ESTIMATION OF THE CLOGGING TIME OF LINEAR FILTERS BASED ON EXPERIMENTALLY DETERMINED PARTICLE-SIZE DISTRIBUTION FUNCTION

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Abstract. Several mathematical models have been designed to described the clogging process in porous media. Most of models use the continuum scale. Because of inherent heterogeneity and interconnectivity existing in porous media several phenomena occurring at
the pore-scale cannot be modeled effectively at the continuum scale. The aim of this work is to use experimentally observed distributions of pores and particles sizes to obtain qualitative and predictive results of clogging filter time τ. For bundle of linear pores we computed the clogging time distribution: $P(\tau) \approx (1/\tau)^{5/4} \exp[-(1/\tau)^{1/4}]$.

Flow and transport in porous media are important in many science and engineering applications such as deep bed filtration, hydrodynamic chromatography, migration of fines, ground water contamination, the flow of dilute stable emulsions, and enhanced oil recovery. In order to understand the behavior of these systems, we needs to know the microscopic characteristics of the suspended particles. Consider the example of deep bed filtration, where a suspension is injected into a filter made of porous material. The suspended particles are collected in the filter while clear fluid passes through. A filtrate particle flowing through the pore space may be trapped by the geometric constraint of reaching a pore smaller than its diameter, or by other adhesive mechanisms. Latham and co-workers [1] surveyed a collection of experimentally established pore and particle size distributions for materials encountered in mining and petroleum industries:

Table 1

Distribution of pore size (equivalent spherical diameter) in microns

d_i	100	117	134	151	168	185	202	219	236	253	270	287	304	321
$P(d_i)$	$\frac{107}{396}$	$\frac{2}{11}$	$\frac{17}{132}$	$\frac{17}{132}$	$\frac{1}{18}$	$\frac{31}{396}$	$\frac{17}{396}$	$\frac{17}{396}$	$\frac{1}{36}$	$\frac{1}{66}$	$\frac{1}{99}$	$\frac{1}{132}$	$\frac{1}{198}$	$\frac{1}{396}$

Based on this work and Table 1, we adopt the exponential distributions of pore $b(r)$ and particle $p(r)$ radii, respectively in the following form:

$$\begin{aligned} b(r) &= \alpha \cdot (r/30) \cdot \exp[-\alpha(r/30)] \\ p(r) &= (r/30) \cdot \exp[-(r/30)] \\ r &> 0 \end{aligned} \qquad (1)$$

Here, s is the ratio between the average bond and particle radii and $\alpha = 1/s^2$ is a basic parameter which determines the nature of the clogging process.

Our objective is to estimate the clogging time of bundle of linear pores as presented in Figure 1.

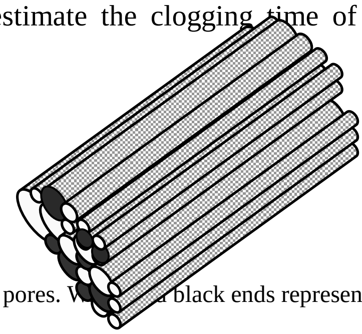

Fig. 1. Linear bundle of pores. White and black ends represent empty and clogged pores

We assume that average pores are smaller than average particles ($s < 1$). In this case each particle injection event leads to the clogging of the first pore entered. The initial pores are blocked quickly, while later pores are blocked more slowly because the overall flow rate decreases significantly near the end of the clogging process. In further calculation we assume that clogging time is dominated by the time of these later blockage events. When only the smallest bonds remain open near clogging, the permeability is determined by these smallest radii. We estimate the radius of the k^{th} smallest bond from the following relation

$$\int_0^{r_k} b(r)dr = \frac{k}{w} \qquad (2)$$

which gives $r_k = 30 s^2 \frac{k}{w}$.

Here we consider the Poiseuille flow, in which the fluid flux passing through a pore of radius r_i is proportional to $r_i^2 \nabla p$, where ∇p is the local pressure gradient when a fixed overall pressure drop is imposed. Then the permeability of a parallel bundle of k smallest pores is

$$\kappa(k) = \sum_{j=1}^{k} r_j^4 \approx 30^4 s^8 \sum_{j=1}^{k} \left(\frac{j}{w}\right)^4 \approx 30^4 s^8 \frac{k^5}{w^4} \qquad (3)$$

Setting $k = w$ above we obtain the initial system permeability $\kappa(w) = 30^4 s^8 w$. As the overall fluid flow is proportional to the permeability for a fixed pressure drop, the time increment t_k between blocking the $(k\text{-}1)^{st}$-smallest and k^{th} - smallest pore behaves as

$$t_k = \frac{\kappa(w)}{w \cdot \kappa(k)} \approx \frac{w^4}{k^3} \tag{4}$$

We find the clogging time distribution in terms of the radius distribution of the smallest bond, since this bond ultimately controls clogging in a simple parallel bond array. For the exponent distribution, the probability that a given bond has a radius greater than or equal to r, $B_>(r)$, is

$$B_>(r) = \int_r^\infty \alpha \exp[-\alpha(r/30)]d(r/30) = \exp[-\alpha(\gamma/30)] \tag{5}$$

Therefore the radius distribution of the smallest bond from among w, $S_w(r)$, is given by

$$S_w(r) = wb(r)\left[B_>(r)\right]^{w-1} = \frac{\alpha}{30} w \exp[-\frac{\alpha}{30} rw] \tag{6}$$

From the connections between permeability, pore radius and time scale (Equations (3) and (4)), and the fact that the clogging time is dominated by t_1, we deduce

$$T \approx t_1 \approx \frac{\kappa(w)}{w\kappa(1)} = 30^4 s^8 \frac{1}{r_1^4} \tag{7}$$

while the clogging time distribution, $P_w(T)$, is directly related to the smallest bond radius distribution through $P_w(T)dT = S_w(r)dr$. From Equations (6) and (7), we obtain the main result of this paper

$$P_w(\tau) \approx w(1/\tau^{1/4})^5 \exp[-w/\tau^{1/4}] \tag{8}$$

Comparing Eq. (8) to the distribution function: $w(1/\tau^{1/2})^3 \exp[-w/\tau^{1/2}]$ obtained in [2] with the use of a theoretical distribution of pore radii: $f(r) \approx r \cdot \exp[-r^2]$.

We conclude that realistic particle size distribution used in our calculation leads to the longer clogging time of filters compare to the clogging time computed with the use of theoretical distribution of pores and particles radii.

References

[1] Latham J.-P., Munjiza A., Lu Y., Powder Technology 2002, 125, 10-27.

[2] Render S., Datta S., Phys. Rev. Letters 2000, 84, 6018-6021.