PSEUDOGROUPS IN PREMANIFOLDS

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Abstract. In [1] it was proved that the set of all diffeomorphisms of a quasi-algebraic space which was introduced by W. Waliszewski is the Ehresmann's pseudogroup. Proving this theorem we did not use properties of a quasi-algebraic space, so it was possible to generalize this theorem and formulate it for any set of functions. It was noticed in [2]. The concept of an analytical premanifold was introduced by Waliszewski in [3]. In [4] he also called it a general differential space. In this paper we show how to use the above theorem in analytical premanifolds.

1. Diffeomorphisms in analytical premanifolds

We can not use the theorem about diffeomorphisms automaticly because in analytical premanifolds W. Waliszewski introduced the concept of *topM*, where M is a set of functions, in a different way. We show in this paper that it does not matter.

In quasi-algebraic spaces, which were used in [1], the smallest topology on

$$\underline{M} = \bigcup_{a \in M} D_a \tag{1}$$

where M is a set of functions, such that all sets D_a, $a \in M$, are open was denoted by *topM.*

It is not the same in analytical premanifolds. On the set $\underline{M}$ is considered the smallest topology, also denoted by *topM* such that all the sets $a^{-1}(A)$, where $a \in M$ and A is open in $K(K = R$ or $K = C)$ are open. The sets M_s and smooth mappings are defined in the same way in both papers.
Let us remind us of these definitions.
For any premanifold M and any subset S of $\underline{M}$ we define

$$M_s = \{b;\ b \text{ is a function and } \bigwedge_{p \in D_b} \ \bigvee_{a \in M} \ \bigvee_{U \in (topM)|S}$$

$$p \in U \subset D_b \cap D_a \wedge b|U = a|U \} \tag{2}$$

Here *(topM)*|S stands for the topology on S induced by *topM* and $b|U$ stands for the restriction of b to U.

Let M and N be analytical premanifolds and f be a function which maps $\underline{M}$ into $\underline{N}$. We say that f maps smothly M into N, what we denote in form

$$f \ : \ M \to N \tag{3}$$

iff for any $b \in N$ we have $b \circ f \in M$.

We say that *f* is a diffeomorphism of *M* onto *N*, what we denote in form

$$f \ : \ M \underset{\approx}{\to} N \tag{4}$$

iff $f \ : \ M \to N$ is one-to-one and $f^{-1} \ : \ N \to M$

2. Pseudogroups in premanifolds

Let us consider the set of all diffeomorphisms of a premanifold. Let *M*, *N* be premanifolds. Let $Diff\ (M, N)$ stands for the set of all diffeomorphisms of *M* onto *N*. Let us denote

$$Diff\ M = \bigcup_{U,V \in topM} Diff\ (M_U,\ M_V) \tag{5}$$

Now we show how we can avoid difficulties. It is enough to join new functions to the set *M* and replace it by a new set *N*, where

$$N \ = \ \{ f \,|U \ : f \ \in \ M \ \wedge \ U \ \in \ topM \} \tag{6}$$

Obviously, we understand *topM* in a new sense. For the set *N* we can use the theorem that the set of all diffeomorphisms is the *Ehresmann's* pseudogrup.

In [1] we used the following definition of a pseudogrup.

A non-empty set Γ of functions for which domains are non-empty, will be called a pseudogrup of functions if it satisfies the following conditions:

1° $\bigwedge_{f,g \in \Gamma} f(D_f) \cap D_g \neq \varnothing \Rightarrow g \ \circ \ f \ \in \ \Gamma$

2° $\bigwedge_{f \in \Gamma} \left(f^{-1} \in \Gamma \right)$

3° $\bigwedge_{\Gamma' \in \langle \Gamma \rangle} \left(\bigcup \ \Gamma' \in \ \Gamma \right)$

where

$\langle \Gamma \rangle = \{ \Gamma';\ \varnothing \neq \Gamma' \subset \Gamma$ and $\bigcup \Gamma'$ is a function and $\bigcup (\Gamma')^{-1}$ is a function$\}$

and

$$(\Gamma')^{-1} = \{ f^{-1};\ f \in \Gamma' \}$$

and f^{-1} denotes an inverse relation.

It was shown in [1] that if Γ is a pseudogrup of functions, then $\left(\Gamma, \left\{D_f ;\ f \in \Gamma\right\} \cup \{\emptyset\}\right)$ is a topological space and Γ is an *Ehresmann* pseudogrup of transformations on this topological space. On the other hand, if Γ is an *Ehresmann* pseudogrup of transformations on a topological space *S*, then Γ is a pseudogrup of functions.

Now we can verify that *topN* in the old sense is equal *topN* in a new sense. We can get it because $D_a = a^{-1}(K)$ is open in **K** for any $a \in M$. So we have the following theorem:

Theorem 1.1. *If M is an analytical premanifold the set DiffM forms an Ehresmann pseudogrup on the topological space (*$\underline{M}$*, topM).*

References

[1] Lipińska J., Diffeomorphisms of quasi-algebraic spaces, Demonstratio Math. 1986, 19, 139-151.

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[4] Walliszewski W., Inducing and coinducing in general differentical spaces, Demonstratio Math. 24(1991), 657-664.