{1cm} (9)
so the equations (7) can be written in the form

\[
\frac{c_e}{\partial t} \left( \frac{D T_e}{D b} \right) = \lambda_e \frac{\partial^2}{\partial x^2} \left( \frac{D T_e}{D b} \right) - 2\lambda_e \frac{\partial^2 T_e}{\partial x^2} \frac{\partial T_e}{\partial x} - \lambda_e \frac{\partial T_e}{\partial x} \frac{\partial^2 v}{\partial x^2} - k_e \frac{D T_e}{D b} \tag{10}
\]

or (c.f. equations (1))

\[
\frac{c_e}{\partial t} \left( \frac{D T_e}{D b} \right) = \lambda_e \frac{\partial^2}{\partial x^2} \left( \frac{D T_e}{D b} \right) - 2 \left[ \frac{c_e}{\partial t} \frac{\partial T_e}{\partial x} - k_e (T_e - T_e) - Q_{me} \right] \frac{\partial v}{\partial x} - \lambda_e \frac{\partial T_e}{\partial x} \frac{\partial^2 v}{\partial x^2} - k_e \frac{D T_e}{D b} \tag{11}
\]

Differentiation of boundary condition (2) leads to the equation

\[
x = 0: \quad \frac{D q_1}{D b} = -\lambda_1 \frac{D}{D b} \left( \frac{\partial T_i}{\partial x} \right) = \frac{D q_e}{D b} = 0 \tag{12}
\]

Taking into account the formula [5]

\[
\frac{D}{D b} \left( \frac{\partial T_e}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{D T_e}{D b} \right) - \frac{\partial T_e}{\partial x} \frac{\partial v}{\partial x} \tag{13}
\]

one has

\[
x = 0: \quad -\lambda_1 \frac{\partial}{\partial x} \left( \frac{D T_i}{D b} \right) + \lambda_1 \frac{\partial T_i}{\partial x} \frac{\partial v}{\partial x} = 0 \tag{14}
\]

We differentiate the continuity conditions (3)

\[
x = L_e : \quad \left\{ \begin{array}{l}
-\lambda_e \frac{D}{D b} \left( \frac{\partial T_e}{\partial x} \right) = -\lambda_{e+1} \frac{D}{D b} \left( \frac{\partial T_{e+1}}{\partial x} \right) \\
\frac{D T_e}{D b} = \frac{D T_{e+1}}{D b}, \quad e = 1, 2
\end{array} \right. \tag{15}
\]

and then (c.f. equation (13))

\[
x = L_e : \quad \left\{ \begin{array}{l}
-\lambda_e \frac{\partial}{\partial x} \left( \frac{D T_e}{D b} \right) + \lambda_e \frac{\partial T_e}{\partial x} \frac{\partial v}{\partial x} = -\lambda_{e+1} \frac{\partial}{\partial x} \left( \frac{D T_{e+1}}{D b} \right) + \lambda_{e+1} \frac{\partial T_{e+1}}{\partial x} \frac{\partial v}{\partial x} \\
\frac{D T_e}{D b} = \frac{D T_{e+1}}{D b}
\end{array} \right. \tag{16}
\]
Differentiation of condition (4) gives

\[ x = L_3 : \quad -\lambda_3 \frac{D}{Db} \left( \frac{\partial T_3}{\partial x} \right) = 0 \]  \quad (17)

or

\[ x = L_3 : \quad -\lambda_3 \frac{\partial}{\partial x} \left( \frac{DT_3}{Db} \right) + \lambda_3 \frac{\partial T_3}{\partial x} \frac{\partial v}{\partial x} = 0 \]  \quad (18)

Finally, the initial condition (5) is differentiated

\[ t = 0: \quad \frac{DT(x, 0)}{Db} = \frac{2(L_2 - x)}{L_3} (T_e - T_v) \nu \]  \quad (19)

We assume that \( b = L_1 \) and [7]

\[ v(x, L_3) = \begin{cases} \frac{x}{L_1}, & 0 \leq x \leq L_1 \\ \frac{L_2 - x}{L_2 - L_1}, & L_1 \leq x \leq L_2 \\ 0, & L_2 \leq x \leq L_3 \end{cases} \]  \quad (20)

The equations (11) take a form

\[ 0 < x < L_1 : \quad c_1 \frac{\partial U_1(x,t)}{\partial t} = \lambda_1 \frac{\partial^2 U_1(x,t)}{\partial x^2} - \frac{2c_1 \partial T_1(x,t)}{L_1} \]  \quad (21)

and

\[ L_1 < x < L_2 : \quad c_2 \frac{\partial U_2(x,t)}{\partial t} = \lambda_2 \frac{\partial^2 U_2(x,t)}{\partial x^2} - k_2 U_2(x,t) + \frac{2}{L_2 - L_1} \left[ c_2 \frac{\partial T_2(x,t)}{\partial t} - k_2 [T_s - T_v(x,t)] - Q_{ac2} \right] \]  \quad (22)

while

\[ L_2 < x < L_3 : \quad c_3 \frac{\partial U_3(x,t)}{\partial t} = \lambda_3 \frac{\partial^2 U_3(x,t)}{\partial x^2} - k_3 U_3(x,t) \]  \quad (23)

where \( U_e = DT_e/Db \) is the sensitivity function.
The boundary conditions (16) can be written as follows

\[
x = L_1 : \begin{cases}
W_1(x,t) - \frac{1}{L_1} q_1(x,t) = W_2(x,t) + \frac{1}{L_2 - L_1} q_2(x,t) \\
U_1(x,t) = U_2(x,t)
\end{cases}
\]  

(24)

and

\[
x = L_2 : \begin{cases}
W_2(x,t) + \frac{1}{L_2 - L_1} q_2(x,t) = W_3(x,t) \\
U_2(x,t) = U_3(x,t)
\end{cases}
\]  

(25)

where: \( q_e(x, t) = -\lambda_e \partial T_e(x, t) / \partial x \), \( W_e(x, t) = -\lambda_e \partial U_e(x, t) / \partial x \).

The remaining boundary conditions are of the form

\[
x = 0 : \quad W_i(x, t) = -\frac{q_L}{L_i}
\]  

(26)

and

\[
x = L_i : \quad W_i(x, t) = 0
\]  

(27)

2. Inverse problem

We assume that the thicknesses of epidermis and dermis of skin tissue are unknown this means the value of \( L_1 \) must be found. Additionally, the time-dependent course of temperature on the skin surface is given

\[
T_d^f = T_i(0, t^f), \quad f = 0, 1, 2, ..., F
\]  

(28)

In order to solve the inverse problem, the least squares criterion is applied [6]

\[
S(L_i) = \sum_{f=1}^{F} \left( T^f - T_d^f \right)^2
\]  

(29)

where \( T^f = T_i(0, t^f) \) is the calculated temperature at the boundary point \( x = 0 \) for time \( t^f \) by using the current available estimate of unknown parameter \( L_i \).

Differentiating the criterion (7) with respect to the unknown co-ordinate \( L_i \) and using the necessary condition of optimum, one obtains

\[
\frac{DS}{DL_i} = 2 \sum_{f=1}^{F} \left( T^f - T_d^f \right) \frac{DT^f}{DL_i} \bigg|_{L_i=L_i} = 0
\]  

(30)
where for \( k = 0 \): \( L_k^1 \) is the arbitrary assumed value of \( L_1 \), while for \( k > 0 \): \( L_k^1 \) results from the previous iteration, \( D(\cdot)/DL_1 \) denotes the material derivative [3, 4]. Function \( T' \) is expanded in a Taylor series about known value of \( L_1^1 \), this means

\[
T' = (T')^k + \left. \frac{DT' }{DL_1} \right|_{L_1^1} (L_1^{k+1} - L_1^k)
\]

or

\[
T' = (T')^k + (U')^k (L_1^{k+1} - L_1^k)
\]

where \((U')^k = DT'/DL_1^k\) is the sensitivity function.

Putting (32) into (30) one has

\[
\sum_{f=1}^{F} \left[ (U')^k \right]^2 (L_1^{k+1} - L_1^k) = \sum_{f=1}^{F} (U')^k (T'_f - (T')^k)
\]

this means

\[
L_1^{k+1} = L_1^k + \frac{\sum_{f=1}^{F} \left[ T'_f - (T')^k \right] (U')^k}{\sum_{f=1}^{F} (U')^k^2}, \quad k = 0, 2, ..., K
\]

This equation allows to find the value of \( L_1^{k+1} \). The iteration process is stopped when the assumed accuracy is achieved.

It should be pointed out that value \( L_1 \) corresponds to the epidermis thickness, while the value \( L_2 - L_1 \) corresponds to the dermis thickness. So, the identification of \( L_1 \) allows to determine both the epidermis and dermis thicknesses.

3. Results of computations

In numerical computations of direct problem the following values of parameters have been assumed [1, 2, 5]: \( \lambda_1 = 0.235 \) W/(mK), \( \lambda_2 = 0.445 \) W/(mK), \( \lambda_3 = 0.185 \) W/(mK), \( c_1 = 4.3068 \) MJ/(m\(^3\)K), \( c_2 = 3.96 \) MJ/(m\(^3\)K), \( c_3 = 2.674 \) MJ/(m\(^3\)K), \( c_B = 3.9962 \) MJ/(m\(^3\)K), \( T_B = 37^\circ\)C, \( G_1 = 0 \), \( G_e = 0.00125 \) (m\(^3\) blood/s)/(m\(^3\) tissue) for \( e = 2, 3 \), \( Q_m = 0 \), \( Q_{me} = 245 \) W/m\(^3\) for \( e = 2, 3 \). The thicknesses of successive skin layers: 0.1, 2 and 10 mm.

The basic problem and additional one resulting from the sensitivity analysis have been solved using the boundary element method [2, 7, 8]. The layers of skin have been divided into 10, 40 and 120 internal cells, time step: \( \Delta t = 0.05 \) s.
It is assumed that skin surface is subjected to the action of boundary heat flux \( q_S = 6000 \text{ W/m}^2 \) (variant 1) and \( q_S = 4000 \text{ W/m}^2 \) (variant 2). In Figure 1 the courses of skin surface for both cases are shown. On the basis of these values of temperature (c.f. equation (28)) the inverse problem has been solved. Figure 2 illustrates the courses of sensitivity function for \( x = 0 \) (skin surface) for two variants of computations and real thicknesses of epidermis and dermis layers.

Fig. 1. Skin surface temperature (1 - \( q_S = 6000 \text{ W/m}^2 \), 2 - \( q_S = 4000 \text{ W/m}^2 \))

Fig. 2. Courses of sensitivity functions at the skin surface (1 - \( q_S = 6000 \text{ W/m}^2 \), 2 - \( q_S = 4000 \text{ W/m}^2 \))
The next figures concern the inverse problem solution. In Figures 3 and 5 the results of co-ordinates $L_1$ identification for different initial values of this parameter are shown, while Figures 4 and 6 illustrate the values of function $S$ (c.f. equation (29)) for both variants of computations.

Summing up, the algorithm presented allows to determine the thicknesses of epidermis and dermis under the assumption that skin surface temperature is known.

Fig. 3. Identification of value $L_1$ ($q_S = 6000 \text{ W/m}^2$)

Fig. 4. The values of function $S$ for successive iterations ($q_S = 6000 \text{ W/m}^2$)
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References


