SENSITIVITY ANALYSIS OF MACROSEGREGATION PROCESS

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Abstract. Heat transfer in the domain of solidifying and cooling metal is accompanied by the process of mass diffusion. Connected with this phenomena changes of alloy component concentration are called the macrosegregation process. In the paper the sensitivity analysis is applied in order to find the mutual connections between the perturbations of partition coefficient value and the course of macrosegregation. On the stage of numerical computations the boundary element method has been used. In the final part of the paper the results obtained are shown.

1. Introduction

Heat transfer in the domain of solidifying and cooling metal is accompanied by the process of the mass diffusion. Connected with this phenomena changes of alloy component concentration are called the macrosegregation process. From the mathematical point of view the macrosegregation process is described by a system of partial differential equations determining the mass transfer in solid and liquid state sub-domains and adequate boundary-initial conditions [1]. The very essential parameter determining the course of the process is called the partition coefficient. It can be found using the equilibrium diagram, but in reality the macrosegregation proceeds in the unbalance conditions. In the paper the mutual connections between the perturbations of partition coefficient value and the course of macrosegregation are discussed, in particular the methods of parameter sensitivity analysis are applied [2]. The numerical model of macrosegregation and the sensitivity one are constructed on the basis of a certain variant of the boundary element method called in literature the BEM using discretization in time [3, 4]. In the final part of the paper the examples of computations are shown.

2. Mathematical model of the process

The directional solidification (1D task) is analyzed. In the coordinate system connected with the solid-liquid interface the diffusion equation for the molten metal sub-domain takes a form

$$\frac{\partial z_L(x,t)}{\partial t} = D_L \frac{\partial^2 z_L(x,t)}{\partial x^2} + \frac{\partial z_L(x,t)}{\partial x} v(t) \tag{1}$$

where z_L is an alloy component concentration, D_L is a diffusion coefficient, v is a solidification rate. Because $D_L >> D_S$ (diffusion coefficient of solid) therefore the mass transfer in solidified part of the metal can be neglected. Taking into account the above assumption the boundary condition for $x = 0$ (moving boundary) is the following

$$-D_L \left. \frac{\partial z_L(x,t)}{\partial x} \right|_{x=0} = v(t)\left[z_L(0,t) - z_S(0,t) \right] \tag{2}$$

or

$$-D_L \left. \frac{\partial z_L(x,t)}{\partial x} \right|_{x=0} = (1-k)v(t) z_L(0,t) \tag{3}$$

where $k = z_S(0, t)/z_L(0, t)$ is a partition coefficient. This value is, as a rule, treated as the constant one and $z_S(0, t) = k z_L(0, t)$.
In the axis of symmetry of domain considered we have $\partial z_L/\partial x = 0$. Additionally the initial condition in the form $t = 0 : z_L(x, 0) = z_0$ is also given.

3. Sensitivity analysis

In order to analyze the influence of parameter k on the course of macrosegregation the diffusion equation and boundary initial conditions should be differentiated with respect to the parameter considered. In this way one obtains the following sensitivity model

$$\frac{\partial U_L(x,t)}{\partial t} = D_L \frac{\partial^2 U_L(x,t)}{\partial x^2} + \frac{\partial U_L(x,t)}{\partial x} v(t) \tag{4}$$

where $U_L = \partial z_L/\partial k$. For $x = 0$

$$-D_L \left. \frac{\partial U_L(x,t)}{\partial x} \right|_{x=0} = (1-k)v(t)\left[U_L(0,t) - \frac{z_L(0,t)}{1-k} \right] \tag{5}$$

Additionally for $x = 0$: $U_S(0, t) = k\, U_L(0, t) + z_L(0, t)$. In the axis of symmetry $\partial U_L/\partial x = 0$, while for $t = 0$: $U_L(x, 0) = 0$.

The basic problem and the sensitivity ones have been solved using the combined variant of the boundary element method [3, 4].

4. Example of computations

The plate of thickness $2G = 0.018$ m made from Al-Si alloy ($z_0 = 0.05$, $k = 0.2$, $D_L = 3 \cdot 10^{-9}$ m^2/s, $v = 10^{-6}$ m/s) has been considered. The basic and sensitivity models have been solved and next the influence of parameter k perturbations (±10%) on the distribution of alloy component has been found.

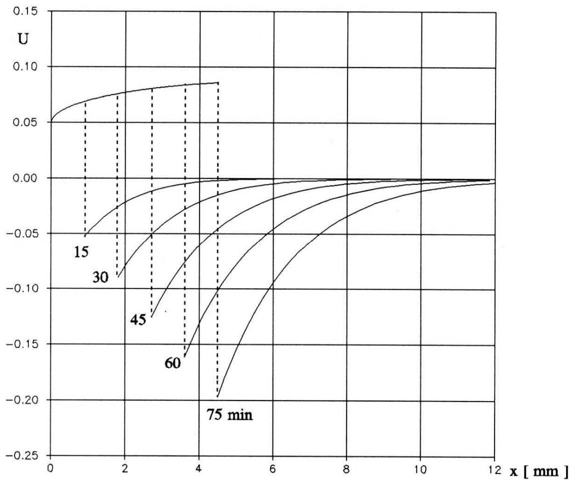

Fig. 1. Distribution of sensitivity U

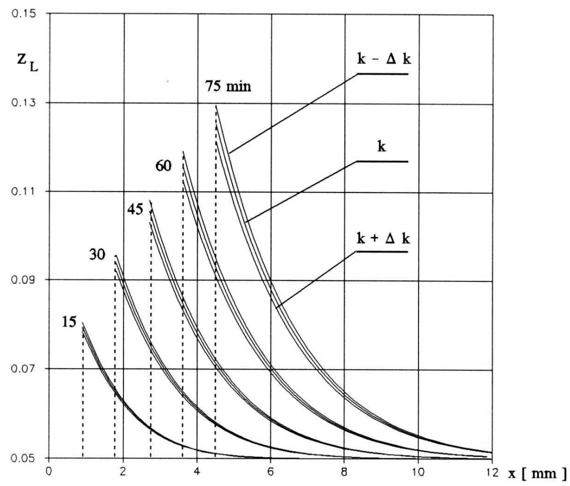

Fig. 2. Influence of k changes $\Delta k = 0.02$

In Figures 1 and 2 the sensitivity fields for times 3, 6, 9, 12 and 15 minutes and the concentration profiles for basic solution and solutions corresponding to $k + \Delta k$ and $k - \Delta k$ are shown.

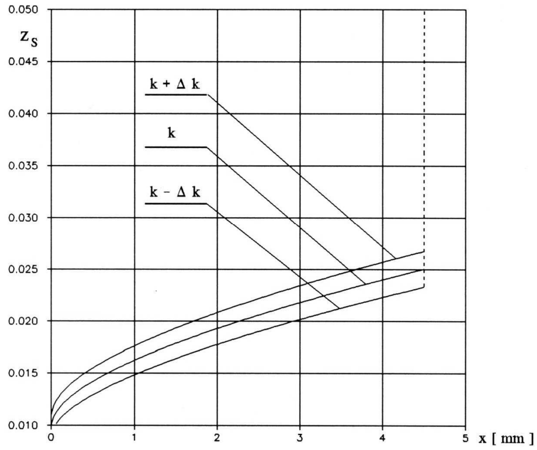

Fig. 3. Distribution of concentration z_S

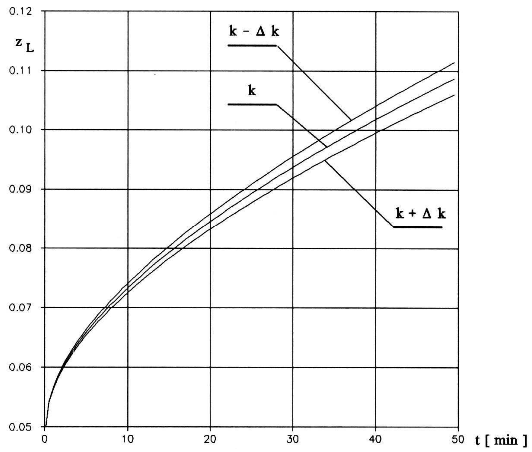

Fig. 4. Concentration z_L (0, t)

Figures 3 and 4 illustrate the distribution of alloy component concentration in sub-domain of solid state and the change of z_L for $x = 0$ (solidification front). It turned out that the results are similar and it seems to be the essential conclusion from the practical point of view.

References

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